

# Mathematics Teacher Knowledge in Iceland

## Historical and Contemporary Perspective

Kristín Bjarnadóttir

University of Iceland – School of Education

### Introduction

The history of mathematics education in Iceland in the 20th century reveals different opinions on the content of mathematics education of teachers and their knowledge. The situation at present becomes more complex when taken into account that more than half the group of mathematics teachers at secondary level do not have special training as teachers of the subject.

Icelandic pupils score slightly above average in international comparison studies, while excellent performance is scarce, and more infrequent than in other countries on similar level. This situation is discussed in the light of theories about teachers' desirable knowledge, as defined by Shulman (1986), and the conclusions Blum, Neubrand et al. (2008) have drawn from their analysis of the relation between *content knowledge* and *pedagogical content knowledge* of German secondary school teachers according to their educational background. An attempt is made to identify teachers' knowledge in Iceland at two different times by their instructional outcomes.

### Theoretical Framework

Shulman (1986) has defined several categories of teachers' knowledge. In particular he distinguished between *subject matter knowledge*, as the amount and organization of knowledge per se in the mind of the teacher, a deep understanding of the domain itself, and *pedagogical content knowledge*, as mastering the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations; that is the ways of representing and formulating the subject that make it comprehensible to others. Thirdly, Shulman defined *curricular knowledge* as a necessary factor of teacher education; that is knowledge of the 'materia medica' of pedagogy, 'invitations into enquiry' and materials under study by his/her students in other subjects they are studying at the same time, familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the material that embody them.

Scholars around the world have been working on identifying and measuring the amount of these different kinds of knowledge and the relation between them. Krauss, Baumert, Blum and Neubrand and their collaborators have made extensive research in this field (Krauss, Baumert & Blum, in print; Neubrand, 2008). Their results reveal a strong correlation between *subject matter knowledge*, or *content knowledge* (CK) by their terms, and *pedagogical content knowledge* (PCK). The PCK profits from solid base of CK, but CK is though only one possible route to PCK. An emphasis on didactics in the initial teacher training may be another route. Their conclusion is a claim for fostering a strongly subject bound, but nevertheless pedagogically oriented education of teachers. They claim that knowledge in psychology or pedagogy remains empty without being bound to the subject and that the mathematics education part of teacher education cannot be realized as just a methods course; rather it needs

reflections on characteristic features of mathematics, be it in the sense of epistemology or by referring to students' ways of thinking.

### **Landmarks in Mathematics Teacher Education in Iceland**

Iceland belonged to the Danish realm until the 1940s, and Icelandic teacher education has its roots in Denmark. Niss and Jensen's (2002) description of the divide between the seminar tradition and university education in teacher education (pp. 81–82), and consequent teacher subcultures (pp. 160–162), applies to Iceland as well.

At the implementation of school legislation in 1907, primary level teachers were needed, so the Iceland Teacher Training College was established in 1908. The student teachers in the first few years might not have had any previous schooling so they had to study elementary arithmetic. Only minimal algebra (equations) and geometry (mainly area, volume and Pythagorean Theorem) were taught at the Teacher Training College until after 1946 and 1962 respectively. Dr. Ó. Danielsson was the first mathematics teacher there, serving in 1908–1920. Some mathematical content knowledge and pedagogical content knowledge of primary-school level student teachers was ensured in Dr. Danielsson's time, while later, until the 1960s, the Teacher Training College was served by part-time mathematics teachers, many of whom were without special training in mathematics. Dr. Danielsson's 1906 arithmetic textbook, modified in 1914 and 1920, and from 1946 his 1927 algebra textbook, were the basis of mathematics teaching at the Teacher Training College until the 1960s. For teachers at lower secondary schools, study of theology or another university subject was the accepted preparation (Bjarnadóttir, 2006a/b: pp. 160–162).

By 1946 Education Legislation Act, some university education, preferably in the teaching subject, was required for tenure at the lower secondary level, and in 1950 a B.A.-programme in mathematics and physical sciences was established as a part of a programme for engineering students at the University of Iceland. Only 15 teachers from the programme ever taught mathematics at the lower secondary level but they proved to be a strong force to build up mathematics education for the college-bound stream of the level (Bjarnadóttir, 2006a/b: pp. 189–191).

By 1974 Education Legislation Act, the requirements for teaching at lower secondary level became a B.Ed. degree from the University of Education or equivalent, preferably with mathematics and mathematics education as one of either one or two electives, and for teaching at the primary school level the same degree with any field of choice. These requirements have remained the same since that time.

