WORKING GROUP 15. Comparative studies in mathematics education **2398**

Comparative Studies in mathematics education – comparing the incomparable? 2399

Birgit Pepin, Eva Jablonka, Richard Cabassut

Development of the mathematics education system in Iceland in the 1960s in comparison to three neighbouring countries 2403

Kristín Bjarnadóttir

Policy change, graphing calculators and 'High stakes examinations': A view across three examination systems 2413

Roger G. Brown

Examples of comparative methods in the teaching of mathematics in France and in Germany 2423

Richard Cabassut

Types of algebraic activities in two classes taught by the same teacher 2433

Tammy Eisenmann, Ruhama Even

Proportion in school mathematics textbooks: A comparative study 2443

João Pedro da Ponte, Sandra Marques

Factors related to students' mathematical literacy in Finland and Sweden 2453

Jukka Törnroos

A comparative study of assessment activity involving 8 pre-service teachers: What referent for the assessor? 2463

Marc Vantourout

The construction of personal meaning – A comparative case study in Hong Kong and Germany 2473

Maike Vollstedt

COMPARATIVE STUDIES IN MATHEMATICS EDUCATION- COMPARING THE INCOMPARABLE?

Birgit Pepin, University of Manchester, UK

And

Eva Jablonka, University of Umeå, Sweden

Richard Cabassut, IUFM d'Alsace, France

OVERVIEW

This Working Group is a newly-established group, and its specific interest was to encourage and represent research in comparative and international mathematics education- a very wide theme to investigate. We invited proposals concerning research on the processes of, and contexts for, teaching, learning, and the relationships among them, in international settings and comparing these. Furthermore, we were also interested in the methodologies and epistemologies used when carrying out such international and comparative studies. In line with the nature and aims of the conference, we wanted to provide a forum for international and comparative mathematics education researchers to discuss and communicate, collaborate and research, in an atmosphere of mutual respect. As expected, the variety of contributions to this Working Group shaped the thematic nature of the debates, and we aimed at developing deeper understandings of each others' research interests and areas.

The participants represented a variety of countries, amongst them Iceland, Israel, Germany, France, Finland, Portugal and the UK. The Group discussed eight papers, and two posters, selected from 11 paper submissions. It appeared that our acceptance rate was lower than for other group, after a rigorous reviewing process. However, those eight accepted papers covered a wide variety of areas and issues in the field of comparative mathematics education. In order to be able to discuss the common concerns, the papers were grouped under four themes:

- Socio-historial view (Bjarnadottir)
- Culture and assessment (Vollstedt, Eisenmann et al, Vantourout)
- Curriculum and policy (Da Ponte et al, Törnroos, Brown)
- Methodology (Cabassut)

The particular papers addressed specific areas, such as

- Examples of comparative methodologies in the teaching of mathematics in France and in Germany;

- Factors related to student's mathematical literacy in Finland and Sweden;
- 'Proportion' in school mathematics textbooks : a comparative study of textbooks in Portugal, Brazil, Spain and the USA;
- A comparative study of assessment activity involving pre-service teachers;
- Types of algebraic activities in two classes taught by the same teacher;
- Meaning-making in mathematics education in Hong-Kong and in Germany;
- Development of the mathematics education system in Iceland in the 1960s in comparison to some neighbouring countries;
- Comparison of three countries' examination systems introduction of graphic calculators.

INDIVIDUAL PAPERS IN SUMMARY

In Cabassut's article he discusses methods and **methodology** used in three comparative studies (comparing France and Germany) and highlights the problems involved in comparing internationally. These concerns are also linked to Eisenmann et al's study where a teacher taught the same curriculum content in two schools with different socio-cultural characteristics. It can be argued that we 'compare the incomparable' at times, and do not pay sufficient attention to variations within a system. Furthermore, and based on Eisenmann et al's study, it can be challenged to what extent we can take "elements", conceptually and methodologically, from international comparative studies ("big cultures") to national comparative studies ("small cultures"). Is it possible to just "zoom in" or zoom out"?

Bjarnadottir takes a socio-historical view to compare the mathematics education system in Iceland with that of Denmark. Similarities were found and it can be argued that these relate to historical developments as well as to cultural traditions. **Culture** is also a theme that is pertinent to, and resonates with, findings from Vollstedt's study who compared students' perceptions of what it means to be a student of mathematics. In theoretical terms it must be asked how we can connect, and interpret, culture and the differences found in different countries. Moreover, and this was a theme for long discussions, it can be argued that we have to develop a more differentiated view of culture, if we are to explore the nuances of this concept. We contend that there are different levels of culture, i.e. classroom culture, teaching culture, a national system's culture, or mathematical culture, and each may need its own way of exploring the inherent phenomena.

The study by Törnroos made use of PISA 2003 data to explore factors that may enhance, or impede, students' mathematical literacy performance in Finland and

Sweden. Interestingly, many of the factors were common to both countries. However, the strengths of the relationships differed, and questions were raised about, for example, the mathematics textbooks used in particular schools. This was a theme that Da Ponte et al explored in the context of Portuguese, Brazilian, Spanish and American middle schools, and with respect to the topic of 'proportions'. Brown, in his paper, used the implementation of the graphic calculator into high school mathematics examinations to compare how different examination authorities (Denmark, Australia, International Baccalaureate Organisation) established policies for the introduction of these.

PERMEATING STRANDS

Whilst there were considerable differences in terms of themes that were developed in those eight papers, there were common concerns related to all studies. The following questions exemplify those issues raised:

- why using a comparative approach?
- what are the issues that arise when using a comparative approach?
- To what extent does the comparative approach help us to reconsider our own practices?
- How can we develop a better understanding of the similarities and differences in terms of 'culture'?

Furthermore, our work in the Working Group 15 can be summarised under the headings of "Curriculum", "Teachers/students", "Assessment" & "Culture" where culture seemed to be a pervasive strand. The following grid summarises how the individual studies relate to those strands.

	Q1 : why using a comparative approach?	Q2 : what are the issues that arise when using a comparative approach?	Q3 : to what extent does the comparative approach help us to reconsider our own practices?	Q4 : how can we develop a better understanding of the similarities and differences in terms of 'culture'?
Curriculum	Da Ponte et al Bjarnadottir Törnroos	Cabassut	Da Ponte et al	Da Ponte et al Bjarnadottir Törnroos (Kandemir et al-poster)
Teachers / students	Eisenmann et al Vollstedt	Vollstedt	Cabassut (Vale& Palhares - poster)	Eisenmann et al Vollstedt
Assessment	Vantourout Törnroos	Vantourout Törnroos	Brown Vantourout	Brown

CONCLUDING REMARKS

Considering the variety of papers and approaches to comparative mathematics education, it is clear that this is too wide a field to be covered in eight papers, or over seven sessions. However, the Working Group provided an opportunity for researchers in the field of comparative mathematics education to examine their findings, theories and underpinning beliefs. Participants were able to develop understandings of, discuss and represent issues related to, and encourage scholarship in their particular area. In addition, it provided a space where members could network and connect to other research groups.

DEVELOPMENT OF THE MATHEMATICS EDUCATION SYSTEM IN ICELAND IN THE 1960S IN COMPARISON TO THREE NEIGHBOURING COUNTRIES

Kristín Bjarnadóttir Ph.D.

Iceland University of Education

Mathematics education in Iceland was behind that of its neighbouring countries up to the 1960s, when radical ideas of implementing logic and set theory into school mathematics reached Iceland, mainly from Denmark. Introduction of 'modern' mathematics in Icelandic schools is compared to its parallels in Denmark, Norway and England. Similarities are found in expectations of social and economic progress, promoted by the OECD, expectations of increased clarity and improved understanding of mathematics, a clash between different cultures of teacher education and egalitarian trends in providing 'education for all,' with the implication 'mathematics for all'. The differences lie mainly in different societal structure, characterized by Iceland's recent independence from Denmark, its sparse population and underdeveloped decision-making structure.

RESEARCH QUESTION AND RESEARCH METHOD

Increasing international influences in Iceland in the 1960s, partly channelled by the OECD, brought international currents of school mathematics reform. This initiated discussions and questions about the situation of Icelandic education, science and mathematics education in particular. The question to be discussed is:

> To what extent did mathematics education in the 1960s develop similarly or differently in Iceland from that in its neighbouring countries, and what explanations can be offered for this?

Three countries have been chosen for comparison: Denmark, due to the long-lasting cultural relationship between the countries, and Norway and Britain, two countries geographically close to Iceland and in cultural contact. The research question was the subject of a recent Ph.D. study by the author of this paper, *Mathematics Education in Iceland in Historical Context – Socio-Economic Demands and Influences* (Bjarnadóttir, 2006). In that thesis a description of Icelandic society and the educational system was provided in order to explain the fundamental reasons for mathematics education and its absence in Iceland at various times, and thus for differences from the neighbouring countries.

The research method was historical. The history of mathematics education was told within the framework of the history of education and the general history of Iceland, traced through scholars' published works, legislation, regulations, reports and documents preserved in official archives. Where applicable, events were explored by referring to contemporary articles in newspapers and journals. Supplementary knowledge was acquired through interviews with persons involved or knowledgeable observers, and from published memoirs, biographies and textbooks, in addition to some personal experiences and a few memoirs of contemporaries.

RESEARCH FRAMEWORK

No other theoretical study than the above mentioned thesis by Bjarnadóttir exists about mathematics education in Iceland and its comparison to other countries. The thesis will therefore be the framework for this study concerning Iceland, together with a textbook and articles written by Guðmundur Arnlaugsson (1966, 1967, 1971), a prime promoter of 'modern' mathematics in Iceland.

In Denmark, an anthology edited by P. Bollerslev (1979): *Den ny Matematik i Danmark,* was written on the theme of 'modern' mathematics. From there J. Høyrup's article: 'Historien om den nye matematik i Danmark – en skitse' will be cited. Another source is a report from a national meeting of the Danish Mathematical Society about the mathematics in Denmark in 1981 (*Rapport fra landsmødet om matematikken i Danmark 1981*).

Gunnar Gjone (1983) has written an account of the 1960s school mathematics reform movement in Norway: *'Moderne matematikk' i skolen. Internasjonale reformbestrebelser og nasjonalt læreplanarbeid,* and Barry Cooper (1985) has made a study of the introduction of 'modern' mathematics to England: *Renegotiating Secondary School Mathematics*. *A Study of Curriculum Change and Stability*.

The OEEC (1961) published a report on its 1959 seminar in Royaumont: *New Thinking in School Mathematics*, containing its conclusions and the results of a questionnaire survey made on the status of mathematical education in the member countries of the OEEC. The report is extremely useful for comparison of the status of the countries in question.

MATHEMATICS EDUCATION IN ICELAND BEFORE 1960S

Iceland is an island in the North Atlantic, similar in size to Ireland. The population did not grow markedly until the 20th century. In 1970 it reached 200,000. It was ruled by Denmark from 1397 until 1944. Cultural relationships were confined to Denmark for most of that period, and Danish influences lasted still longer.

From the early 19^{th} century until 1930 there was only one 'learned school' in Iceland, which was situated in Reykjavík, the capital, from 1846. Danish school authorities offered a choice of a mathematics-science stream and a language-history stream at learned schools in Denmark from the 1870s. Icelandic school authorities chose the language-history stream exclusively for the Reykjavík School, due to the small number of pupils attending the school. A mathematics-science stream was first established in 1919. This decision was to cause a chronic lack of mathematically-educated people and mathematics teachers well into the 20^{th} century. All university education in mathematics had to be acquired abroad until 1941, and after that only within a programme for engineering students. Mathematics education at the Teacher Training College, established in 1908, declined during the period 1920–1960 due to lack of tenured teachers. Primary-school teachers were not accepted at the university, and so had no opportunity of further education except abroad.

The mathematics education of teachers was therefore meagre and the tradition of mathematics education in the country is extremely short, compared for example to a widespread public tradition of enjoying literature (Arnlaugsson, 1971). By the mid-1960s a number of young intellectuals held up constructive criticism on the Icelandic educational system (Hannibalsson, 1965–1967), while university mathematics teachers had become aware of international reform trends (Arnlaugsson, 1967).

SCHOOL MATHEMATICS REFORMS IN THE 1950S AND 1960S

Questions arose in many countries in the 1950s about mathematics teaching at the upper secondary school level. There was discontent with mathematics teaching in the United States after the World War II. Induction testing for the war had presented evidence that many young people were incompetent in mathematics. The war focused national attention on the growing need for trained personnel to serve an emerging technological society (Osborne and Crosswhite, 1970, pp. 231–238), involving problem solving, such as making and cracking of codes. This led to growth in the field of discrete mathematics, probability and statistics and operational research, which again led attention to school mathematics (Gjone, vol. 1, p. 1).

An international reform movement in mathematics education had at least three points of origin. During the 1950s several important school mathematics projects were launched in the United States. There was also a broad reform movement in French-speaking Europe in the mid-1950s (Gjone, vol. 2, pp. 8–62) and another from 1957 in England, where the School Mathematics Project (SMP) was developed (Cooper).

From 1959 the reform started to expand – psychologists and pedagogues became more interested in mathematics and natural science teaching – to new pupil-groups and new grades. OEEC experts found that reform was necessary within the member countries to meet demands from industry and its new techniques. The experts knew about the movement in the USA, and wished to implement a reform of a similar kind in Europe (Gjone, vol. 2, pp. ii–iii). An important seminar on new thinking in school mathematics was held by the OEEC at Royaumont, France, in November 1959. The member countries and the United States and Canada were invited to send three delegates: an outstanding mathematician, a mathematics educator or person in charge of mathematics at the Ministry of Education, and an outstanding secondary school teacher of mathematics. The seminar was attended by representatives from all the invited countries except Portugal, Spain and Iceland (OEEC, 1961, pp. 7, 213–219).

While originally the intention was to place increased emphasis on applied mathematics, there had also been discussions amongst mathematics educators on the relations between the ideas of the French group of mathematicians, Bourbaki, on unifying mathematics, and the theories of the Swiss psychologist Piaget, who wrote in his 'Comments on mathematical education' (in Howson, A.G. (ed.), 1973, *Developments in Mathematical Education*, Cambridge Univ. Press, Cambridge):

> … having established the continuity between the spontaneous actions of the child and his reflexive thought, it can be seen from this that the essential notions which characterize modern mathematics are much closer to the structures of 'natural' thought than are the concepts used in traditional mathematics (Gjone, Vol. 2, p. 54).

At the Royaumont seminar these theories won support and its final recommendations included a combined syllabus of applied mathematics and modern algebra, and that modern algebra should be the basic and unifying item in the subject of mathematics. In the teaching of all secondary school mathematics, modern symbolism (i.e. from logic and set theory) should be introduced as early as possible, as it represented concepts that bring clarity and conciseness to thinking. The reforms were primarily conceived for a select group of pupils, but there are indications that a broader group was also borne in mind (OEEC, pp. 105–125). The above description of school mathematics will be called hereafter 'modern' mathematics.

The Royaumont Seminar was a central event for the Nordic countries. Their participants organized cooperation on reform of mathematics teaching, and the Nordic Council set up a committee under its Culture Commission (Gjone, 1983, Vol. 2, 62). Each of four countries – Denmark, Finland, Norway and Sweden – appointed four persons to the committee, *Nordiska kommittén for modernisering af Matematik-undervisningen* (The Nordic Committee for Modernizing Mathematics Teaching), NKMM, which dominated mathematics instruction in the Nordic countries for most of the 1960s (Gjone, vol. 2, p. 78). Iceland did not participate in the NKMM cooperation, but all the Danish representatives made an impact in Iceland through their writings. The prime promoter of 'modern' mathematics in Denmark (Høyrup, p. 57), Svend Bundgaard, a guest speaker at the Royaumont seminar, was also to exert influence in Iceland through his personal contact with Guðmundur Arnlaugsson, the prime promoter in Iceland (Bjarnadóttir, pp. 267–268).

MODERN MATHEMATICS IN ICELAND COMPARED TO DENMARK, NORWAY AND ENGLAND

Status in the early 1960s

An OEEC questionnaire survey in connection with the Royaumont seminar in 1959 shows that the content of mathematics education in all the countries in question included the same topics, in spite of different educational systems, while they were not all taught at the same age. Iceland was in most respects a year later than the other countries and Icelandic students completed matriculation examinations at the age of 20 (OEEC, pp.187–206, 233–237).

In the early 1960s mathematics education in Iceland was in most respects similar to what it had been since the 1920s, except that a greater number of people were receiving instruction. The focus was on pupils aiming at further education, while others received no detailed attention. No development took place and there was little initiative in compulsory education. The upper secondary level still adhered to the requirements and standards of the Danish school system (Bjarnadóttir, pp. 366–377).

The OEEC questionnaire survey reveals that only 30% of secondary mathematics teachers in Iceland had full certification requirements, while the corresponding numbers were 95% for Denmark, 100% in Norway and 80% in the United Kingdom (OEEC, p. 158). Considerable class stratification existed in schools at the secondary level in Iceland between the grammar schools and the general lower secondary schools. It was demonstrated by differently rigid syllabi and different requirements for qualifications of their teachers. At the grammar schools and their entrance examination grade, university education was a requirement for teachers, and fulfilled if possible, while teacher training college education was more likely to be accepted at the general lower secondary level (Bjarnadóttir, pp. 189–191).

By the 1950s there were two broad traditions in England, of selective and non-selective secondary school mathematics. Two versions of mathematics were taught to two different categories of pupils, largely in two different types of schools, by teachers who, broadly speaking, had been educated in two different types of post-school institution: the university and the teacher training college (Cooper, p. 63). The curriculum of the selective schools was an amalgam of ‘academic’ and ‘practical’ mathematics, with more emphasis on classical mathematics rather than arithmetic, preparing pupils for further study of mathematics and science. The non-selective schools were concerned almost entirely with the ‘practical’ (Cooper, pp. 36–42).