In the academic year 2003–2004, 33% of mathematics teachers in grades 8–10 of compulsory school, the lower secondary level, had a B.Ed. degree with mathematics as an elective and 2% had a B.Sc. degree in mathematics in addition to fulfilling requirements in pedagogy and didactics. At the upper secondary level, 46% had a B.Sc. degree or higher qualification in mathematics (Menntamálaráðuneytið, February 2005, p. 15).

The majority of teacher students at the University of Education are graduates from social science stream at the upper secondary school level, a stream without specialization in mathematics. In 2005 only 18% of first year students had completed 18 mathematics credits or more out of 140 credits earned for a four year study in an upper secondary school. In 1999 the Ministry of Education published a new national upper-secondary-level curriculum, which lowered the minimum number of mathematics credits required at all streams. The number of students completing less than 12 credits in mathematics rose from 12% to 39% among students enrolled at the University of Education in 2004, according to whether they graduated

from upper secondary school before 2003 or later, when the new national curriculum had come into effect (Bjarnadóttir, 2006c).

### **Instructional Outcomes as Indicators of Teachers' Knowledge**

In the following we shall look into available data from 1967–1973 and from 2003–2004 in order to identify relations between teachers' mathematical knowledge and instructional outcomes in Iceland, based on indicators such as university degrees and national and international tests/surveys.

In the 1960s a divide between the seminar tradition and university education in teacher education was clear in Iceland, that is between the B.A. university educated teachers and those trained at the Teacher Training College. At that time the 'New Math' movement brought new currents in mathematics education. Mathematics teachers were differently prepared. Already a year after the reform began in Iceland, there were indications that teachers did not perceive what the reform was about; if there were mainly new calculation methods being introduced or something more going on. The leader of the reform movement, G. Arnlaugsson, an upper secondary school and university teacher, wrote:

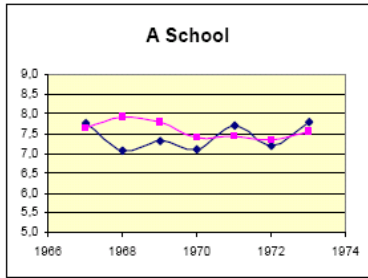
Many teachers in the primary and lower secondary schools have never in their studies met mathematical thinking. ... Mathematics ... should ... be the tool to train the child in logical thinking. If this is clear to the teacher, and he/she has an overview of the coherence of the topics of arithmetic that he/she is teaching, he/she could doubtless achieve a better result than ... now, even if there were few actual changes in the syllabus (Arnlaugsson, 1967: 43–44).

Around 1970 there were 11 lower secondary mathematics teachers who had the required B.A. degree in mathematics as major or minor, in addition to general pedagogy and didactics, but no specialisation in mathematics education. A study was made in 2005 (Bjarnadóttir, 2006a/b: pp.286-289), comparing the mathematics grade average of lower-secondary-school pupils to their overall grade point average at an entrance examination into the upper secondary level. In years when the mathematics national grade average was lower than the national overall average, the mathematics grades were corrected by the difference. The relation between teachers' *content knowledge*, as measured by their degrees in mathematics, and their pupils' performance in 1967–1973, indicating their *pedagogical content knowledge*, was investigated in seven schools, of which six teachers in five schools had a B.A. degree in mathematics. The schools chosen were two rural boarding schools, three urban schools in coastal towns, and two schools in the capital area where the teachers could be identified. The total number of schools offering the national entrance examination in this period was around 30.

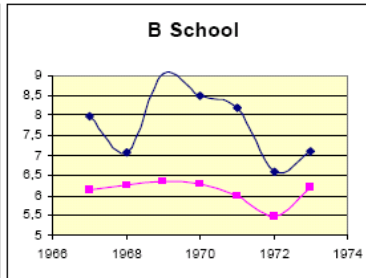
The results of the study are shown on the graph below. The performance in mathematics compared to the corrected overall average was generally better by the B.A.-degree teachers and better than in the two other schools in the study, D and R. The results indicate that formal education of teachers in mathematics led to better results of the students, even if that mathematics education was primarily aimed at engineers. Other factors, such as illnesses, as in school P, and frequent shifts of teachers, such as in school R and in school A before 1970, were also shown to influence results to the worse. However, in interviews in the 2000s, headmasters of schools B, P and S seemed not to have realized their teachers' expertise and thanked the good performance to the teachers' personal qualities.

# Mathematics compared to Grade Point Average in Grade 9 National Examination 1967–1973.

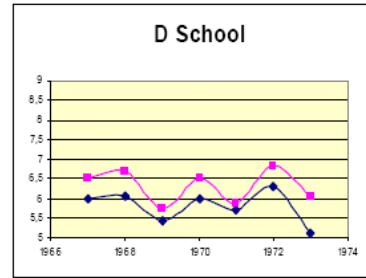
**Red:** Overall Grade Point Average  
**Blue:** Mathematics Average



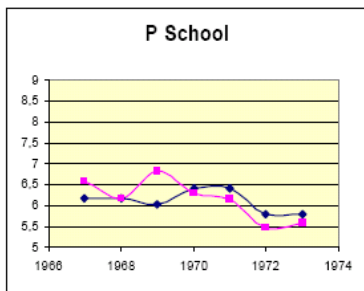
Instability in teaching force 1967–1969.  
B.A. qualified teacher from 1970



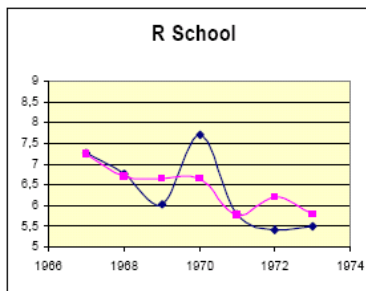
B.A.-qualified teacher 1967–1973



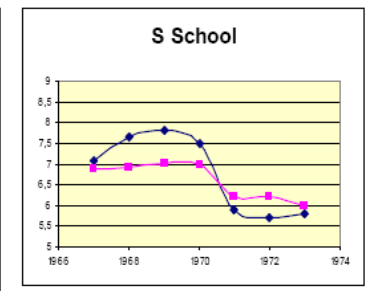
Two teachers without qualifications in mathematics



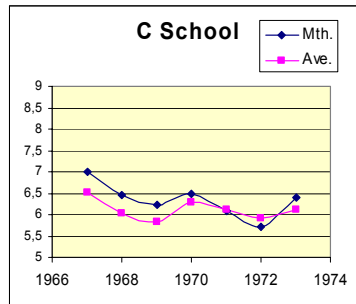
B.A. qualified teacher in 1962–73.  
Illness in 1967 and 1969



Instability in teaching force



B.A.-qualified teacher in 1966–1970



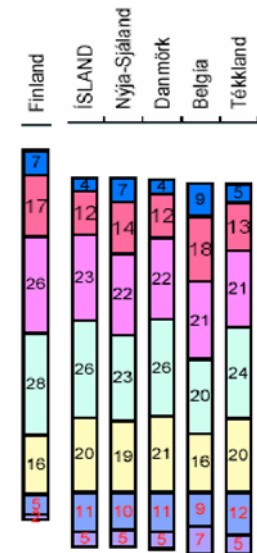
Two B.A. qualified teachers 1967–1973

The number of pupils that took the entrance examination increased rapidly during 1967–1973, especially in towns, or from 1175 to 1627, while the number of pupils attending the schools of this particular study increased from 210 to 273 or around 17% of the whole group.

## PISA 2003 and Education of Teachers

The second example relates to Icelandic pupils' performance in the 2003 PISA study. A lack of mathematical training of lower secondary school teachers may be reflected in the PISA 2003 results. The scores of Icelandic pupils placed them 10<sup>th</sup>–14<sup>th</sup> of 29 countries, and 14<sup>th</sup>–17<sup>th</sup> of 41 countries, similar to Danish and Czech pupils. Iceland's above-average performance was mainly based on a relatively large group at level 4, 23.2% (OECD, 2004).