In Norway also there were two directions within the school system, each with a long tradition, a movement originating from ‘below’ in beginners’ education, and a movement from ‘above’ from higher education, each supported by its own teacher organization (Gjone, vol. 8, pp. 14–15, 18–19).

Another trait in common was an emerging expansion of upper secondary education. A demand for ‘education for all’ was manifested in alternatives to the selective grammar school structure, in Denmark by a Higher Preparation Programme, HF (*Rapport fra landsmødet*, p. 195–196), in England by the GCE, General Certification of Education programme (Cooper, p. 42), and in Iceland by lower secondary school continuation departments and upper secondary modular schools (Bjarnadóttir, p. 322–330). These structures naturally implied ‘mathematics for all’.

Reasons for Introducing ‘Modern’ Mathematics

In the late 1950s the government and opposition in England were increasingly concerned with the adequacy of arrangements for teaching and research in scientific and technological areas, and in particular with potential shortages of manpower in these fields. Many politicians and commentators assumed that Britain’s economic success would depend on scientific research on industrial processes. Concern was expressed by many in educational organizations about possible and perceived shortages of specialist teachers of the subject, and about the mathematical education of non-specialists (Cooper, p. 91). In Norway general optimism that technology would be conducive to economic development of society – strongly emphasized by OECD – was an important factor in bringing the authorities' attention to what was happening in other countries (Gjone, vol. VIII, p. 13). A sign of international reform trends in Denmark was demands from the technical and industrial sphere for a better-qualified working force. A need for increased expertise was emphasised, simultaneously with an economic up-swing (*Rapport fra landsmødet*, p. 193). In all the countries, proposed changes were legitimised by reference to the nations' need for scientific and technological manpower. There was no pressure in Iceland from any industry, but there was an obvious lack of mathematically trained teachers.

Fear of being left behind seems to have been common to the countries in question. The implementation of 'modern' mathematics reform in Norway was influenced by the view that Norway could not stay outside the development going on in Europe and USA (Gjone, vol. 8, p. 8). In Iceland, Guðmundur Arnlaugsson had made a survey which he interpreted as demonstrating poor standing of children and adolescents in mathematics (Bjarnadóttir, p. 252), and this was confirmed by a survey made by physicist S. Björnsson (1966) indicating that the lower secondary syllabus in mathematics, physics and chemistry was markedly behind that in the Norway and Denmark. Nor did the British want to be left behind, as stated in a quotation from the editorial of the British journal *Mathematics Teaching* in April 1958:

> … much of the psychological work of Piaget suggests that many of the essential notions of modern algebra (which are regarded as a university study) have to form in the pupil's mind before he is even ready to undertake the study of number … Such topics as the algebra of sets or relations might be taught with a profit not merely in the sixth form but lower down the school as well. In other countries they are learning how to do this, and unless we learn too we shall be left behind.
>
> Of course, such ideas have to be presented in a suitable way. The formal axiomatic way … presented ... at university would never do in school. The idea must be presented in terms of concrete applications with a similar structure (Cooper, p. 76).

The quotation leads attention to the Piagetian theories. Arnlaugsson expressed in his writings (1966, 1967) expectations that the new concepts would facilitate deeper understanding of arithmetic and mathematics in general. In the foreword of his textbook (1966) he stated a clear echo from the Piagetian theories:

> The emphasis on skills and mechanical ways of work has moved aside for demands for increased understanding. This development has pushed several basic concepts from logic, set theory and algebra down to primary level. The experience from many places indicates that children – even very young children – can easily adopt these concepts, which previously were only introduced at university level, and enjoy them. Furthermore, they seem to be conducive to increased clarity and exactness in thinking and arithmetic (Arnlaugsson, 1966, pp. 4–5).

Arnlaugsson also recommended to teachers readings by psychologist Jerome S Bruner (Arnlaugsson, 1971; Bjarnadóttir, p. 422) who was influenced by Piaget (Gjone, vol. 2, p. 30). Bruner's theories, especially on discovery learning, and those of Piaget seem to have been the main impetus of the educators in their hope that the concepts of 'modern' mathematics would lead to better understanding.

Two OECD experts presented to Icelandic educators in 1965 the idea that education, especially in mathematical subjects, was considered central to social and economic progress (Bjarnadóttir, pp. 27–28). The initiation of 'modern' mathematics experiments in Iceland was wholeheartedly supported by the Minister of Education, who also was minister of OEEC/OECD affairs. The minister succeeded in convincing the parliament to allocate funds for the reform experiments within the framework of overall school research and reform on the initiative of OECD (*Alþingistíðindi* A 1966: p. 99). In Denmark, the OECD theories led to the view that it was necessary to improve mathematics teaching as early as in primary school. This demanded intensive re-training of primary school teachers, put into practice at the Royal Danish School of Educational Studies/*Danmarks Lærerhøjskole*. As early as 1958 an extensive programme for retraining of teachers was established (Høyrup, pp. 56–57), to which Reykjavík education authorities later sent their teacher-trainers.

Implementation

The international initiative for implementing 'modern' mathematics into schools came from university educators, and university people had most to say about the content. This was also the case in the countries in question here. In England in 1957, a conference was held on a personal initiative for the purpose of bringing together, for the first time, those who taught mathematics in schools and universities and those who used mathematics in real life (Cooper, p. 91). In Norway only a small number of individuals were involved in implementing school mathematics reform, among them participants at the Royaumont seminar (Gjone, vol. 8, p. 15–16). In Denmark there were only a few initiators (*Rapport fra landsmødet*, p. 198).

The Royaumont resolutions reached Icelanders mainly through personal contacts with Danish participants at the Royaumont seminar. The reform experiments at all three school levels were essentially initiated by one man, in cooperation with his colleagues: grammar school and university teacher Guðmundur Arnlaugsson. From 1964 he experimented with using American 'modern' mathematics textbooks at Reykjavík Grammar School, and in 1966 he wrote a new mathematics textbook (Arnlaugsson, 1966) on numbers and sets for the lower secondary preparation grade for grammar school. That same year, experiments began with translated Danish textbooks by A. Bundgaard et al. (1967–1972) for the primary level, created within the Nordic NKMM cooperation and channelled to Iceland through Arnlaugsson's personal contact with A. Bundgaard's brother, Svend Bundgaard. This material, chosen in some haste without knowledge of the content for the later grades, turned out to be extremely orthodox modern mathematics (Høyrup, p. 59), and its implementation became controversial. It spread rapidly and reached the majority of the Icelandic age cohorts born in 1962–1965.

The Icelandic and Norwegian educational contexts were similar in their centrally organized structure of textbooks, curricula, law and regulations (Gjone, vol. 8, p. 8). The differences in reactions to foreign educational currents lay in the decision-making process. The proposals for ‘modern’ mathematics reform in Norway went into a process which lasted several years. There was a developed process, from controlled experiments within a limited number of schools, to proposals from a subject committee, to a proposal from a curriculum plan board to the School Council, reconsideration, and subsequent debate in newspapers and parliament. This went on while the worldwide excitement about modern mathematics reached its peak. Final decisions were not taken until after that, and ‘modern’ mathematics was first formally introduced nationwide when the curriculum plans had undergone this process. At that time the most abstract concepts had retreated into the background (Gjone, vol. 8, pp. 7–10). In Iceland important steps in the implementation process were missing. The decision-making process was underdeveloped, few people had relevant knowledge, and fewer still were involved. The process went from one experimental stage to another, while one might conjecture that the process in itself created more knowledgeable personnel, who were to lead the developmental work of the following decade. The primary school experiment went out of control, and no national curriculum document existed until a preliminary one was produced at about the time that the experiment was coming to an end (Bjarnadóttir, p. 388–389).

The success of the reform

Generally, the redefinition for the university-bound streams went on without major difficulties, and is e.g. in Norway evaluated as a necessary adjustment to university mathematics (Gjone, vol. 8, p. 11). The mathematics teachers at the Reykjavík Grammar School and in the first-year courses at the University were the same individuals, Guðmundur Arnlaugsson and his colleagues, and their intention was to ensure coherence between the school levels. The upper secondary level went through a process of implementing and developing a ‘modern’ mathematics syllabus, and its subsequent retreat, without any major conflict. The redefinition of the syllabus contributed to a wider variety, meeting different demands from an explosively growing number of pupils attending upper secondary school (Bjarnadóttir, p. 378).

The problems emerged when the ‘modern’ mathematics was implemented ‘lower down’. The set-theoretical mathematics syllabus in Icelandic primary schools aroused debates and reactions. Parents and the public realized that school mathematics teaching was changing radically. Different computation algorithms were one of the side effects. In many cases Icelandic school teachers missed the point of the reform, and saw only yet another method, in addition to the old ones (Arnlaugsson, 1967, p. 43). And the public saw cumbersome methods, wordy explanations and a decline in computation skills. Parents had difficulties in assisting their children, and indeed were not expected to. More problems emerged when new teachers, who were not familiar with the material and the mathematical language, took over grades four to six (Bjarnadóttir, pp. 296–299). The age level 11–13 was a common vulnerable area. In this section there was a clash between the perspectives of the two types of teachers belonging to the two subcultures, trained at universities vs. teacher training colleges, where the former were the initiators and the latter were expected to implement the university version of mathematics (Cooper, pp. 265–266, 282; Gjone, vol. 8, pp. 14–15, 18–19; Høyrup, pp. 55–59). In fact, similar problems occurred in other countries. Introduction of "modern" mathematics in primary schools in the USA proved to be the beginning of the end (Gjone, vol. 1, p. 53).

However, the implementation of 'modern' mathematics also contributed to a dialogue between the subcultures of teachers and dissolution of the border between them. Only five years later domestic primary mathematics study material was created, turning away from 'modern' mathematics to tasks of a more investigational nature. This material was created by young teachers, the majority of whom were women, while earlier no-one was considered able to take on such a task. The strenuous experience of introducing the highly theoretical material thus released teachers' creativity and initiative, however disturbing it may have been for the young children to adapt to.

SUMMING UP

The experiences from the World War II created a demand for different content of mathematics in the western world, but also an increased demand for education for all. Mathematics reform in Iceland and its neighbouring countries, Denmark and Norway, was embedded in general school reforms. The currents to dissolve social stratification, to improve public education and to improve and alter mathematics education, were realized in the implementation of 'modern' mathematics. Its implementation thus caused a clash between two cultures of teachers and schools in all the countries in question, but it also contributed to dialogue across social borders, and thus may have contributed to dissolution of stratification in the educational systems.

The reactions were in many respects similar in the four countries. The changes aroused expectations of economic and social progress and improvement in understanding of mathematics. These expectations turned out to be too high in most cases. The social and economic progress took a long time to develop. Curriculum change innovations operate at many levels. Even if those involved were concerned with content, pedagogy and the 'attitudes' established, the redefinition achieved was probably primarily one of content. Mathematics teachers remained 'transmission' oriented but new content was, in many cases, being transmitted (Cooper, p. 281).

In Iceland, however, where school mathematics had not received any attention since the 1920s, the implementation of 'modern' mathematics in the context of a meeting of different educational currents, however unfortunate in many respects, contributed to the creation of a long-needed channel for initiative and creativity on the part of the teachers belonging to both cultures.

REFERENCES

Alþingistíðindi / *Parliamentary report*: 1966, Frumvarp til fjárlaga / Budget Bill, A, no.1, Menntamálaráðuneytið, Reykjavík.

Arnlaugsson, G.: 1966, *Tölur og mengi*. Ríkisútgáfa námsbóka, Reykjavík.

Arnlaugsson, G.: 1967, 'Ný viðhorf í reikningskennslu', *Menntamál 40*(1), 40–51.

Arnlaugsson, G.: 1971, 'Stærðfræði', in Matthías Jónasson: *Nám og kennsla*, pp. 296–320, Heimskringla, Reykjavík.

Bjarnadóttir, K.: 2006, *Mathematical education in Iceland in historical context – Socio-economic demands and influences,* IMFUFA tekst no. 456/2006, Roskilde University, Roskilde.

Björnsson, S.: 1966, 'Samanburður á námi í stærðfræði, eðlisfræði og efnafræði í dönskum, norskum og íslenzkum unglinga- og gagnfræðaskólum', *Menntamál 39*(2), 100–121.

Bundgaard, A. et al.: 1967–1972, *Stærðfræði. Reikningur*, Ríkisútgáfa námsbóka, Reykjavík.

Cooper, B.: 1985, *Renegotiating secondary school mathematics*. *A study of curriculum change and stability*, The Falmer Press, London.

Gjone, G.: 1983, *"Moderne matematikk" i skolen. Internasjonale reformbestrebelser og nasjonalt læreplanarbeid*, I–VIII, Oslo.

Hannibalsson, A. 1965–1967, a series of columns in *Frjáls Þjóð*, Reykjavík.

Høyrup, J.: 1979, 'Historien om den nye matematik i Danmark – en skitse', in Bollerslev, P. (ed.): *Den ny matematik i Danmark – En essaysamling*, pp. 49–65, Gyldendal, Copenhagen.

Osborne, A. R. and F. J Crosswhite.: 1970, 'Forces and Issues Related to Curriculum and Instruction, 7–12', in *A history of mathematics education in the United States and Canada*, Thirty-second Yearbook, pp. 155–297, NCTM, Wash. D.C.

OEEC: 1961, *New Thinking in School Mathematics*, 2nd ed., Paris.

POLICY CHANGE, GRAPHING CALCULATORS AND 'HIGH STAKES EXAMINATIONS: A VIEW ACROSS THREE EXAMINATION SYSTEMS

R G Brown
Visiting Research Fellow
Department of Education, University of Bath

This paper focuses on policy changes brought about by the implementation of the graphics calculator into high stakes end of high school mathematics examinations. The paper uses a comparative approach to consider how two examination authorities, located in Denmark, Australia along with an International examination authority went about establishing policies for the introduction of the graphics calculator and later how these authorities had to adapt these policies to meet changing needs. Similarities and differences in the implementation process are also described.

INTRODUCTION

In the early 1990's the graphing calculator (hereafter GC) first began to appear in mathematics classrooms in the USA, Australia and many European countries. These calculators provided the rapid production of graphs and incorporated all the functionality of the scientific calculators that were already being used in high school mathematics. The "early adopters" in many countries felt that these calculators would allow students to develop a better conceptual understanding of mathematics by supporting an investigative approach to mathematics (Dunham & Dick, 1994; Penglase & Arnold, 1996). The introduction of the GC soon led to calls for their use in examinations (Harvey, 1992). These calls immediately provided challenges for those responsible for setting examinations (hereafter known as Examining Authorities, EA) as the GC had the capacity to complete many of the traditional pencil and paper test items with the push of a button. The EA's needed to develop policies that took account of a wide range of competing requirements in mathematics examinations, such as equity, the style of questioning, the rules concerning what was an acceptable written solution, whether questions should be GC active or should be excluded all together. In conjunction with, a recognition of the marked increase in the repertoire of techniques and skills a student was required to assimilate (Drijvers & Doorman, 1996).

This paper, reports on part of a larger comparative study by (Brown, 2005), and has its focus on the changes in policy within three "national" EA's as they went about implementing the introduction of the GC into their system wide 'high stakes' end of secondary school mathematics examinations. The three authorities are the Danish Ministry of Education (DME), Denmark, the International Baccalaureate Organization (IBO) and the Victorian Curriculum and Assessment Authority (VCAA), Victoria, Australia are described in the following section. Followed by a description of the research project and the policy initiatives enacted by the EA's. The paper concludes with a discussion of these policy initiatives.

EXAMINATION AUTHORITIES (EA)

The Danish Ministry of Education (DME)

The Danish Gymnasium programme is a three-year course leading to the Upper Secondary Leaving Examination. At the time of the study there were two courses of study from which the students choose one: the language line and the mathematics line, the focus of this study. For the mathematics line all students must complete at least B-level mathematics a (2 year course) whilst the majority choose A-level mathematics, either as a three-year course, or as a one-year course after B-level.

International Baccalaureate Organization (IBO)

The International Baccalaureate Diploma Programme caters for over 1800 schools in more than 124 countries (IBO, 2006) and is an internationally recognized pre-university qualification. The International Baccalaureate therefore provides an interesting contrast to a national system, its cross cultural mix of students, teachers and examiners as well as its three different languages provides a contrasting set of values to those, which appear in a national system. Like the other examination boards described in this paper all students must select at least one mathematics course from the 3 programmes offered.

Victorian Curriculum and Assessment Authority (VCAA)

The Victorian Curriculum and Assessment Authority (VCAA, formerly the Victorian Board of Studies (VBOS)) administer the Victorian Certificate of Education (VCE). The aim of the VCE programme is to provide students with a qualification, giving them access to universities. The course of study is a two-year course leading to the Victorian Certificate of Education. There are three courses of study in mathematics from which the students may choose one or two.

Differences

There are a number of structural differences between the three examination boards. The government of Denmark, through the Minister of Education is directly responsible for the management and administration of examinations and curricula development. Whereas in Victoria, the VCAA is a statuary authority, which reports directly to the Minister of Education, but retains some independence from the government. Whilst the IBO is a non-profit educational foundation governed by a Council of Foundation located in Switzerland. Thus whilst in the cases of the VCAA and the DME there is governmental monitoring of the educational administration, in the case of the IBO it is managed by elected representatives from each of the regions.

METHODOLOGY

A descriptive multiple case study (Yin, 1994) was used for the larger study as it met the requirement of being able to take account of a wide range of variables within the contemporary context of the study of policy implementation. The criteria established relating to the selection of EA's for the study were;

- Similar curricula and a final high school 'high stakes' mathematics examinations prior to university entrance
- Two stage process for the implementation of the GC into the mathematics examinations, that is allowed use followed by required use
- Examination authorities at similar stages of the implementation i.e. the GC adopted at similar times

The relevant EA documents relating to the policies, curriculum and examinations that accompanied the introduction of the GC into the EA's 'high stakes' examinations formed the data for this study and included

- Curriculum documents for each of the 3 EA's (DME, 1993, 1999b; IBO, 1987, 1997b, 1998; VBOS, 1996c, 1999b)
- Examinations from each EA (DME, 2007; IBO, 2007; VCAA, 2007a; VCAA 2007b)
- Other policy statements issued in respect to the use of technology and conduct of examinations (DME, 1996, 1997a, 1997b, 1998, 1999a, 1999c; IBO, 1992, 1995, 1997a, 1997c, 1999; VBOS, 1995, 1996a, 1996b, 1996d, 1997, 1998, 1999a, 1999c)
- Interviews with the question writers for each of the EA's regarding the setting of examinations in a GC assumed environment with a follow up survey of the questions writers.

This study was partly historical as it considered the changes that had been put in place prior to the introduction of the GC and then considered how these policies were modified to take account of the skills to be assessed with pencil and paper versus those where students could use technology along with newer models of the GC. As a consequence the study was not affected by policy changes during the data collection phase, in contrast to Paechter's (2000) study, where the researcher had to incorporate ongoing changes of policy.

RESULTS: COMMONALITIES

This paper will describe the policy decisions that took place as a response to or aligned with the introduction of the GC into 'high stakes' examinations. These are described in the following section.

Mathematics content changes and graphing calculator specifications

In all cases there was minimal changes to the curriculum and in each case these changes were not directly attributable to the introduction of the GC.

Each authority established its own requirements concerning the types of GC allowed in the examinations. These decisions were driven by a number of factors including the functionality of the GC and the availability of various brands and models (especially relevant for the IBO). There were two approaches to such decisions, either an open approach with restrictions on maximum capabilities (e.g. GC could not have symbolic manipulation capabilities (DME, 1999b; IBO, 1999a; VBOS, 2000)) or to provide minimum specifications, e.g. GC must have the following capabilities (or functionality)

- decimal logarithms, values of x^y and $x^{1/y}$, value of π, trigonometric and inverse trigonometric functions, natural logarithms, values of e^x
(IBO, 1999b)

Statements in curriculum guides on use of technology in Mathematics

In the case of the VCAA there were outcome statements that specifically indicated the expectation regarding the use of technology (including the GC) which stated that

> Outcome 3
> On completion of each unit the student should be able to select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches. (VBOS, 1999b, pp. 161- 162)

Further descriptions of the how this Outcome Statement could be achieved were also provided (VBOS, 1999c). However, for the IBO there were minimal statements on the use of the GC within the curricula and assessment materials. The DME Mathematics Faculty consultant stated that

> … we do not require students to do very much with a graphic calculator simply because the way that the law is written is that they should just have a graphic calculator. We don't have requirements that they should have a TI83 (Texas Instruments GC) or whatever. So all we can build on is they are able to draw graphs and so on (DME3, 2001).

This lack of specification was of concern to the faculty consultant and indicates the difficulties of introducing technology without setting standards for that technology.

Use in all examinations

In the cases of the IBO and the VCAA, the GC was required in all mathematics examinations, however for the DME only one of the examinations required the GC, the other was technology free. The reason for this is not directly related to the introduction of the GC but instead is a result of government legislation. The Danish government legislated that at least one of the examinations should be technology free (DME, 1997), as a result of publicly expressed concerns regarding the skill level of students in mathematics at the end of gymnasium level (Christoffersen & Svaneborg, 1996).

Implicit or explicit statements on the use of the GC in an examination question.

Examiners responsible for writing questions struggled with the setting of questions in a GC assumed environment, and it soon became evident that some questions could be solved with the push of a button whilst others were unaffected by the GC. So examiners, and EA's, resorted to ways to ensure that students either did not waste their time trying to solve a problem algebraically when a GC solution was more appropriate or the skill being tested was to be done without the GC. Each of the EA's in this study developed statements to restrict or encourage the use of the GC these are summarized in the table 2.

EA	Graphics Calculator Excluded	Graphic Calculator Active
DME	Solve by calculation; Calculate Find equation of tangent Use definite integral	Use your graphics calculator
IBO	Find exact	Use your graphics calculator; Write down approximated coordinates; Find to an accuracy of six significant figures
VCAA	Use Calculus; Find exact	Find to an accuracy of three significant figures

Table 2: Statements used on examinations to indicate the use (or non use) of the GC in a particular question.

Interestingly the restriction of the use of the GC excluded questions ranged from 0% for the IBO to 47% for one mathematics subject in the VCAA (Brown, 2005). Perhaps indicating that for some examiners they were still focused on the assessment of skills that had been automated by the GC.

Guidelines for acceptable graphics calculator based solutions.

Each of the boards provided rules for what constituted an acceptable GC based solution, which are summarised in the following table, Table 3.

EA	Working and Marking instructions
DME (DME, 1999a)	A mark would be awarded for a correct answer and possibly incorrect one (but close), but without an explanation of the method used and information included, such as a sketch of the graph (including indicating the window dimensions), it would not be possible to obtain full marks.
IBO (IBO, 1999b)	Where candidates are asked to show, prove or justify their answer, then correct mathematical reasoning must be used. A reference to a calculator operation such as "I used the Solve command to find that …" would be insufficient. When candidates are answering questions they will be expected to demonstrate their mathematical set up of the solution before using the GDC. That is, candidates need to demonstrate their thought processes in the development of their solutions. Correct mathematical terminology must be used to gain method marks If candidates are required to find the solution to a problem which can be solved using the inbuilt functions of the GC, other than those normally found on a scientific calculator (eg sin, cos, tan), they are required to show all the steps in the solution
VCAA (VBOS, 2000a)	Where a numerical answer to a question, or part of a question, is required, this may be obtained using any of analytical, numeric or algebraic approaches as appropriate unless instructed otherwise.

Table 3: Instructions to students on an expected response to an examination question

It can be seen that the EA's have slightly different policies regarding the instructions given for a GC based solution. In the case of the DME and the IBO these instructions specify that working must be shown, whereas for the VCAA the instructions indicate that when a GC is used in a question then any solution can be found by any method.

The DME first published their instructions after the completion of the standard-level mathematics written examinations in 1999. These instructions were intended to indicate the mark allocation for differing GC based solutions ranging from zero marks to full marks for a complete solution with all working including a description the GC window. Whereas, the IBO has specified that working must be shown and the use of correct mathematical notation is required. In contrast the VCAA has focused on describing when and when not to use a GC solution. It is apparent therefore, that the examining boards have different expectations on what a GC based solution should look like.

DISCUSSION

This study considered the policy changes that coincided with the introduction of the GC, as well as those implemented prior to the first GC required examinations. The initial policy announcements by the EA's indicated that the GC would be required in examinations from the year 2000. These announcements were followed by curricula and assessment documents, which recognised the new policy but provided little evidence of change in the content of the curriculum or the structure of the examinations.

However, as the first examinations requiring the GC approached further policy initiatives were introduced. These included

- Setting minimum specifications of the functionality of the GC
- Indicating when, and when not, to use the GC in examination questions
- Describing what constitutes an appropriate GC based solution
- Use of no GC examinations

The realisation of the need for these changes can be seen as a consequence of two issues surrounding the GC. Firstly, the capability of the technology and secondly, the need for fairness for all students sitting the examinations.

In terms of the GC's functionality prior to the introduction of the GC many mathematics questions could only be completed with the use of a standard algorithm, which the question writers wanted assess a students' ability to use. However, the GC opened up a multiplicity of methods to the student (Arnold & Aus, 1997a, 1997b; Ruthven, 1996), thereby making difficult for question writers to assess a particular skill. The introduction of technology has led to a debate on mathematics skills (see Gardiner, 1995; Ralston, 1999; Waits & Demana, 1998; Wong, 2003; Wu, 1998, for a wider discussion), which the question writers inadvertently become part of when writing examination questions. The EA's, in their attempt to side step the mathematics skills issue, endeavoured to follow a middle path and tried to balance these competing solution methods by using key words that restricted the solution method to a particular problem or left it open to the student to choose. Within the parliament in Denmark a debate had ensued on the skill level of students leaving the Gymnasium and as a consequence a "technology free" examination paper was introduced (DME, 1997a). The implementation of this examination paper, however, was not directly attributable to the introduction of the GC but part of the ongoing mathematics skills debate in that country. The use of such a paper did not limit the use of questions in the technology allowed paper where the wording indicated when the GC was not to be used.

Many of the problems that question writers faced from a technological standpoint can be explained by (Kaput, 1998) who stated that "The computational medium alters the growth of mathematical content, changes which content is important and for whom, changes the means by which it can be known, taught or learned …" (p. 1) As stated earlier for all EA's, the GC had been introduced into a virtually unchanged mathematics curriculum, furthermore, only limited changes to examination procedures were deemed necessary to accommodate the GC. The minimal changes to the curriculum and assessment models were admirable, and undoubtedly intended to help teachers feel less threatened by the introduction of the GC. However, it left question writers with the challenge of 'retrofitting' a new technology to an older curriculum, especially given recognition on the part of the question writers, and others (Kieran & Drijvers, 2006), of the difference between GC and pencil and paper techniques.

High stakes examinations are intended for the purposes of 'certification, selection and motivation' (Peterson, 1987) and as a consequence these examinations must ensure that they are valid assessments of the content of the curriculum as well as ensuring that the assessments are fair to all. Where fairness of assessment implies that "the test results neither overestimate nor underestimate the knowledge and skills of members of a particular group … Fairness also implies that the test measures the same construct across groups." (Gollub, Bertenthal, Labov, & Curtis, 2002, p.143). Question writers are therefore bound to ensure that there is a 'level playing field' for all students sitting the examinations as well as ensuring that the examination is an assessment of the curriculum. To ensure fairness the EA, whilst assuming that all students have covered the content of the curriculum, are required to take account of the differences in the types of technology available as well as capability of such technologies. Thus the EA's have felt obliged to set minimum requirements for the technology as well as excluding some types of GC from examinations. The difficulties associated with not establishing minimum requirements are clearly indicated by the faculty consultant in Denmark who stated, "it is up to the teachers to decide how much they want to use the facilities of the GC other than the most basic ones."(DME3, 2001), thus presenting a dilemma for question writers, how much do they assume that the teachers and students know about the functionality of the GC.

CONCLUSION

Each of the three examination boards saw the need to develop additional policies that accompanied the introduction of the GC. In particular these policies were required to take account of differing functionalities within available technologies, endeavouring to come to terms with "what mathematics skills should be tested", (an unresolved debate (Forster, Flynn, Frid, & Sparrow, 2004). And combined with minimal change to the curricula content and the structure of the examinations then question writers resorted to the use of particular instructions so that they could assess students knowledge of a particular skill or concept.

It is apparent therefore that Examination Authorities considering introducing hand held or possibly computer based technologies into their high stakes examination systems need to take the following into account

- how they will take account of the range of capabilities of the allowed technologies,
- how they will ensure that the examination assess the skills as laid down in the curriculum,
- whether they need to rewrite their mathematics curricula,
- whether the current examination structure is appropriate.

In conclusion EA's will need to recognise that technologies continue to develop and they will need to establish policy structures that allow changes to take place as the availability and affordability of advanced technologies places them in the hands of students.

REFERENCES

Arnold, S., & Aus, B. (1997a). Graphics Calculators: Why Should I? Part 1. *The Australian Mathematics Teacher, 53*(2), 8-11.

Arnold, S., & Aus, B. (1997b). Graphics Calculators: Why Should I? Part 2. *The Australian Mathematics Teacher, 53*(3), 6-9.

Brown, R. (2005). *The impact of introduction of the graphics calculator into "high stakes" mathematics examinations at the system wide level*, Swinburne University of Technology, Hawthorn, Victoria, Australia.

Christoffersen, T., & Svaneborg, V. (1996). Folketingets forespørgselsdebat 30. jan 1996 - og nogle konsekvenser heraf. *LMFK, 1996*(4), 2.

DME3. (2001). Interview.

DME. (1993). Gymnasiebekendtgørelsen Bilag 22 Matematik (pp. 11). Copenhagen: Danish Ministry of Education.

DME. (1996). *Eksamensbekendtgørelsen*. Copenhagen: Danish Ministry of Education.

DME. (1997a, 26. maj 1997). Bekendtgørelse om ændring af bekendtgørelse om gymnasiet, studenterkursus og enkeltfagsstudentereksamen nr 354. Retrieved 3/04/07, from http://www.retsinfo.dk/_GETDOC_/ACCN/B19970035405-REGL

DME. (1997b, 24. November 1997 af Jørgen Elgaard Larsen). Bilag til Beretning 1997. Retrieved July 4, 2005, from http://pub.uvm.dk/beret97b/hel.htm#matematik

DME. (1998). *Bilag til Beretning 1998*. Copenhagen: Danish Ministry of Education.

DME. (1999a). Executive order no. 411 of 31 May 1999. Retrieved 3/04/07, from http://www.uvm.dk/gymnasie/almen/lov/bek/exorder.html
DME. (2007). GYM/HF: Centralt stillede skriftlige opgavesæt. Retrieved 3/04/07, from http://us.uvm.dk/gymnasie/almen/eksamen/opgaver/
Drijvers, P., & Doorman, M. (1996). The Graphics Calculator in Mathematics Education. *Journal of Mathematical Behavior, 15*, 425 - 440.
Dunham, P., & Dick, T. (1994). Research on Graphing Calculators. *The Mathematics Teacher, 87*(6), 440-445.
Forster, P., Flynn, P., Frid, S., & Sparrow, L. (2004). Calculators and computer algebra systems. In B. Perry, G. Anthony & C. Diezmann (Eds.), *Research in Mathematics Education 2000 - 2003*. Flaxton: MERGA.
Gardiner, T. (1995). Wrong Way, Go Back! *Mathematical Gazette, 79*(July), 335-346.
Gollub, J. P., Bertenthal, M. W., Labov, J. B., & Curtis, P. C. (Eds.). (2002). *Learning and Understanding: Improving Advanced Study of Mathematics and Science in U.S. High Schools*. Washington DC: National Academy Press.
Harvey, J. G. (1992). Mathematics testing with calculators: Ransoming the hostages. In T. A. Romberg (Ed.), *Mathematics Assessment and Evaluation* (pp. 139-168). Albany: State University of New York Press.
IBO. (1987). *Mathematics Subject Guide*. Geneva: IBO.
IBO. (1992). Groups 3-6: Use of Calculators in Examinations. Examination Sessions: May 1994 onwards. *IB Coordinator Notes.* (12).
IBO. (1993a). Mathematics Subsidiary Level: Paper 1, *Paper 1* (pp. 5). Geneva: IBO.
IBO. (1993b). Mathematics Subsidiary Level: Paper 2, *Paper 2* (pp. 14). Geneva: IBO.
IBO. (1993c). Mathematics with Further Mathematics. Higher Level: Paper 1, *May Paper 1* (pp. 3). Geneva: IBO.
IBO. (1993d). Mathematics with Further Mathematics. Higher Level: Paper 2, *May Paper 2* (pp. 10). Geneva: IBO.
IBO. (1995). Calculators. *IB Coordinator Notes.*(8).
IBO. (1997a). Calculator Use. *IB Coordinator Notes.*(9).
IBO. (1997b). *Mathematical Methods Standard Level*. Geneva: IBO
IBO. (1997c). Update on the Use of Calculators. *IB Coordinator Notes.* (12).
IBO. (1998). *Mathematical Higher Level*. Geneva: IBO.
IBO. (1999). *Graphic Display Calculators*. Geneva: IBO.
IBO. (2007). IB Questionbank - Mathematics (HL, SL and mathematical studies). Retrieved April 3, 2007, from http://store.ibo.org/index.php?cPath=23_32
Kaput, J. (1998). Technology as a Transformative Force. Retrieved 3/04/07, from http://www.simcalc.umassd.edu/downloads/kaputtechpaper.pdf
Kieran, C., & Drijvers, P. (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: a study of CAS use in secondary school algebra. *International Journal of Computers and Mathematics Learning*, 205-263.