Low percentage of Icelandic pupils on two uppermost performance levels causes concern. The OECD average for level 6 of highest score was 4.0% (SD = 0.1) and the average sum on levels 5 and 6 was 13.1%. Comparison to pupils in Denmark and Czech Republic, which total scores were similar to Iceland, and to Finland, is shown below (Data from Námsmatsstofnun, the Icelandic Assessment Institute):



(Halldórsson et al. 2004)

Country	Level 6	Levels 5 and 6	Level 1 and below
Iceland	3.7% (SE = 0.4)	15.4%	15.0%
Denmark	4.1% (SE = 0.5)	15.9%	15.4%
Czech Republic	5.3% (SE = 0.5)	18.2%	16.6%
Finland	6.7% (SE = 0.5)	23.4%	6.8%

Looking at performances in the three countries, Iceland, Denmark and Czech Republic, Icelandic pupils score lowest at levels 5 and 6, while at the two lowest levels the order is reverse but closer. In Finland the score is totally different (OECD, 2004: p. 354). Considering the similar social status of the Nordic countries raises questions why pupils in Iceland, and in Denmark for the same reason, are not on a level similar to Finland in the international PISA survey. Is it related to the teacher education, teachers' ways of working or their working conditions? It is a well-known fact that many of the University of Education graduates do not stay in teaching for a long time if they enter the teaching profession at all.

Pupils in Belgium and New-Zealand scored significantly higher than pupils in Iceland, due to higher scores on the two highest performance levels, 26.5% and 20.7% against 15.4% in Iceland.

The population of Icelandic pupils participating in PISA 2003 was 3350 pupils in 126 schools.

There were

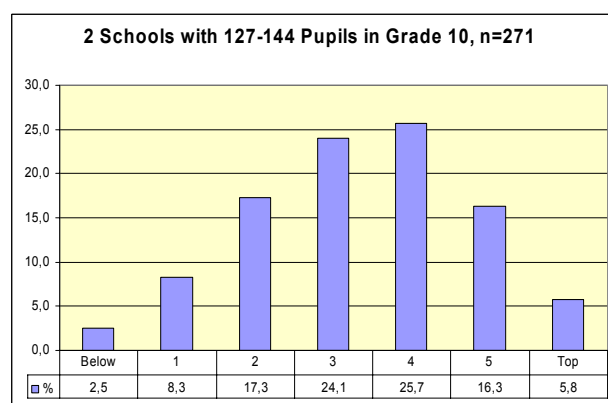
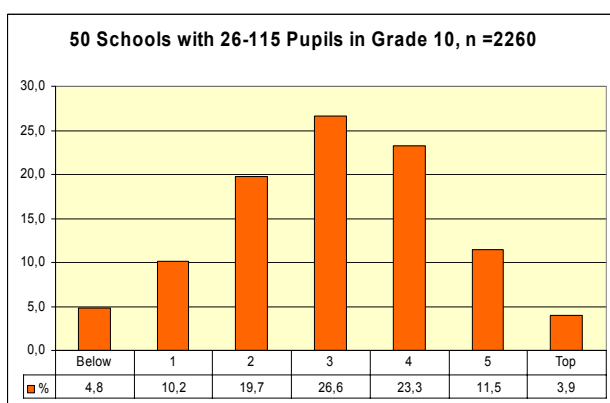
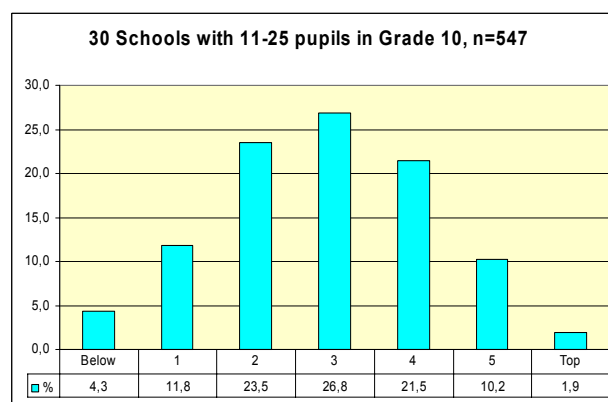
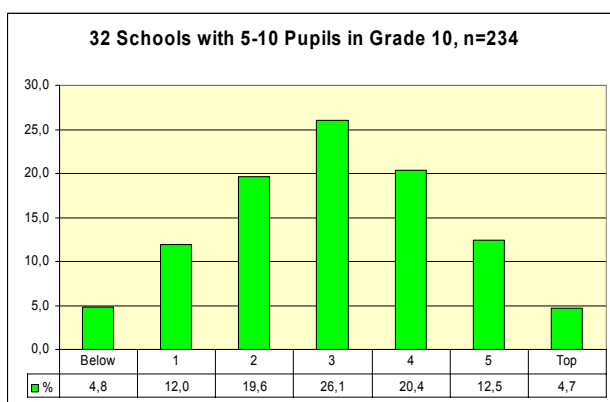
- 12 very small schools with 1–5 pupils, average 3 pupils (n=38)
- 32 small schools with 5–10 pupils, average 7 pupils (n=234)
- 30 medium size with 11–25 pupils, average 18 pupils (n=547)
- 50 large schools with 26–126 pupils, average 45 pupils (n=2260)
- 2 largest schools with 127-144 pupils, average 135 pupils (n=271).