Paechter, C. (2000). Moving the Goalposts: carrying out curriculum research in a period of constant change. *British Education Research Journal, 26*(1), 25-37.
Penglase, M., & Arnold, S. (1996). The Graphics Calculator in Mathematics Education: A Critical Review of Recent Research. *Mathematics Education Research Journal, 8*(1), 58-90.
Peterson, A. D. C. (1987). *Schools Across Frontiers. The story of the International Baccalaureate and the United World Colleges*. La Salle. Illinois: Open Court.
Ralston, A. (1999). Lets Abolish Pencil and Paper Arithmetic. *Journal of Computers in Mathematics and Science Teaching, 18*(2), 173-194.
Ruthven, K. (1996). Calculators in the Mathematics Curriculum: the Scope of Personal Computational Technology. In A. J. Bishop, K. Clements, C. Kietel, J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education* (First ed., Vol. 1, pp. 435-468). Dordrecht: Kluwer Academic Publishers.
VBOS. (1995). Graphics Calculators in VCE examinations. *VCE Bulletin, (3),* 13-14.
VBOS. (1996a). Approved Calculators in VCE examinations. *VCE Bulletin, (9),* 9.
VBOS. (1996b). Approved Graphics Calculator list for 1997. *VCE Bulletin, 1996,* 9-11.
VBOS. (1996c). *Mathematics Study Design.* Carlton: Victorian Board of Studies.
VBOS. (1996d). Some Possible calculator 'active' questions. *VCE Bulletin, 1996,* 7.
VBOS. (1997). Use of calculators in examinations. *VCE Bulletin, (9)*.
VBOS. (1998). Mathematics Graphic Calculators. *VCE Bulletin, (10) 1998,* 7-8.
VBOS. (1999a). Graphics Calculators. *VCE Bulletin, 1999,* 6-7.
VBOS. (1999b). *Mathematics Study Design*. Melbourne: Victorian Board of Studies.
VBOS. (1999c). Use of graphics calculators in examinations for the revised VCE Mathematics Study - 2000. *VCE Bulletin, 1999,* 5-6.
VCAA. (2007a). Mathematical Methods. Retrieved 3/04/07, from http://www.vcaa.vic.edu.au/vce/studies/mathematics/methods/exams.html
VCAA. (2007b). Specialist Mathematics. Retrieved 3/04/07, from http://www.vcaa.vic.edu.au/vce/studies/mathematics/specialist/exams.html
Waits, B., & Demana, F. (1998). Reform Backlash. Retrieved 25/4/03, from http://www.math.ohio-state.edu~waitsb/papers/reformbacklash.pdf
Wong, N. Y. (2003). Influences of technology on the mathematics curriculum. In A. J. Bishop, M. A. Clements, C. Kietel, J. Kilpatrick & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education* (Second ed., Vol. 1, pp. 271-322). Dordrecht: Kluwer Academic Publishers.
Wu, H. (1998). The mathematics education reform: What is it and why should you care? Retrieved 14/01/03, from http://math.berkeley.edu/~wu/
Yin, R. K. (1994). *Case Study Research: Design and Methods* (Second ed.). Thousand Oaks: Sage Publications.

EXAMPLES OF COMPARATIVE METHODS IN THE TEACHING OF MATHEMATICS IN FRANCE AND IN GERMANY

Richard CABASSUT, richard.cabassut@alsace.iufm.fr

IUFM d'Alsace, Didirem Paris 7

Abstract: In this article we propose a theoretical and philosophical discussion on some methodos used in examples about French-German comparative studies from Knipping, Cabassut and TIMSS. The statement of this discussion is that comparative studies are complex because they involve various variables making difficult to take into account what is specific or generic in the study and what is the role of data structure. To conclude we discuss on how the relation between aims and methods in the comparative studies serve very different philosophical conceptions.

I FOCUS OF THE PAPER

The focus of this paper is to provide a discussion on methods used in comparative studies. This paper is not a research paper on comparative methodology. It provides only three examples to illustrate a discussion on comparative methods. We do not pretend to exhaustivity in the questions and in the problematics that could be the aim of another paper written by another writer.We limit our three examples to French-German comparisons. This limitation is explained by the writer's domain of research: French-German comparison. We have chosen these examples because firstly we know these examples. Secondly we think that every example brings an interesting illustration for the discussion. Thirdly to limit to France and Germany will limit the considerations about cultural, societal and curriculum influences. More various are the countries, more difficult is to take in consideration all the influences in the discussion.

II QUALITATIVE METHOD : DESCRIPTION OF PROTOTYPES

Knipping (2003) studies and compares processes of proof starting from observation of lessons on the theorem of Pythagoras. Three German classes were observed: two Klasse 9 (14-15 years) of a secondary school of Hamburg and one Klasse 8 (13-14 years) of the French-German secondary school of Buc). Three French classes were observed : two quatrième (13-14 years) of Parisian secondary schools and one quatrième of the French-German secondary school of Buc. An empirical method was applied, founded on a qualitative analysis of observed cases via audio recording and photographs of the blackboard. Knipping explains that the statistical surveys seek to obtain general results on the basis of representative samples whereas in a case study, the samples are not representative from a statistical point of view. The objective is to find and to understand new phenomena revealed by a detailed analysis of each case. Clearly qualitative approach is particularly adapted to the study case as it enables one to foreground data which is not amenable to numerical analysis. For each class Knipping begins with a contextual analysis which consists of a general description of the teaching unit, and a bipartite didactic analysis of the contents of the proofs and their related exercises. From the studied cases Knipping then defines prototypes of the proving processes in teaching resulting. An analysis of the argumentation is carried out according to a local analysis followed by a reconstruction of the total proof using an arborescent diagram. In order to analyse the discourse of proof, the total discourse is divided in individual scenes. A comparative analysis of the argumentations allows Knipping to define prototypes of the discourses of proof.

Two types of discourse of proof have been identified. The first type of discourse, referred to by Knipping as contemplative, consists mainly of argumentations based on the contemplation of the geometrical figures.

> 'The diversity of justifications characterizes an argumentation structure with parallel streams and diverse classes […] These streams do not link statements and conclusions into a chain of arguments, but rather they nourish the global stream of argumentation as many springs nourish a stream [source-structure] […] The correct drawing of the proof figure is claimed as a warrant for the fact that the inner quadrilateral is a square[…] [it] involves a different type of argument that can be described as visual contemplative' [Knipping 2003, 2-5].

This type of discourse is rather associated with the German lessons. The second type of discourse, official justification, is marked by a sequence of clearly justified arguments: in particular the various parts of an argument, data, conclusion and warrant, are clearly quoted. 'Argumentations with a reservoir-like structure flow towards intermediate target conclusions that structure the whole argumentation into parts that are distinct and self-contained' [ibid.]. The sequence of the arguments leads to the final conclusion. The validation of the arguments is discussed and officialized on the blackboard. This type of discourse is rather associated with the French lessons. 'What distinguishes the reservoir like structure from a simple chain of deductive arguments is that abduction allows for moving backwards in a logical structure and then moving forward in deductions again' [ibid.].

Discussion: These different types of discourses can help to understand the different ways of learning of proof and the students' difficulties with proof. The advantage of the method is that it enables to describe precisely a prototype of discourse of proof, that it would be difficult to describe with quantitative method. Especially the observation was partially based on analysis of oral discourse, what is difficult to analyse on a quantitative way, particularly when discourses are produced in different mother tongues. However it would be interesting to know if there is a correlation between these types of discourses and national considerations : curricula, organisation of pre-service teachers' training, instructional practice. If we know what national parameters are connected to these types of discourse, and what ways of learning and learners' difficulties are linked to these types of discourse, we could relate the first ones to the second ones in order to improve learning and to reduce difficulties. A quantitative study could try to answer this question, which would necessitate the collection and codification of a large body of data in order to show if the observed differences are statistically significant. Let us consider an example of a quantitative study.

III QUANTITATIVE METHODS: THE INFLUENCE OF THE DIFFERENCES IN STRUCTURES

The study [Mullis et am. 1998] carried out in May June 1994-95, except for the German pupils of Gymnasium, studied in 1996 [Ibid p.14]. The part that we have considered in this paper "was designed to measure the mathematics and science learning of all final year students regardless of their school curriculum" [Ibid p.31]. In the study, the French pupils have repeated no classes. They attend the final year (grade 12) of technological or general secondary schools, or the final year (grade 13) of vocational secondary schools. A further possibility is that the pupils are in the short vocational class 'BEP' or 'CAP'. German pupils are from higher secondary schools or from vocational programs (Klasse 13 or last year) in Länder of the former West-Germany, or from Klasse 12 or last year in Länder of the former East Germany. The French populations sample is indicated by F and the German one by G. In France the academic programs relate to the classes S (science), ES (economy) and L (literature) of the upper secondary schools, the technical programs relate to technological classes STT and other classes, and the vocational programs relate to vocational baccalaureats, the 'BEP' and the 'CAP'. In Germany, the academic programs relate to Gymnasia or to general schools (Gesamtschule or Integrierte Gesamtschule). The technical programs relate to the technical or vocational programs or to applied sciences in Fachgymnasia or in Fachoberschulen. The vocational programs relate to the full-time training (Berufschulen) and vocational schools (Berufsfachschulen) [Ibid p.84]. The measurement of the results of F or G tries to take into account not only the correction of the answers to the items proposed in the evaluation but also the difficulty of the items within the framework of the Item Response Theory [Ibid p. 31]. In a table [Ibid p.32] countries are ranked depending on their mean and F has a better mean than G. But if we take in consideration the educational programs (academic, technical or vocational) the means for G is the three cases greater than for F. In this case the results are presented by the study in a table [Ibid p.83] where the countries are ranked depending on the alphabetic order. We indicate below the synthesis of the two tables on the results in sciences and mathematics.

program		academic		technical		vocational	
country	total mean	pupils %	mean achievement	pupils %	mean achievement	pupils%	mean achievement
F	505	54	534	34	486	12	435
G	496	26	567	11	502	63	466

Discussion: We have here an effect of structure, which is an important problem in comparative studies. It will be noticed that in study TIMSS the classification suggested takes only into account of the general average. The effect of structure is not taken into account. We see here advantages of quantitative comparison : it enables on an easier way to compare different countries. It can show difference in the structure of the population and their roles in the performance. But different problems remain. Statistically it is difficult to get a representative sample of the population. In TIMSS, France has not satisfied the guidelines for sample participation rates and Germany has unapproved student sampling [Ibid. p.32]. And in our example there is no statistically difference between France and Germany about the general mean. Another problem is how to interpret these results. We know that France and Germany have a different structure in education. France has mainly a comprehensive school until the age of 15 in comparison with Germany used to differentiate the schools after primary school in most of the Länder of West Germany. In Germany the final level reached in professional training is more discriminant than the level reached in the general education: it is the contrary in France [Maurice et al 1982]. “Basic vocational education leads to a higher earnings premium in Germany than in France. This points to the better efficiency of the German system of vocational education compared to the low-status vocational education in France, which remains rather theoretical, less connected to the job market, and signals failure in general education” [Lauer 2004 p.537]. The interpretation of the results depending on the structure of the school system is related to the societal conception of the school : What relation between success and standards? What relation between democratisation and differenciation? What relation between education and professional life?

The aggregation of the answers from a particular country leads to the identification of a coherent body of practice whereas further studies show important fluctuations of the scores according to sociological variables within the same national group. It would be particularly the case in Germany where primary and secondary educational system are regionally organised (per Länd). For example the performance in Germany at the international survey Pisa 2000 about mathematical literacy are for Bavaria significantly above the OECD average and for Brandbourg significantly below the OECD average. In France the percentage of success for “baccalaureat” (examination to enter University) in “Academie de Creteil” (region at the border of

Paris) was 70.6% in 2004 and in Académie de Rennes (in Britany region) 86.4%. In both countries sociological differences between regions could help to understand the differences in performance maybe better than a reference to Finland for which the performance in mathematic literacy at Pisa 2000 is the best of European countries.

IV CONVERGENCE OF THE METHODS IN A MULTIPLE APPROACH

[Cabassut 2005] compares the teaching of the proof in France and Baden-Württemberg. He multiplies objects and methods of studies.

Qualitative study of mathematic syllabus: First a qualitative study of mathematic syllabus in France and Baden-Würtemberg is realised. A diachronic study from 1970 to 2003 compares the syllabus for two curricula preparing the entrance to University for academic studies in lower and upper general secondary school (in the most scientific branch for upper secondary school) : "Gymnasium" for Baden-Württemberg and "collège" and "lycée général" for France. The study collects the references to proof teaching by reading systematicaly all the syllabus text and analyses the development of this topic in syllabus. A synchronic study observes in detail the place of proof in the syllabus valid at the time of the research, and for Germany in all kind of secondary schools. To analyse the syllabus we use different typologies of : arguments (Cabassut 2005, p.185), techniques (way to prove) and technologies (justification of the techniques) (Ibid; p.195; Bosch et al 2006); functions of proof (Ibid. p.107; De Villiers 1990). The analysis gives the following results.

Proof is explicitly an object to be taught up to grade 8 (about 13-14 years old) in general secondary schools in both countries (it was not always the case in France and it is not the case in Realschule and Hauptschule in Baden-Württemberg).
In Baden-Württemberg pragmatic and semantic argumentations become stronger than formal argumentation (in comparison with previous syllabus). In France additionaly to formal arguments (used in the previous syllabus), semantic, grafic, or intuitive arguments could be used.
Mathematic technologies (for example definition, theorems or properties used to prove) are not always available at the same grade in both countries.
The verification of the plausibility is emphasised in Baden-Würtemberg, and mainly in discovery phase in France where the function of verification of the thruth remains strong.
The function of explanation are valorised in both countries (it was not the case in the syllabus of years 70). But in Baden-Württemberg there exists a propedeutic function of the proof to prepare a future teaching (for example teaching of limits prepared in grade 10 and taught in grade 11).
The communication function is very discursive in France with emphasis on written communication.
The whole branch Gymnasium works a global systematisation function while

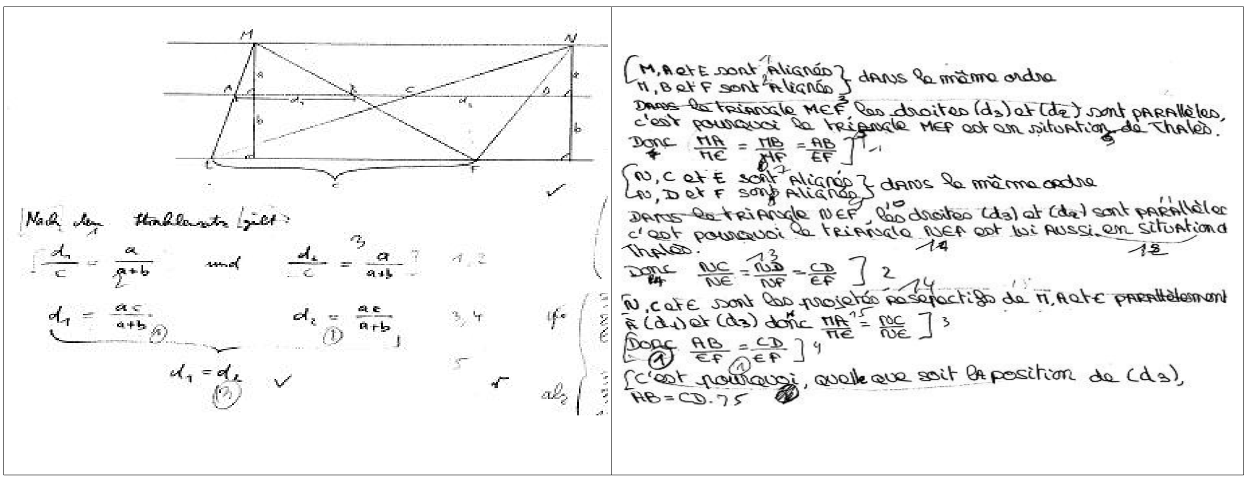
"collège" (that goes on with very differentiated branches in "Lycée") works a local systematisation function.

Qualitative study of mathematic textbooks: (Pepin et al. 2001) have shown how "textbooks occupy an important position in English, French and German classrooms and education. One could argue that textbooks would ideally be resources for teachers to use in designing their lessons and for pupils to enhance their learning"[Ibid. p.172]. In a first study it is shown that proof is taught in a specific lesson in a German textbook (the most used in Baden-Württemberg, from Lambacher Schweitzer editor, Klasse 8) and in a French textbook (from Hatier editor, collection Triangle, quatrième) of grade 8 (13-14 years old). Grade 8 is the grade where proof is introduced as an object to be taught in the curriculum. The method is clinical because these lessons will be compared on a clinical way, taking these particular lesson examples not as representative of all the books from both countries but as lesson example where proof is a taught object. To compare the lessons we use anthropological analysis (Bosch et al 2005) in term of type of tasks to do to learn proof, techniques (way of doing the task), technology (way to justify the technique) and theory (way to justify the technology). All the tasks appearing in the lessons are analysed. The different type of tasks (to discover, to control, to change semiotic register (for example to go from a written geometric construction program to the drawing), to prove) are present in both textbooks with the presence of similar techniques, technologies and functions of the proof. The type of tasks to discover or to control and the number domain are less present in the German textbook. Examples of everyday life are present in the German textbook nut not in the French one. The French lesson is more developed than the German one, as usual for other lessons. It is difficult to conclude if the observed differences are characteristic of a national style or are only natural fluctuation from a book to the other, as usual also in books from the same country. The main conclusion is that in every country there exists a textbook where proof to be taught in the curriculum is effectively taught in the textbook and developing specific tasks, techniques and technologies. Not mathematic technology (for example visual) and mathematic technology are used.

In a second study, examples of proof of theorems are analysed in the most used textbook in Baden-Württemberg. The examples are chosen depending on the class where they are introduced (at the same class level than in France, earlier or later), on the domain (Geometry, measure, function, equation), the position in connection with the year (grade 8) where proof is introduced (same year, earlier, later).Then we look for a French textbook where a similar proof exists and we analyse for these similar proof the differences. When we don't find similar proofs we compare different proofs.

We observe (Cabassut 2005, 2005b) similarities in proofs: use of plausible and pragmatic arguments (even when formal arguments are available), use of visual technique, mathematic technology available or not (both cases are illustrated), use of mathematic and not mathematic technologies in the same proof (for example: visual or inductive technology and mathematic technology together used). These uses can be explained through the functions of the different proofs. For some proofs, techniques are different because the didactical contracts are different (for unequation or function, use of grafic in the Baden-Württemberg textbook while use of algebra properties in France textbook) or because the functions of the proof are different. The explanation and the propedeutic functions are more developed in the Baden-Württemberg textbook and the function of verification and of written communication seems more developed in French textbooks as confirmed by syllabus. Knipping's contemplative proof is not clearly related to the studied German textbook in Geometry domain. However in the domain of function, equation and unequation, the proof of German textbook use graphic contemplation while the French proofs use algebra that is clearly related to official justification discourse mentioned by Knipping. It is difficult to check if the observations are linked to national style : but the previous results on the study of syllabus are confirmed by the textbook study.

Quantitative and qualitative study of pupils' proofs: The last study concerns pupils' proofs for which in both countries the same mathematic technologies are available (here Thales theorem geometry and algebra properties). It was requested from French classes (troisième and seconde) in Alsace and from German classes (Klasse 10 and 11) in Baden-Württemberg to write a proof corresponding to the same problem formulated in the respective mother tongues. The choice of the problem was carried out by a French-German commission which ensured itself that the pupils had the mathematical techniques to write this proof. Each work corresponds to a different class (and thus to different teachers). Each work was corrected by two teachers. Then the 23 best works from each country were extracted (among the hundreds of works of each country). We will call A the French sample and B the German one, in order to reduce the temptation to extend A to all the French pupils and B to German ones. The idea is to study the correct proofs and not to study the errors produced by the pupils. A qualitative study makes it possible to give off two prototypes of proof.



One observes in the B sample the domination of the symbolic algebraic register and the recourse to the figure, with coding of the figure. In the A sample, one observes the absence of figure and the importance of the recourse to the written register and a great expansion of the discourse. One wishes to check by quantitative methods if these characteristics are nationally representative. Indeed, one finds copies of the two types in each country. The statistical study shows that the recourse to the figure is significantly more frequent in B work than A. It is necessary to define a measurement of the expansion of the discourse which is not based on the number of words. For example "the theorem of Thalès" counts for four French words (« le théorème de Thalès ») and two German words ("der Strahlensatz"). The linguistic differences require a supra-linguistic method of discourse analysis. One distinguishes the following proving units in the formulation of an argument : context of the argument (for example: in triangle MEF), the entries of the arguments or data (for example: "points M, A, E are on a straight line", "(AB)//(EF)", the quotation of the rule of argument (for example: according to the theorem of Thalès), an exit of the argument or conclusion (for example: " MA/ME = MB/MF=AB/EF" , the introduction of a new data (for example a new point necessary for a geometrical construction).

The statistical study shows that between the A sample and the B sample the difference of the number of proving units in geometry is statistically significant, whereas it is not in algebra. In geometry, the quotation of the context, the entry of argument are significantly more frequent while the variation is not statistically significant for the quotation of the rule of argument. On the other hand in algebra, the quotation of the rule of argument is significantly more frequent in the B sample. The use of a drawing is significantly more frequent in the B sample. In the A sample the discourse in Geometry is more expanded (number of proving units) and more precise (the status of a proving unit is marked) : here the function of verification and the function of written communication are more emphasised in accordance with French syllabus putting Geometry domain as the main domain for proof learning. In the B sample the discourse is less expanded and has a greater use of drawing: the function of explanation is more stressed.

Discussion: These differences illustrate different didactical contract which develop different techniques and functions of the proof in a case where the available mathematic technologies are the same for both samples. On a convergent way, these studies of syllabus, textbooks and pupils' proofs have shown similarities and differences in proof teaching. However it can be a distance between syllabus textbook and what pupils can produce, between what is to be taught, what is effectively taught and what is learnt. And it can be also no convergence between different studies. For example we have tried to quantify Knipping 's prototypes (reservoir-like structure, source structure) but we have not observed statistically significant difference between A and B samples. It is also difficult to know if the observations on these pupils' samples can be considered as linked to a national didactical contract or if there are very related to the context of the specific exercise of the study.

V FINAL DISCUSSION AND CONCLUSION

We have seen in the different examples how difficult it is to control the variables and to extend the information given by the study of limited cases or samples. Even when a quantitative method shows correlations or significant statistical differences, it is difficult to interpret. To help to interpret, different studies can converge to the same interpretation but without warranty for the interpretation.

Methods serve different aims in the comparative studies. Some studies try to give a better knowledge of an object. It can be to improve teaching and learning. It can be to look for similarities and differences to improve the relations between two countries like in the following cases. France and Germany propose in three secondary schools (Versailles, Saarbrücken and Freiburg) a common French-German examination to enter the University. France proposes in the German border area an initial training for mathematic teachers with a service in a German school.

TIMSS or PISA studies oblige to study the structure and the organisation of the populations. But the interpretations are not so easy, even with a question like: is it relevant to improve the performance mean? The educational reform of 2004 in Baden-Württemberg has taken into account TIMSS and PISA results and decided to change the mathematic allocation of hours in the two last years of secondary school : it was 5 hours per week for advanced mathematic branch and three hours for the basic branch : now there is only one branch with four hours per week. We will see if there is a consequence in the next studies.

The interpretation of the results given by the comparison will depend on philosophical conceptions, for example on the following notions: difference, variation, performance, comparability, mathematic education. [Clarke 2004 p.11] has for example pointed two conceptions:

> Globalisation seeks to minimise international differences (whether by consensus or imposition) whereas internationalisation seeks to celebrate both the similarities and the differences and to learn from them. This difference can be illustrated by comparing the goal of aspiring to standardize instructional practice in mathematics classrooms internationally and the goal of aspiring to optimize local practice through critical reflection stimulated by consideration of best practice elsewhere.

The comparative method seems to be a major tool in clinical questioning, making it possible to break with the apparent naturality of observations in each country, which encourages the constitution of multinational teams of research. The methodological difficulty lies in the articulation between qualitative methods which make it possible to get conjectures of research and quantitative methods which make it possible to control these conjectures. The results of this comparative research then become a stake of formation for the teachers with the ecological questioning: "Why this? Why not that?"

REFERENCES

Bosch,,M., Chevallard, Y., Gascón, J. : 2005, 'Science or magic? The use of models and theories in didactics of mathematics'. *Proceedings of 4th Cerme* (congress of European society for research in mathematics education) , Sant Feliu de Guixols; on line http://cerme4.crm.es/Papers%20definitius/11/paperswg11.htm

Cabassut, R : 2005, *Démonstration, raisonnement et validation dans l'enseignement secondaire des mathématiques en France et en Allemagne*, Thesis, Université Paris 7, Online October 2006 [http://tel.ccsd.cnrs.fr/tel-00009716]

Cabassut, R: 2005b, 'Argumentation and proof in examples taken from French and German textbooks'. *Proceedings of 4th Cerme* (congress of European society for research in mathematics education), Sant Feliu de Guixols; online http://cerme4.crm.es/Papers%20definitius/4/wg4listofpapers.htm

Clarke, D.: 2004 'Issues of Voice and Variation : The problematics of International Comparative Research in Mathematics Education'. *International comparisons in mathematics education (ICME 10)*, Copenhagen, Danemark, July 4-11, 2004. paper submitted to discussion group 11.

De Villiers, M. : 1990, 'The role and the function of proof in mathematics'. *Pythagoras n°24*, pp.17-24.

Knipping, C. : 2003, 'Argumentation structures in classroom proving situations'. *Proceedings of the Third Conference of the European Society for Research in Mathematics Education*, Online October 2006 [http: //www.lettredelapreuve.it/CERME3Papers/Knipping-paper.pdf]

Mullis I., Martin M., begaton A., Gonzales E., Kelly D., Smith T.: 1998. 'Mathematics and Achievement in the final Year of Secondary School'. *TIMSS Third International Mathematics and Science Study* .Boston College, USA.

Lauer C. : 2004. 'Education, gender and earnings in France and Germany: Level and dispersion effects'. *Cahiers économiques de Bruxelles.Vol. 47 (3-4).* University libre de Bruxelles. pp. 505-541

Maurice M. , Sellier F. Silvestre J.-J. : 1982. *Politique d'éducation et organisation industrielle en France et en R.F.A.: Essai d'analyse sociétale* .Paris, P.U.F.

Pepin B., Haggarty L., Keynes M. : 2001. 'Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures'. *ZDM vol. 33/5* pp.158-175.

TYPES OF ALGEBRAIC ACTIVITIES IN TWO CLASSES TAUGHT BY THE SAME TEACHER

Tammy Eisenmann and **Ruhama Even**

Department of Science Teaching, Weizmann institute of science, Israel

This study investigates the role played by the classroom in shaping curriculum enactment. Using Kieran's (2004) categorization of school algebra activities this study examines how a teacher enacted the same intended algebra curriculum in two schools with different socio-cultural characteristics. The results show that by means of types of algebraic activity, students in the two schools worked on assignments and tasks with relatively similar potential. However, there was a significant difference in the types enacted during whole class work that seems to be connected to the different contexts in which the teacher worked.

One of the interesting findings of recent international comparative studies is that mathematics teaching is dependant on socio-cultural aspects (e.g., Stigler et al., 1999). This study has the same research assumption but it 'zooms in' and compares algebra teaching in two schools (vs. countries) with different socio-cultural characteristics, where the same teacher taught the same curriculum materials.

BACKGROUND

Many of the studies in the last three decades show that the enacted curriculum is not identical to the curriculum materials (e.g., Cohen & Ball, 2001). Recently, studies of curriculum enactment explain these differences by giving a prominent role to thoughtful teacher decision-making, which depends greatly on the context in which the teacher works (e.g., Remillard, 2005). But not much is known about the significance of the context in shaping the enacted curriculum (Squire, MaKinster, Barnett, Luehmann & Barab, 2003). To examine this, there is a need to study curriculum enactment by the same teacher in different contexts. This study examines this, focusing on the teaching of the topic *equivalent algebraic expressions*, using Kieran's (2004) model for conceptualizing algebraic activity.

Kieran distinguishes between three types of school algebra activities:

- *Generational activities*. These activities involve the forming of expressions and equations that are the objects of algebra (e.g., writing a rule for a geometric pattern). The focus of generational activities is the representation and interpretation of situations, properties, patterns, and relations. A lot of the initial meaning making of algebra (i.e., developing meaning for the objects of algebra) occurs within generational activities.

- *Transformational activities*. These include 'rule-based' algebraic activities (e.g., collecting like terms, factoring, substituting). Transformational activities often involve the changing of the form of an expression or equation in order to maintain equivalence. It is important to note that meaning building is not related solely to generational activities, as transformational activities involve meaning building for equivalence, and for the use of properties and axioms in the manipulative processes.

- *Global/meta-level activities*. These are activities that are not exclusive to algebra. They suggest more general mathematical processes and activity. In those activities algebra is used as a tool. They include problem solving, modeling, generalizing, predicting, justifying, proving, and so on.

RESEARCH DESIGN AND PROCEDURE

This is a case study of one teacher, Sarah (pseudonym), who taught two 7th grade classes, each from a school with different socio-cultural characteristics, Carmel and Tavor (pseudonyms). Sarah used the same curriculum materials (i.e., textbook and teacher guide) in both schools.

Carmel is a selective single-gender (girls only) Jewish religious school. The 7th grade class (with 20 students) that participated in the research was characterized by a learning atmosphere with rich and meaningful classroom talk. Sarah, as other teachers in the school, had autonomy in planning her lessons and exams.

Tavor is a secular junior-high school. Mathematics lessons in the 7th grade class (with 27 students) which participated in the research were characterized by lack of cooperation – the class was very noisy and there were many disciplinary problems. In Tavor, it was the head of the mathematics department's responsibility to plan the teaching sequence for all the mathematics classes, and to construct uniform exams that were taken at the same time by all classes in the same grade level. Thus, in Tavor Sarah had less autonomy in planning her lessons and exams.

The curriculum materials Sarah used in both classes are part of *Everyone Learns Mathematics* (Robinson & Taizi, 1995), one of the innovative 7th grade mathematics curricula developed in the 1990's in Israel, which includes many of the characteristics common nowadays to reform curricula (Even, Robinson, & Carmeli, 2003).

Data collection was conducted during one school year. The main data source includes observations of the teaching of the topic *equivalent algebraic expressions* – 19 45-minute lessons in Carmel, and 15 45-minute lessons in Tavor. Additional data sources include observations of other lessons and various school activities; occasional informal conversations with the teacher, students and school staff; and an individual semi-structured interview with Sarah on her view of the curriculum materials and the enacted curriculum, and of the two research classes and the differences between them.

The first stage in data analysis was to code all assignments in the 11 units used for teaching the topic of equivalent algebraic expressions, at least in one of the classrooms, into one or more of the following categories: generational, transformational and global/meta-level algebraic activity. Since each assignment is composed of several related tasks we also coded all tasks into one or more of the above categories. Four other researchers in mathematics education participated in the categorization of about 15% of the data. All disagreements were resolved by discussion so that consensus was reached. We then created three measures: (1) the total number of assignments in each category, (2) the total time specified in the textbook for the assignments in each category, and (3) the total number of tasks in each category.

The next stage of data analysis was to identify, measure the duration, and create the above three measures for the enacted assignments and tasks in each of the two classes. Assignments and tasks not included in the curriculum materials that the teacher used were also categorized and included in the summaries. Because the measures were a priori, this categorization first took into consideration the *potential* algebraic type of the assignments and tasks, but not the *actual* algebraic type of the classroom activity. However, this potential may not be realized in class. The nature of the data collected during small group work did not allow us to examine the actual enactment. However, we were able to analyze the actual classroom activity assignments and tasks enacted during whole class work, which reflects the main mathematical ideas of the unit and what the teacher considered to be important. Using Chi-square test we then compared between the distributions of algebraic activity types: (1) in the curriculum materials and in the enacted curriculum, for each of the two classes, and (2) in the enacted curricula in two classes.

THE ENACTED CURRICULA – CARMEL VS. TAVOR

Analysis of the enacted curricula in each of the two classes shows that not all the assignments and tasks from the curriculum materials were enacted. Indeed, both the number of assignments and the number of tasks enacted in each class during the study of the entire topic are lower than the number of assignments and tasks included in the curriculum materials. Moreover, in Carmel more assignments and tasks were enacted, and the duration of teaching this topic was longer than in Tavor. Surprisingly, in spite of these quantitative differences, the relative distribution of the potential types of algebraic activities enacted is similar in the two classes and it is also similar to the distribution in the curriculum materials. Thus, quality-wise, we may say that by means of the three types of algebraic activity, students in the two classes worked on assignments and tasks with relatively similar potential.

In contrast with the teaching of the entire topic, when examining the whole class components only, the two schools are rather similar quantity-wise. That means that no significant differences are found between the numbers of enacted assignments, the time devoted to teach them, and the numbers of tasks. However, analysis shows significant differences in the distributions of the three types of algebraic activity for one measure: the distributions of the tasks.

Figure 1 presents the distributions of the three types of algebraic activities (generational, transformational and global/meta-level). Three measures are presented: the total assignments in each category (1st line), the total time devoted to the assignments in each category (2nd line), and the total tasks in each category (3rd line). For each of these measures the numerical and proportion of each category is given.

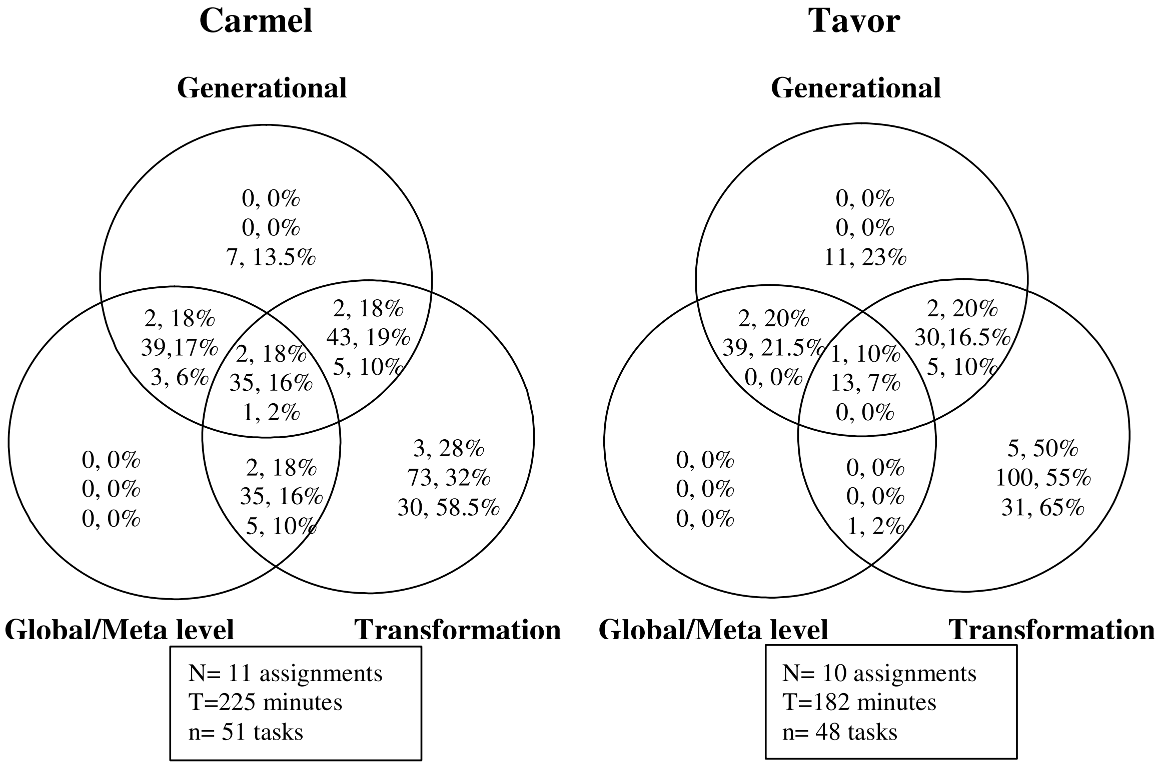


Figure 1 – Distribution of assignments, time and tasks in the whole class components

Examination of the sources of the differences shows that it is related only to the significantly greater percentage of global/meta-level activities enacted in Carmel during whole group work (six out of 11 assignments, occupying 49% of the total time, and 18% of the tasks – nine out of 51) compared with Tavor (three out of 10 assignments, occupying 28.5% of the total time, and 2% of the tasks – one out of 48). The global/meta-level assignments and task enacted in Tavor were enacted also in Carmel. They were from units 1-3 and 5 – about half way into the teaching of the topic *equivalent algebraic expression*. In Carmel global/meta-level activities were, in addition to the above-mentioned units, from units 6 and 9. The latter, unit 9, is an advanced investigation unit that Sarah enacted only in Carmel and not in Tavor, and it includes more than one-half of the global/meta-level enacted tasks (five tasks). Thus, Carmel's students not only worked on more global/meta-level activities than

Tavor's students, but they did it throughout the teaching of the topic, and not only at the beginning.