A small study was made in order to link the performance of Icelandic pupils in the 2003 PISA survey to available data about mathematics teachers on lower secondary school level. As indicated earlier, only 121 out of 369 or 33% of mathematics teachers in grades 8–10 had a B.Ed. degree with a specialization in mathematics, and 9 teachers or 2% a B.Sc.-degree in mathematics in the academic year 2003–2004. Out of the remaining group of teachers, 49%

had a B.Ed. degree with different specialization, which should supply the *pedagogical knowledge* and *curricular knowledge*, while *content knowledge* is lacking, and 17% had diverse background other than the above mentioned.

Acknowledging that the data on the education of teachers apply to the academic year after PISA 2003, one may though assume that the situation did not change considerably between the years and that the teaching force remained largely the same. It seems also natural to assume that teachers with specialization in mathematics, i.e. *more content knowledge*, are preferably employed at larger schools that have possibility for more specialization.

In the graphs below, the results in the PISA-study 2003 are grouped into four categories of schools by their size. The 12 smallest schools with 5 pupils or less in grade 10 (n = 38) were left out of this study.



The profile of the largest sample, 50 schools with 26–115 pupils, n=2260, is similar to the profile of the whole population, n=3350. In this category of schools the education of this large sample of teachers is presumably similar to the education of Icelandic teachers as a whole. In 10 schools or 20%, the score at level 6, the highest performance level, is even to or above 6.7%, the average score in Finland. A total of 5 schools (10%) reached Finland’s average score, 23.4, on levels 5 and 6.

In the 32 smallest schools, 5–10 pupils, n = 234, with slightly above-average results on the highest performance levels, one may expect more personal guidance than elsewhere, whichever education the teachers have. In 9 out of the 32 schools or 28%, a score even to or above 6.7% at level 6 is found, while the standard deviation is high. In 12 schools (38%) the total score at levels 5 and 6 was at or above 23.4%.

Relatively poor results in the 30 schools with 11–25 pupils,  $n = 547$ , might be due to relatively few teachers with specialization in mathematics within that category. In 2 of the 30 schools (6.7%) a score even to or above 6.7% at level 6 is found, and in 2 (other) schools the total score in level 5 and 6 reached above 23.4%.

The relative above-average performance in the two largest schools with 127–144 pupils,  $n = 271$ , could be due to more specialization and stability in the teaching staff than in smaller schools. In one of the two schools (50%) the score in the highest performance level is even to or above 6.7% and there the total score in levels 5 and 6 is 26%.

The number of pupils for ‘large schools’ could be set lower, for example at 80 pupils, which would increase the number of large schools to 4 and include more than half the group of compulsory schools which only contain grades 8–10 and where the teaching is generally more subject specialized. However, a special effect of boys’ extremely low performance in those schools, which probably is a social phenomenon not related to teacher education, led to the decision to leave them in the large group of schools with 26–115 pupils.

No grouping by size reveals high scores at the highest performance levels. This causes concern and may point to lack of subject specialized training of the teachers, supplying *content knowledge*, and does not indicate that general *pedagogical knowledge* of teachers or *curricular knowledge* suffice to bring out excellent performance by their pupils.

## Summary

The above described two studies do not prove a correlation between teachers’ *content knowledge*, CK, and their pupils’ performances, which should indicate their *pedagogical content knowledge*, PCK, but they suggest that teachers’ substantial mathematics education matters in promoting pupils’ performances.

The main problem in promoting substantial mathematics education at the lower secondary school level in Iceland is lack of formal education of mathematics teachers, both in mathematics, supplying CK, and mathematics education, supporting PCK. The majority of mathematics teachers at lower secondary level have a teacher certificate and *curricular knowledge*, while only a third of the group has mathematics and mathematics education as their field of specialization. The majority of the teachers did not either choose mathematics as their electives at upper secondary school.

There are indications that schools with a stable staff of formally trained teachers were awarded with above average results in the PISA2003, while other factors affect the results, such as social phenomena and the number of pupils in class. The highest performance levels though have only been achieved by few, which is a source of severe concern.

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