Interestingly, there were cases when the same assignment or task was enacted in one class as a global/meta-level activity but not so in the other class. For example, in both schools, the students investigated in small groups the relationship between the number of matches and the length of a “train” for different numbers of matches and trains (see examples of “trains” in Figure 2).

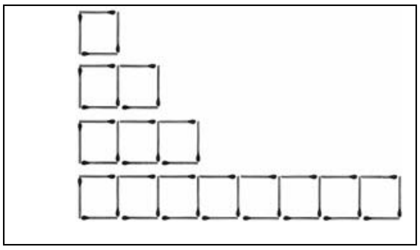


Figure 2 – Examples of "Match trains"

Then, in line with the recommendation of the curriculum materials, Sarah started the whole group work in both schools with the following task (Robinson & Taizi, 1997, p. 10):

> *Doron said: "For the number of matches required to build a train with r squares, the algebraic expression 4+3*r is suitable."*
> *Is this algebraic expression suitable?*
> *Use substitution to check.*
> *How many numbers need to be substituted to determine that this algebraic expression is not suitable?*

As has often happened, this task was enacted differently in the two schools. In the following we describe the main mathematical teaching sequence of each enactment and analyze it by means of the algebraic types of the classroom activity.

In Carmel Sarah started the whole group work by posting on the board three five-wagon trains that illustrate different ways of counting the number of matches (Figure 3), and invited students to post their small group products (algebraic expressions for the number of matches in a train with r squares). Students posted 11 algebraic expressions (e.g., 1 + 3*r, 4 + 3*(r-1), 2*r + 1*r +1).

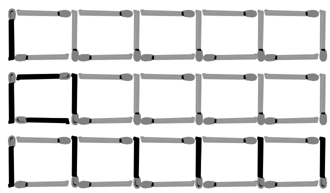


Figure 3 – Different ways of counting the number of matches of a five-wagon train

Sarah then asked the students what they thought about Doron's claim that the algebraic expression: $4+3*r$ is suitable. One student, Sapir, objected almost immediately, suggesting to use, as an example, the case of a five-wagon train to show that it was wrong. Sapir claimed that if one removed one match from the train in Figure 3a then one should multiply the number of wagons by three, and then Doron has one extra square: "So I can take off from this four this one, and then I get three, and then I multiply it by five, and in this there is one more square."

The teacher used Sapir's suggestion to check the specific case of five to explain an important mathematical principle – the role of counter example in refutation:

> O.K., let's check Sapir's answer. She said correctly but I want us to explain... The method Sapir used is correct. It is called, when we want to prove that something is incorrect, I can give a counter example. Counter example means that I, it is enough that I provide one example where this is not correct, in this case what Doron says, then, Shani, it is sufficient for saying that it does not work out, that it is probably wrong. And in the example that Sapir said, if indeed we have five wagons [writes 5 above Doron's algebraic expression: $4+3*r$], then we have, according to that [Doron's algebraic expression], three times five, which is fifteen, plus four it is nineteen. Do five wagons have nineteen matches?

Several students immediately shouted "No!" and claimed that the number of matches in a five wagon train was 16. The class then analyzed Doron's mistake and constructed a suitable algebraic expression:

> T: Then what is Doron's mistake?
>
> S: You have to take 1 off r, because r …
>
> T: …r is related to that there are here five wagons. What is four matches? It is actually the first wagon. According to what Doron says, there is one wagon that I count twice. I count the first wagon both as four separate matches and also as one of these five wagons. Therefore this is wrong. If you want to do it like Doron then you really have to

S: Take off one.

T: Take off one, and say, here, I took the first wagon separately. This is the first wagon. I already counted it. Therefore, I'll multiply the three with one less wagon, not these five wagons, but four. And this is what you actually wrote in this algebraic expression [points at 4 + 3*(r-1) which is one of the algebraic expressions on the board]. Which group wrote this expression?

S: We did.

T: Great. Then this algebraic expression is what is described here… This is what we say, that we have four separate wagons [matches] and we add to them, ah, the three matches that repeat themselves one time less than the number of wagons.

The whole group work in Carmel described above includes all three algebraic activity types. Led by the teacher, the class examined a situation, formed suitable expressions and by analyzing the hypothetical process Doron used to form his algebraic expression, showed that his suggestion is inappropriate – generational. Working on the task also included substitution in Doron's expression (r=5) to enable a comparison between the numerical result of the substitution (19) and the actual number of matches in a five-wagon train (16) – transformational. Finally, the teacher explained and named an important method of refutation in mathematics (counter example) – global/meta-level.

While enacting this task at Tavor, Sarah, again, invited students to post on the board their small group products. This time, only four expressions were posted (r*3 + 1, 4 + 3*(r-1), r*4 – (r-1), 3*r + 1). In contrast with Carmel, Sarah did not post ready-made wagon trains that illustrate different ways of counting the number of matches. Instead, she stated that there was a problem with the expression Doron suggested, and started to explain the hypothetical process Doron used to form his algebraic expression: four matches for the first wagon, and three for each of the other wagons. Sarah accompanied her explanation with a drawing of a six-wagon train (similar to Figure 3b), using blue for the first square and red for the others, emphasizing the addition of three matches for each additional wagon. Throughout Sarah's teaching the class was very noisy, and Sarah continuously stopped her talk to deal with disciplinary problems, hardly completing her sentences. Sarah concluded like Doron: "Therefore, this is 4 + 3*r" and immediately questioned this conclusion: "Then what Doron says is fine?" She then used the six-wagon train to examine this:

r is the number of wagons. Here I have one, two, three, four, five, six – six wagons. And then we do four plus three times six. This is the number of matches. Does everyone agree with this?...

If I have here four and three times the number of wagons, three times six, eighteen, plus four. How much is it? Twenty-two.

Sarah invited one student to come to the board to count the number of matches used for the six-wagon train drawn on the board. To the surprise of some students, the counting resulted with the number 19: “Yhu, how come this is wrong?” one student asked. The teacher pressed for a decision: 22 or 19? One student suggested that the first square was counted twice, and the teacher explained that this was true. It was once counted as “4” and then also when the total number of wagons (6) was multiplied by 3. Realizing that in the case of a six-wagon train they needed to multiply three by five and not by six, the class reached the expression: $4 + 3*(r - 1)$. Again, the students were very noisy and Sarah often stopped her talk to deal with disciplinary problems.

In contrast with Carmel, in Tavor the whole group work on this task included only two algebraic activity types. Again, led by the teacher, the class examined a situation, formed suitable expressions and by analyzing the hypothetical process Doron used to form his algebraic expression, showed that his suggestion was inappropriate – generational. An important component of the work on the task in Tavor was substitution in Doron’s expression (r=6) to enable a comparison between the numerical result of the substitution (22) and the result of the actual counting of the number of matches in a six wagon train (19) – transformational. However, unlike the work in Carmel, the class activity did not include a global/meta-level aspect. Neither the teacher nor the students incorporated more general mathematical processes and activity, such as the role of examples in mathematical proof and refutation.

DISCUSSION

Sarah taught the topic *equivalent algebraic expressions*, using the same curriculum materials, in two schools with different socio-cultural characteristics, By means of the three types of algebraic activity, students in the two classes worked on assignments and tasks with relatively similar potential. However, greater percentage of global/meta-level activities was enacted in Carmel during whole class work. This difference between the two classes seems to be connected to the different contexts in which the teacher worked.

An important factor is time constraint in Tavor. In Carmel, where the teacher was relatively autonomous and enjoyed a lot of freedom in planning and conducting her teaching, 19 (45-minute) lessons were devoted to the teaching of the topic equivalent algebraic expressions, whereas in Tavor, where Sarah did not have this freedom, due to school constraints, only 15 lessons were devoted to the teaching of the topic.

Moreover, observations at Tavor indicated that during the small group work, the class was very noisy and the students barely worked on any mathematics assignments/tasks (in contrast to Carmel). During the whole class work, there were many discipline problems that caused interruptions in the mathematics activity. Figure 4 demonstrates the percentage of time in the whole class work devoted to mathematical activity vs. non-mathematical activity (mainly discipline problems). As we can see, in Carmel, there were rarely any discipline problems that caused interruptions in the mathematical activities, accounting for only 2% of the whole class time. In Tavor, the case was quite different; in every lesson during the whole class work, there were few interruptions to the mathematical activities, totalling 20% of the whole class time. Thus, not only was the same topic taught for four fewer lessons in Tavor than in Carmel, but the time spent on non-mathematical activities in Tavor was much greater.

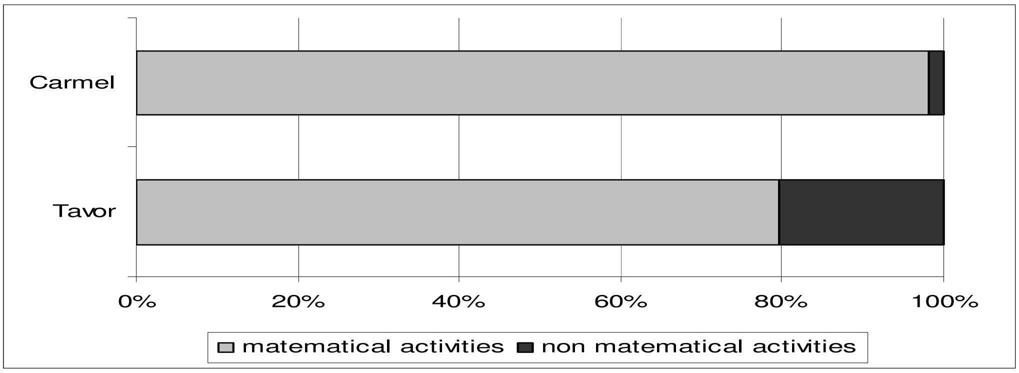


Figure 4- mathematical vs. non-mathematical activities during whole class components

Furthermore, due to lack of cooperation, Tavor's students, in contrast to Carmel's, often did not complete the assigned small group work. Therefore, during the whole class work, tasks intended for the small group work were repeated. Since mathematical work was interrupted many times, complicated tasks, multi-stage mathematical moves and tasks that require higher-order thinking were more difficult to enact. Some of these tasks were of the global/meta-level type, such as refutation by using counter examples.

Another important factor is that Carmel is a school for which excellence is extremely important. Parents and school administration kept talking about this issue and repeatedly approached the teacher and inquired whether the curriculum materials gave enough challenges to advance excellent students. Having the flexibility to plan the teaching of the topic in Carmel, both time-wise and content-wise, and enjoying students' cooperation, the teacher responded to these pressures by making sure that throughout the teaching of the topic she offered Carmel students demanding mathematical problems (part of them were global/meta-level tasks) without deviation from the curriculum materials. Thus, for example, she taught unit 9, which accounts for one-half of the global/meta-level tasks, only in Carmel. Facing lack of cooperation and many discipline problems in Tavor, and having less time to teach the whole topic, Sarah hardly enacted any global/meta-level tasks during whole class work in Tavor. Therefore, most of the global/meta-level activities were enacted only in Carmel and, as we saw earlier, there were cases when the same assignments or tasks were enacted in Carmel as a global/meta-level activity but not so in Tavor.

By means of the types of school algebra activities (Kieran, 2004) there was a great deal of similarity between the two classes. Whereas Carmel class covered more material and worked more time on the topic, both generational and transformational activities were given a relatively similar emphasis in the two classes. These two types of algebraic activities are often considered to be the heart of school algebra. Thus, at first glance it may seem that the case of one teacher who teaches in two schools using the same curriculum materials results in students in the two schools being provided with similar algebraic activities. However, as we saw earlier, Tavor students had limited opportunities to engage in global/meta-level algebraic activities. This type of activities is an integral component of algebra. Knowledge about mathematics is not separated but rather it is an essential aspect of knowledge of any concept or topic (Even, 1990). Thus, by not working on such activities, Tavor students were actually learning a different algebra; algebra that, in contrast with Carmel's algebra, did not include hypothesizing, justifying, and proving. Consequently, the case of Sarah suggests that the context of curriculum enactment plays a significant role in curriculum enactment.

REFERENCES

Cohen, D. K., and Ball, D. L.: 2001, 'Making change: Instruction and its improvement', *Phi Delta Kappan* 83(1), 73-77.

Even, R.: 1990, 'Subject matter knowledge for teaching and the case of functions', *Educational Studies in Mathematics* 21, 521-544.

Even, R., Robinson, N., and Carmeli, M.: 2003, 'The work of providers of professional development for teachers of mathematics: Two case studies of experienced practitioners', *International Journal of Science and Mathematics Education* 1, 227-249.

Kieran, C.: 2004, 'The core of algebra: Reflections on its main activities', In K. Stacey, H. Chick and M. Kendal (eds.), *The future of the teaching and learning of algebra; The 12th ICME study*, Kluwer, Boston, MA.

Remillard, J. T.: 2005, 'Examining key concepts in research on teachers' use of mathematics curricula', *Review of Educational Research*, 75, 211-246.

Robinson, N., and Taizi, N.: 1997, *On algebraic expressions 1*, The Weizmann Institute of Science, Rehovot, Israel (in Hebrew).

Stigler, J. W., Gonzales, P. A., Kawanka, T., Knoll, S., and Serrano, A.: 1999, *The TIMSS videotape classroom study: Methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States*, http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=1999074

Squire, K. D., MaKinster, J. G., Barnett, M., Luehmann, A. L., and Barab, S. L.: 2003, 'Designed curriculum and local culture: Acknowledging the primacy of classroom culture', *Science Education* 87, 468-489.

PROPORTION IN SCHOOL MATHEMATICS TEXTBOOKS: A COMPARATIVE STUDY

João Pedro da Ponte
Faculdade de Ciências da Universidade de Lisboa
Sandra Marques
Escola Básica 2, 3 das Piscinas, Lisboa

This paper analyses how middle school mathematics textbooks of Portugal, Brazil, Spain, and USA present the topic of proportion. The analysis addresses (i) the nature of the approach and (ii) the cognitive demand, structure, and context of the tasks. The results show that textbooks tend to present tasks at an intermediate level of cognitive demand and with a closed structure. Most textbooks stress non mathematical contexts, follow a spiral approach, and present chapters with a similar structure. However, there are significant differences in the way the textbooks approach the conceptual and procedural aspects of proportion and establish a conversation with the students. The impact of such differences in students' mathematics learning should be object of further study.

Textbooks have a strong influence in mathematics teaching and learning. Giving the importance of this educational artefact, its study may be considered a relatively underdeveloped field. This paper analyses how the topic of proportion – a central notion in the middle school mathematics curriculum – is presented in textbooks of four different countries: Portugal, Brazil, Spain, and USA. We are particularly interested in how textbooks introduce and develop the notion of direct proportion and in the tasks that they present to students in order to develop and consolidate their knowledge, noting the similarities and differences among textbooks. The focus of analysis is the nature of the tasks proposed, given their key role in structuring the students' learning activities (Christiansen & Walther, 1986).

PROPORTION IN THE IN MATHEMATICS EDUCATION CURRICULUM

Proportion plays an important role in the curriculum of the four countries represented in this study. In Portugal, the syllabus and the national curriculum documents (Ministério da Educação, 1991, 2001) recommend the development of the concept of direct proportion through real life situations at grade 6. In Brazil, the curriculum document *Parâmetros Curriculares Nacionais* (Ministério da Educação e Ciência, 1998) starts the preparation for the study of proportion at grade 5, through rational numbers and percents, but only at grades 6 and 7 it addresses the direct variation of proportional magnitudes. In Spain, the curriculum documents (*Reais Decretos 830/2003 and 116/2004*), mandate the study of direct proportional magnitudes at grade 7 (the first year of ESO, *enseñanza secundaria obligatoria*); however, percent is studied in grade 6 (last year of EP, *enseñanza primaria*). In the USA there is no official curriculum but we can have an idea of the trends in this country by looking at the documents issued by the NCTM (1989, 2000, 2006). These documents present proportion as a central notion that interrelates numbers and other topics such as algebra and geometry. They also suggest that students must have experiences that prepare their understanding this concept since the first years of school through the study of patterns and regularities, fractions, decimals, and percents. NCTM (2006) indicates "Connecting ratio and rate to multiplication and division" as a curriculum focal point for grade 6 and "Developing an understanding of and applying proportionality, including similarity" as a curriculum focal point for grade 8.

One of the most important research programs in the field of proportional reasoning was developed by Lesh, Post and Behr (1988). The authors regard proportion as the capstone of learning of numbers and operations and as an essential basis for learning algebra and other topics. They emphasize that proportional reasoning evolves through the development of local competencies based on contextualized knowledge. In their view, the student attains certain level of proportional reasoning when he or she is able to reason based on global relationships between rational expressions (fractions, quotients, ratios, and rates).

The way textbooks present direct proportion was addressed in several studies such as Cabrita (1996), Ruggiero and Basso (2003), and Shield and Dole (2002). Two main approaches have been identified. One uses the so called "rule of three simple" or "cross product", stating that if "*a* is for *b* such as *c* is for *d*,

$$a \text{ ---- } b$$

$$c \text{ ---- } d$$

then a, b, c and d are in proportion and $ad = bc$". The other approach emphasizes the "fundamental property of proportions", stating that "if $\frac{a}{b} = \frac{c}{d}$ is a proportion, then $ad = bc$". The main difference is that the rule of three simple or cross product is just a relationship involving four numbers whereas the fundamental property of proportions involves the notions of ratio and equation.

Cabrita (1996) indicates that Portuguese textbooks tend to follow a "linear approach", that is, they address this topic only at a single chapter. She also indicates that, to solve problems, some textbooks use the fundamental property of proportions and others use the rule of three simple. Shield and Dole (2002) analysed two chapters ("ratio and proportion" and "ratio and rates") of an Australian grade 7 textbook. They concluded that the most common method of solving proportion tasks is the "cross product". In their view, textbooks fail to promote the development of proportional reasoning. Furthermore, they contend that textbooks do not make a proper distinction between

(i) ratio and (ii) fraction and percent. Finally, Ruggiero and Basso (2003) analyzed a Brazilian textbook that received the higher rank in a national textbook assessment. They conclude that the distribution of topics is horizontal, but the textbook fails to promote learning with understanding.

MATHEMATICS TASKS AND CONNECTIONS

Mathematical tasks and connections play a central role in this study. Christiansen and Walther (1986) discuss the relationship between task and activity: The task is the starting point for the activity and is external to the student; the activity is what the student really does. The NCTM (2000) also refers to tasks as something constructed by the teacher, to be proposed to the student. In a similar perspective, Skovsmose (2000) says that the teacher “invites” the student to get involved in the task. The way the student responds to such invitation is his or her activity. Students learn mathematics through different kinds of activity – thinking, reflecting, communicating, and arguing. Therefore, the nature of the tasks in which such activity is based assumes a critical role.

Several frameworks have been proposed to categorize tasks. For example, Gimeno (1998) indicates that tasks need to be regarded with attention to the cognitive processes that they promote (memorization, comprehension, opinion, or discovery). In the PISA study (OCDE, 2004) there are three different kinds of tasks, according to the level of cognitive demand: reproduction, connection, and reflection. In a similar direction, Smith and Stein (1998) and Stein and Smith (1998) speak of routine and non-routine tasks. They suggest that routine tasks include memorization tasks and tasks with no connections and non-routine tasks include tasks with connections and “doing mathematics”. Ponte (2005) proposes a model which differentiates tasks according to the degree of structure and challenge, comprising exercises, explorations, problems, and investigations.

Furthermore, the NCTM (1991) stresses that tasks must support students in developing their ability to formulate and solve problems, communicate ideas, and establish mathematical connections. In another document, the NCTM (2000) indicates that in designing a task one needs to take into account the content, the level of difficulty, its routine or non-routine nature, the complexity, and the degree of openness.

The context is an important feature of any task, including tasks dealing with situations of direct proportion (Lesh, Post & Behr, 1988). Task contexts may be familiar, vaguely familiar or non familiar to students. Skovsmose (2000) indicates that contexts may be of reality, semi-reality or purely mathematical; he suggests that semi-reality situations may look like reality at first glance but, in fact, they may be meaningless for the students. The PISA study (OCDE, 2004) gives a particular attention to the context underlying each task.

Related to the notion of context is the notion of connection. The NCTM (1991, 2000) indicates mathematical connections as a basic standard in mathematics learning, from the elementary to the secondary school; this includes connections within mathematics and connections of mathematics with other fields. The importance of connections within and outside mathematics done by textbooks is also underlined by Pepin and Haggarty (2004). Portuguese documents (ME, 1991, 2001) indicate the need of interrelating unities and developing transversal themes.

METHODOLOGY

This study is based in documental analysis and uses a technique of content analysis. First we choose the topic. We selected proportion because it is a central topic of the middle school mathematics curriculum in most countries, but it allows for interesting variations according to cultural traditions and emphasis. The countries to study were selected based in our interest in comparing them with Portugal. We chose: (i) Brazil, as in both countries the same language is spoken; (ii) Spain, as in both countries a similar language is spoken and they have a common cultural root; and (iii) the USA, because it plays a leading role in setting the international mathematics curriculum. In fact, the NCTM (1989, 1991, 2000) documents have been quite influential in Portugal. In the four countries studied, we selected textbooks that had a strong share of the market.

Six textbooks were selected (see table 1). In Brazil and Spain proportion has an important place in the textbooks at two grade levels. Therefore, in Brazil, we considered two textbooks, one for grade 5 and another for grade 6, and in Spain we considered two textbooks, one for grade 6 and another for grade 7. In Portugal and in the USA we only considered one textbook, since in both cases it included a comprehensive treatment of proportion.

The six textbooks were analysed using an instrument consisting on a framework for global analysis and another framework for the analysis of tasks. The first framework takes into consideration the structure, organization, content, and graphical aspects of the textbook. The second framework is specifically related to the tasks and includes three points: cognitive demand, structure, and context.

Following closely the PISA framework (OCDE, 2004), cognitive demand was classified as reproduction, connection, and reflection:

- *Reproduction tasks* are routine tasks that involve the use of knowledge previously acquired and practiced, have a low degree of mathematical complexity and the response does not ask for arguments. Their interpretation is straightforward and they do not require the use of different kinds of representation. Furthermore, reproduction tasks tend to be quite structured and to be presented in a simple and familiar context.

- *Connection tasks* require the establishment of relationships or chains of reasoning, procedures, or computations and require a certain level of interpretation. They may include a request for justification or a simple explanation. Furthermore, they tend to have a closed structure and to be presented in a familiar or almost familiar context.

- *Reflection tasks* are more complex and require a high level of interpretation and reasoning. They ask for an answer that involves the coordination of several steps and often demand a response with some written explanation and argumentation. Their structure is often open or semi open and they are generally presented in less familiar situations.

Country	Textbook
Portugal	Neves, M. A., Faria, L., & Azevedo, A. (2000). *Matemática* (6.º ano, 2.ª parte). Porto: Porto Editora.
Spain	Colera, J., & Gaztelu, I. (2005). *Matemáticas* (Educación Secundaria – 1). Madrid: Anaya. Ferrero, L., Gaztelu, I., Martín. P., & Martínez, L. (2005). *La Tira de Colores – Matemáticas* (Tercer Ciclo de Primaria – 6). Madrid: Anaya.
Brazil	Lopes, A. J. (2000). *Matemática hoje é feita assim* (5.ª série). São Paulo: FTD. Lopes, A. J. (2000). *Matemática hoje é feita assim* (6.ª série). São Paulo: FTD.
USA	Maletsky et al. (2007). *Math*. Orlando: Harcourt Brace & Company.

Table 1 – Textbooks selected for the study.

The structure of the tasks was classified in three subcategories: closed, semi open and open (Ponte, 2005). In *closed* tasks the mathematical givens, goals, and conditions are clearly indicted. On the other hand, in *open* tasks it is necessary that the student provides some further specification of givens, goals, and conditions. *Semi open* tasks lie in between these two.

Finally, we classified the context of tasks using a framework also adapted from PISA (OCDE, 2004). Such context may be mathematical or non mathematical. Non mathematical contexts include six subcategories:

- *Daily life situations* involve personal situations or situations directly related to students' daily activities;

- *School situations* refer to activities and processes that occur in the school context;

- *Professional situations* correspond to a professional activity that students may be involved in the future;

- *Life in society* concern tasks related to life in community and in society;

- *Other areas of knowledge* include tasks from subjects such as physics, geography, sports, language and so on;

- *Imagination/fiction* concerns tasks drawn in a fantasy world.

Furthermore, intra mathematical contexts concerns situations devoid of explicit non-mathematical elements; they are *between topics* if they refer explicitly to concepts taught in other chapters; otherwise, they are on the *same topic*.

RESULTS

General aspects. Textbooks present in the topic of direct proportion in different ways. The Portuguese textbook includes a chapter on this notion. The USA textbook deals with direct proportion in two chapters. In Spain, direct proportion is addressed in one chapter in the textbook for grade 6 and in another chapter in the textbook for grade 7. And in the Brazilian textbooks analyzed, proportion appears in one chapter in the textbook for grade 5 and in two chapters in the textbook for grade 6. This provides a first idea of the different ways textbooks deal with this mathematical topic.

The introduction to the concept is also done in different ways. In the Portuguese and Spanish textbooks the concept is approached based in the study of rational numbers, whereas in the Brazilian and the American textbooks it is approached based in the study of equations and patterns and regularities. Furthermore, in the Brazilian and Spanish textbooks, the study of proportion is prepared by the previous study of percent.

All the textbooks include introductory tasks, application tasks, and consolidation tasks in different quantities and levels of complexity. All the textbooks, except the Brazilian, present revision tasks at the beginning of a new chapter. The organization of the chapters follows a similar pattern in all textbooks. In introducing new concepts, the Portuguese and Spanish textbooks begin by presenting a problematic situation and its solution, explain the concepts that follow from that example, present a synthesis, and, finally, propose a battery of tasks for practice. The American textbook follows the same logic, but it stresses even more the need for revision introducing at the beginning of each new section a small set of routine tasks to review the concepts already studied (figure 1).

The Brazilian textbooks are noteworthy because they do not include an early formalization of the concepts and because they begin with problematic situations in which the solution is made through a “conversation” among cartoon characters. Only afterwards these textbooks propose some tasks for the students to solve.

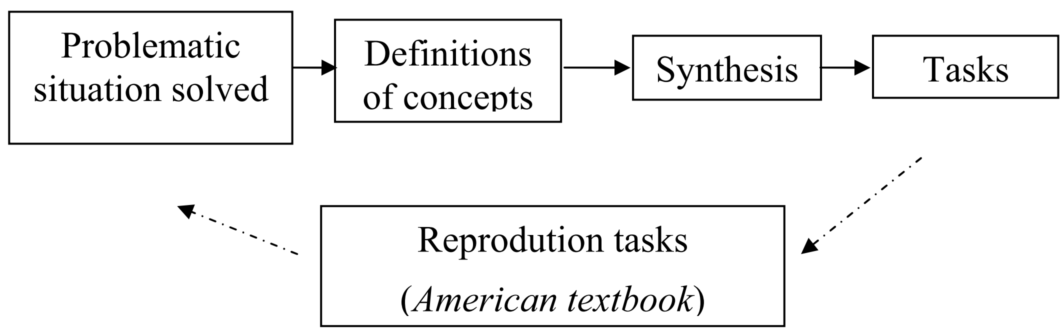


Figure 1 – Pattern of presenting new concepts in the textbooks analyzed.

Approach and structure. All the textbooks, except the Spanish, begin the direct proportion chapter with the concept of ratio. The Spanish textbook (grade 7) begins this chapter with "proportional relationships between magnitudes". All the textbooks follow a spiral approach, coming back to the notion of direct proportion at least a second time, except the Portuguese textbook that addresses this notion only in one chapter, thus following a linear approach. All the textbooks present the cross product but only the Portuguese and the Brazilian textbooks state the fundamental property of proportions.

Tasks. As table 2 shows, connection tasks predominate in the textbooks of the four countries. Reproduction tasks appear in second place and reflection tasks are far less in third place. The textbook with a higher level of reflection tasks (USA) has also the higher level of reproduction tasks. In the other three textbooks connection tasks constitute about 2/3 of the proposed tasks.

In all textbooks, the overwhelming majority of the tasks have a closed structure. Open tasks were only identified in the Brazilian and the USA textbooks. Semi-open tasks were identified in these textbooks and also in the Portuguese textbook, although at much lower level.

Most tasks refer to non mathematical contexts. The most common subcategory is life in society. In tasks that refer to intra mathematical contexts, the most common are those that refer to concepts within a single topic.

Common and different aspects. The textbooks analyzed share several common features. The distribution of the cognitive demand of tasks is similar (with emphasis on connection tasks) and the structure of the tasks is also similar (with emphasis on closed tasks). All the textbooks make little reference to the history of mathematics or to the use of the computer or the calculator.

Furthermore, three textbooks (all except the Brazilian) have their chapters organized with a rather similar structure. Three textbooks (all except the Portuguese) follow a spiral approach. Also, three textbooks (all except the American) emphasize non mathematical contexts.

		Portugal %	Spain %	Brazil %	USA %
Cognitive demand	Reproduction	34	29	22	41
	Connection	62	68	68	47
	Reflection	4	3	10	12
Structure	Open	-	-	3	1
	Semi-open	1	-	9	9
	Closed	99	100	89	90
Context	Non mathematical	65	69	66	17
	Intra mathematical	35	31	34	83

Table 2 - Cognitive demand, structure and context of tasks presented in textbooks (percents over the total number of tasks in the chapter).

The most marked differences among the textbooks concern the way they approach the mathematical notions and procedures. In some cases the study of proportion is essentially based on the previous study of rational numbers. In other cases it is also based on the previous study of percent, equations, and patterns. Only in two cases there is mention to the fundamental property of proportions. The way the students are addressed also varies, ranging from a questioning and problem solving style (in the Brazilian textbook), to an explaining/practicing style (in the textbooks of the other three countries). These two different styles of addressing students may support rather different kinds of student activity. However, only observing students using textbooks one may know if that is really the case.

CONCLUSION

This study shows how research about mathematics textbooks may use instruments of analysis specifically oriented towards the teaching of this discipline. In this case, the was analysis was carried out according to the nature of the approach and to the tasks proposed, what showed to be a productive and relevant perspective of analysis from the point of view of curriculum orientations.

We found notable similarities in the textbooks analysed. Most textbooks follow a spiral approach, present tasks with a similar distribution of levels of cognitive demand and structure, and emphasize non mathematical contexts. We see some connection between the approach used in these textbooks and the curriculum documents that stress the need to offer students a variety of learning experiences and value activity based in familiar real world contexts.

However, we also note important differences. First, the form of presenting proportions as a special case of equations is mathematically more sophisticated – and this only used in the Portuguese and Brazilian textbooks. Second, the notion of proportion is approached based in a wide range of previous notions (rational numbers, percent, equation, patterns and regularities) in the Brazilian, Spanish, and American textbooks but not in the Portuguese one. Third, the questioning and problem solving style of addressing the student was only noticed in the Brazilian textbook. Further studies may investigate if any of these features makes a strong difference in student learning. It would be particularly interesting to know how much of the learning opportunities provided by the textbooks that stress a questioning and problem solving style and present more open and higher cognitive demand tasks gets enacted in classroom practice and, most especially, influence student learning.

REFERENCES

Cabrita, I.: 1996, 'A proporcionalidade directa à luz dos manuais escolares', in CO-VI-SIEM (Ed.), *Actas do SIEM VI - Seminário de Investigação em Educação Matemática* (pp. 95-128). Lisboa: APM.

Christiansen, B. and Walther, G.: 1986, 'Task and activity', in B. Christiansen, A. G. Howson and M. Otte (Eds.), *Perspectives on Mathematics Education*. D. Reidel, Dordrecht, 243-307.

Gimeno, J.: 1989, *El Curriculum: Una Reflexión sobre la Pratica*. Morata, Madrid.

Lesh, R., Post, T. and Behr, M.: 1988, 'Proportional reasoning', in J. Hiebert and M. Behr (eds.), *Number Concepts and Operations in the Middle Grades*. NCTM and Lawrence Erlbaum, Reston, VA, 93-118.

Ministério da Educação: 1991, *Programa de Matemática: Plano de Organização do Ensino-Aprendizagem (2º ciclo do ensino básico)*. Imprensa Nacional Casa da Moeda, Lisboa. (Portugal)

Ministério da Educação: 2001, *Currículo Nacional do Ensino Básico: Competências Essenciais*. Ministério da Educação, Departamento de Educação Básica, Lisboa. (Portugal)

Ministério da Educação e Ciência: 1998, *Parâmetros Curriculares Nacionais (5ª a 8ª séries)*. Ministério da Educação e Ciência, Secretaria de Educação Fundamental, Brasília. (Brazil)

NCTM: 1989, *Curriculum and Evaluation Standards for School Mathematics*. : NCTM, Reston, VA.

NCTM: 1991, *Professional Standards for Teaching Mathematics*. NCTM, Reston, VA.

NCTM: 2000, *Principles and Standards for School Mathematics*. NCTM, Reston, VA.

NCTM: 2006, *Curriculum Focal Points for Prekindergarden to Grade 8 Mathematics*. NCTM, Reston, VA.

OCDE: 2004, *Learning for Tomorrow's World: First results from PISA 2003*. OCDE, Paris.

Pepin, B. and Haggarty, L.: 2004, 'Mathematics textbooks and their use in secondary classrooms in England, France and Germany: Connections, quality and entitlement', in *Proceedings of the 28th PME International Conference*, 391.

Ponte, J.P.: 2005, 'Gestão curricular em Matemática', in GTI (Ed.), *O Professor e o Desenvolvimento curricular*. APM, Lisboa, 11-34.

Real Decreto 116/2004 de 23 de Janeiro: Ministério da Educação e Ciência de Espanha.

Real Decreto 830/2003, de 27 de Junho: Ministério da Educação e Ciência de Espanha.

Ruggiero, M.A. and Basso, I. S.: 2003, 'A Matemática no livro didáctico: Uma reflexão crítica na perspectiva histórico-cultural' *BOLEMA, 20*, 17-36.

Shield, M. and Dole, S.: 2002, 'Investigating textbook presentations of ratio and proportion', in *Proceedings of the Twenty-Fifth Annual Conference of the Mathematics Education Research Group of Australasia*: MERGA, Aukland, NZ.

Skovsmose, O.: 2000, 'Cenários para investigação', *BOLEMA, 14*, 66-91.

Smith, M.S. and Stein, M.K.: 1998: 'Selecting and creating mathematical tasks: From research to practice', *Mathematics Teaching in the Middle School, 3*(5), 344-350.

Stein, M.K. and Smith, M.S.: 1998, 'Mathematical tasks as a framework for reflection: From research to practice', *Mathematics Teaching in the Middle School, 3*(4), 268-275.

FACTORS RELATED TO STUDENTS' MATHEMATICAL LITERACY IN FINLAND AND SWEDEN

Jukka Törnroos

Institute for Educational Research, University of Jyväskylä, Finland

The present study made use of the PISA 2003 database in exploring factors that are connected with students' mathematical literacy performance in Finland and Sweden. Hierarchical linear models constructed in the study showed that most of the statistically significant factors related to students' performance were common to both countries. However, many of the factor coefficients differed markedly between the two countries thus indicating differences in the relative importance of the factors within their respective educational systems. The models raised several questions related, for example, to immigrant students, students' sense of belonging at school, and the textbooks used in the Swedish-speaking schools in Finland. Some of these questions will be addressed in further studies.

INTRODUCTION

The use of hierarchical linear models (or multilevel models) in connection with international assessments, such as TIMSS[1] and PISA[2], has increased substantially during the last decade. In addition to cognitive tests, these assessments include questionnaires targeted at students, teachers (in TIMSS only) and school leaders, and thus offer rich data, for example, on students' home background, attitudes and interests as well as on the different policies employed by schools. All this data can be used in seeking to explain for student outcomes in the cognitive tests.

The present study used the PISA 2003 database to examine student- and school-level factors related to 15-year-old students' mathematical literacy in Finland and Sweden. Hierarchical linear models were constructed for both countries and the findings obtained on the basis of these models are briefly discussed in the paper.

MULTILEVEL MODELLING AND INTERNATIONAL ASSESSMENTS

Since our focus is on mathematics education PISA and TIMSS would seem to be the large-scale international assessments of primary interest. Several reports have appeared on the application of multilevel modelling to the data obtained from

---

[1] TIMSS: the Third International Mathematics and Science Study in 1995 and 1999, and later the Trends in Mathematics and Science Study initiated by IEA (International Association for the Evaluation of Educational Achievement)

[2] PISA: Programme for International Student Assessment initiated by the OECD (Organisation for Economic Co-operation and Development).

different PISA and TIMSS studies. For example, a recent report on further analyses of TIMSS 1999 data includes several chapters on multilevel models in relation to mathematics achievement (for example, Howie, 2006; Kupari, 2006; Park & Park, 2006; Ramírez, 2006). Furthermore, Malin (2005), Marks, Cresswell and Ainsley (2006), and Thorpe (2006), among others, have used PISA 2000 data in their hierarchical multilevel analyses. However, in PISA 2000 reading literacy was the main focus of the assessment and thus also of the related articles. Mathematical literacy was the main area in PISA 2003 and multilevel analysis reports on these data have yet to appear. In one of the early reports Törnroos, Ingemansson, Pettersson and Kupari (2006) used hierarchical linear modelling as one of their analytical tools in looking at the connections between affective factors and students' mathematical literacy in the Nordic countries.

As the present study deals with the Finnish and Swedish PISA 2003 mathematical literacy results, the models constructed by Kupari (2006) and Törnroos and others (2006) provide an interesting reference point for comparisons. According to Kupari (2006), students' self-concept in mathematics was the strongest predictor of their performance in the TIMSS 1999 test in Finland. In the study of Törnroos and others (2006) students' self-concept in mathematics was also very strongly associated with students' mathematical literacy performance in both Sweden and Finland.

The present study investigates the following two research questions:

1. What student- and school-level factors are associated with 15 years old students' mathematical literacy performance in Finland and in Sweden?
2. How much of the variance in students' performance is explained by these factors?

Both research questions also include comparison of the Finnish and Swedish models.

DATA AND ANALYTICAL PROCEDURES APPLIED

The study made use of Finnish and Swedish PISA 2003 data in the international database (available at the PISA website[3]). The data are drawn from the answers of 5796 students in 197 schools in Finland and 4624 students in 184 schools in Sweden. The data consist of student- and school-level files, and include information from the students' cognitive tests and questionnaires as well as the questionnaires addressed to schools.

The analytical tools used in analyses of data of this kind have to cater for the tendency that the results for students within the same school tend to bear a closer resemblance than those for students across different schools (the so called intra-cluster correlation; see e.g. Malin 2005, p. 58). There are several statistical software

[3] http://www.pisa.oecd.org

packages designed for the analysis of such multilevel data (for instance, MLwiN and HLM). Here, the HLM 5 programme was used.

The study made use of all the ready-made student and school-level indices that were available in the international data files and were applicable in Finland or Sweden (OECD, 2005a; 2005b). Altogether 48 student-level indices and an additional seven individual student questionnaire items were used in the analyses. The school-level data files included 29 indices applicable in the Finnish and Swedish school systems. In addition to these ready-made indices all the student-level variables were aggregated to the school-level and then used as contextual school-level variables. Furthermore, 13 national variables were included in the Finnish data to describe the regions the schools were in and the mathematics textbooks used in those schools.

Complete descriptions of the variables are presented in the PISA 2003 data analysis manual (OECD, 2005a) and the PISA 2003 technical report (OECD, 2005b). Here it is sufficient to give a more general description of the variables. The student-level variables were divided into the following categories, which differ in some extent from the ones used in the Data analysis manual (OECD, 2005a).

1. *Student's characteristics:* For example, age, grade level, and gender.
2. *Home background:* For example, family structure, parents' education and occupation, immigration status, and educational and cultural resources at home.
3. *Students, school, and mathematics instruction:* For example, student's expected educational level, attitudes towards school, sense of belonging, teacher support in mathematics lessons, disciplinary climate in mathematics lessons, instructional time in mathematics, and effort invested in the PISA test.
4. *Students and mathematics:* For example, the strategies students use in studying mathematics, and the amount of time they spend on mathematics homework.
5. *Student's interest and self-confidence in mathematics:* For example, student's interest and motivation to study mathematics, and self-efficacy, self-concept and anxiety in mathematics.
6. *Student and ICT (information and communications technology):* For example, student's attitudes toward and self-confidence in the use of ICT.

The school-level variables were categorised as follows (OECD, 2005a):

1. School characteristics: For example, school size, proportion of girls, and the school type (private or public).
2. School resources: For example, the availability of computers, the quality and quantity of teachers, and the educational resources available at the school.
3. Instructional context: For example, the use of assessment and ability grouping, and possible extra mathematics activities or courses offered to students.
4. School climate: For example, school principals' perceptions of students' and teachers' working morale and behaviour.

5. School management: For example, the autonomy of the schools in decision making with respect to resources and the curriculum, and teacher participation in decision making.

Students' mathematical literacy in PISA 2003 was described through five plausible values computed with item-response scaling models (OECD, 2005b). The OECD mean of these plausible values was 500 and standard deviation 100.

The models were constructed in several steps and statistically insignificant variables were discarded at each step of the process. Correlation statistics were used throughout to recognise faulty coefficients caused by collinearity (too high correlations between variables).

RESULTS

After the identification of statistically significant factors these were grouped and entered by stages into four models: Variables describing students' basic information and their family background were entered into the first model. The second model included variables describing students' attitudes and experience related to school and studying mathematics. Variables related to students' confidence in their mathematics abilities and their use of computers were added into the third model and, finally, all the statistically significant school-level variables were added into the model.

Table 1 shows the models for Finland. The final model contained 21 student-level variables and 10 school-level variables. As in the study by Kupari (2006) on the TIMSS 1999 assessment, the variables related to students' confidence in mathematics, especially students' self-concept and self-efficacy in mathematics, were most strongly associated with performance. Some of the associations may look surprising; for example, interest in mathematics was negatively connected with students' performance. However, it is important not to make any causal interpretations of these coefficients but instead try to find explanations in the data. For example, in the case of interest in mathematics it seems that the variables related to self-concept and self-efficacy also account for the positive connections between performance and interest in mathematics. The negative coefficient indicates the existence of, on the one hand, high performing students who are not interested in mathematics and, on the other hand, low performing students who are very interested in mathematics.

One of the most interesting findings in this study is related to the school-level variables in the Finnish model. Two textbooks used by the Swedish-speaking schools in Finland were negatively associated with performance. The effect of these variables was so strong that the difference between Finnish and Swedish speaking schools in general became insignificant.

The respective Swedish models are presented in table 2. The variables related to students' interest and self-confidence in mathematics were again very strongly associated with performance, but they were not as dominant as in the Finnish model. Instead, the coefficients related to school and the learning of mathematics, for instance effort invested in the PISA test and use of control strategies, were somewhat higher (in their absolute values) in the Swedish than Finnish model.

	Model 1	Model 2	Model 3	Final
Intercept	543.7	543.5	544.4	542.4
Student-level				
Student's characteristics				
Grade	39.8**	37.0**	33.2**	32.8**
Birth month	-1.2**	-1.1**	-0.8*	-0.8*
Gender (boy)	12.3**	17.0**	-10.7**	-10.4**
Repeated grade in primary school	-59.3**	-53.2**	-34.8	-34.9**
Home background				
Mother's occupational level	0.6**	0.4**	0.3**	0.3**
Father's occupational level	0.7**	0.5*	0.3**	0.3**
Cultural possessions	11.5**	7.1**	4.2**	4.1**
Foreign language spoken at home	-51.7**	-60.9**	-54.2**	-54.4**
Students & school				
Student's expected educational level		12.8**	5.7**	5.7**
Effort invested in the PISA test		5.4**	3.0**	3.0**
Teacher support		(2.2)	-6.5**	-6.4**
Size of mathematics class		2.9**	2.0**	2.0**
Students & mathematics				
Use of control strategies		4.1	-4.1*	-3.9*
Use of memorisation strategies		(-0.6)	-6.8**	-6.9**
Relative time spent on maths homework		-29.3**	-16.9**	-17.0**
Interest and confidence in mathematics				
Interest in mathematics			-3.6*	-3.3
Self-concept in mathematics			32.2**	31.8**
Anxiety in mathematics			-5.4**	-5.6*
Self-efficacy in mathematics			18.6**	18.7**
Student and ICT				
Confidence in ICT routine tasks			13.0**	12.7**
Confidence in ICT high-level tasks			-10.4**	-10.5**
School-level				
School type				9.5
Proportion of certified teachers				32.9
Poor student-teacher relations				-135.3*
Students' sense of belonging				-20.5**
Teacher support in mathematics lessons				-10.1
Disciplinary climate in maths lessons				9.3
Students' attitudes towards computers	** p<0.001			-10.9

Textbook Matematikboken	* p<0.01			-17.1*
Textbook Matematikens värld	( ) p>0.05			-17.4*
NUTS 2 large area Mid-Finland				-9.7

Table 1. HLM model coefficients for Finnish PISA 2003 data.

	Model 1	Model 2	Model 3	Final
Intercept	508.5	508.2	508.9	508.7
Student-level				
Student's characteristics				
Grade	63.0**	52.0**	40.3**	40.0**
Age	(8.1)	(8.4)	9.7	9.9
Gender (boy)	9.4*	22.7**	7.7	7.3
Home background				
Parents' highest occupational level	1.0**	0.6**	0.4**	0.3**
Computer facilities at home	(-1.9)	-4.5	-4.3*	-4.4*
Home educational resources	11.2**	6.0**	3.6*	3.6*
Cultural possessions	15.0**	10.2**	7.6**	7.4**
Immigration status	-29.1**	-26.8**	-25.6**	-24.3**
Students & school				
Expected educational level		15.4**	8.3**	8.1**
Effort invested in the PISA test		10.2**	7.1**	7.1**
Student's sense of belonging		-4.2*	-8.0**	-8.2**
Teacher support		(0.6)	-4.8**	-4.9**
Relative instructional time on maths		-77.1**	-67.7*	-64.5*
Size of mathematics class		2.7**	1.7**	1.8**
Students & mathematics				
Use of control strategies		-8.1**	-9.2**	-9.1**
Use of elaboration strategies		7.8**	-4.6*	-4.6*
Relative time spent on maths homework		-51.6**	-31.4**	-29.7**
Interest and confidence in mathematics				
Interest in mathematics			-5.3*	-5.2*
Instrumental motivation in mathematics			3.8*	4.0*
Self-concept in mathematics			18.4**	18.4**
Anxiety in mathematics			-7.1**	-7.0**
Self-efficacy in mathematics			28.9**	28.6**
Student and ICT				
ICT internet and entertainment use			-4.0*	-3.8*
Confidence in ICT routine tasks			15.8**	15.8**
Confidence in ICT high-level tasks			-11.0**	-10.6**
School-level				
Poor student-teacher relations	** p<0.001			-125.2*
Parents' highest occupational category	* p<0.01			-13.4*

Relative time spent on maths homework	( ) p>0.05			-43.0*
Students' attitudes towards computers				-8.5

Table 2. HLM model coefficients for Swedish PISA 2003 data.

The Swedish model included a couple of student-level variables related to school that were not included in the Finnish model. The negative coefficient for relative instructional time in mathematics may seem surprising, but it may have a natural explanation: students in Sweden can nowadays choose to devote more time to studying mathematics if they so wish, and this may be a possibility that the weaker students have utilised. Students' sense of belonging seemed to be negatively connected with performance in both Finland and Sweden, but the nature of the relationship seemed to differ between the countries. In Sweden the connection was located on the student-level when higher performing students within schools had more feelings such as loneliness and awkwardness than lower performing students. In Finland, on the contrary, the negative association was found on the school-level so that students in higher performing schools had on average a lower sense of belonging than students in lower performing schools.

One of the greatest differences between the Finnish and Swedish models concerned gender. In the final model the gender coefficient in the Swedish model was positive, favouring boys, whereas in Finland the gender coefficient was negative, favouring girls. Remembering the fact that in both countries the real gender differences were 7 points in favour of boys (OECD, 2004) these coefficients are worth some exploration. From the Finnish models (table 1) we see that the gender coefficient was positive in models 1 and 2, but negative in model 3. This rather dramatic change implies that at least some of the factors added to model 3 are strongly gender-biased. This is reasonable since boys in Finland (and elsewhere, too) generally have more positive attitudes towards mathematics than girls. In terms of the PISA constructs boys, for example, have clearly stronger self-concept and self-efficacy in mathematics than girls and they are also more confident with their ICT skills than girls. Controlling for these factors switched the sign of the gender coefficient in favour of girls. So, if a girl and a boy in Finland had equal attitudes and self-concept in mathematics, the girl performed in average about 10 points higher than the boy.

The gender pattern in the Swedish models (table 2) was markedly different from the Finnish one. In Sweden, as in Finland, boys had more positive attitudes towards mathematics but the influence of attitudes was neutralised by girls' more ambitious educational expectations. The models showed this by the very high gender coefficient (22.7) in model 2, which then fell back to 8 points in model 3. Also in Finland girls had slightly higher educational expectations than boys but, the difference was so small that it did not equalise the effects of the factors related to boys' attitudes and confidence in mathematics and ICT.

Some of the differences between the Finnish and Swedish HLM models are best highlighted by the variances explained by the different models. The most distinctive difference in this respect was seen in the proportions that student characteristics and home background explained of the between-schools variance (tables 3 and 4). In Finland these variables explained about 30 percent of the between-schools variance, whereas in Sweden this proportion was 81 percent. This extremely high proportion was mostly accounted for by the variable immigration status of the student. A closer look at this variable revealed that the proportion of immigrant students varies greatly across Swedish schools and, according to the model, immigrant students, and especially non-native immigrant students (both student and parents born outside the country) showed substantially lower performance than native students.

Finland	Between-school variance			347	4.9 %
	Within-school variance			6659	95.1 %
	Total			7006	
Variance explained by models					
		Model 1	Model 2	Model 3	Final
Between schools		30.3 %	39.0 %	61.4 %	74.7 %
Within schools		19.3 %	35.9 %	60.6 %	60.6 %
Total		19.9 %	36.1 %	60.6 %	61.3 %

Table 3. Variance accounted for by the Finnish models.

Sweden	Between-school variance			977	10.9 %
	Within-school variance			8026	89.1 %
	Total			9003	
Variance explained by models					
		Model 1	Model 2	Model 3	Final
Between schools		81.4 %	91.4 %	91.2 %	93.3 %
Within schools		20.6 %	44.6 %	64.5 %	64.7 %
Total		27.2 %	49.7 %	67.4 %	67.8 %

Table 4. Variance accounted for by the Swedish models.

The variance tables also clearly show the difference in the roles the variables related to school and mathematics learning, on the one hand, and the variables related to interest and self-confidence in mathematics, on the other, play in Finnish and Swedish schools. In Sweden (table 4) the former produced the greatest increase of explanatory power while in Finland (table 3) the latter variables caused the greatest addition in the explanatory power of the model.

The contribution of the school-level variables to the explanatory power of the models was small, especially in Sweden (tables 3 and 4). However, many of the statistically significant school-level variables point to important educational features. For example, in both Sweden and Finland bad teacher-student relations were significantly associated with lower student performance. It is also interesting to notice that more positive attitudes towards computers were also negatively connected with performance on the school-level in both countries. On the first glance this seems like an undesirable result in today's school world, where the use of computers is emphasised.

DISCUSSION

The models constructed for the Finnish and Swedish PISA 2003 results in the present study show that most of the statistically significant factors associated with students' mathematical literacy performance are common to both countries. However, the strengths of the relationships differ to some extent between the two countries.

Many of the findings deserve deeper elaboration. For example, a couple of years ago the Swedish National Agency for Education sparked a lively debate when their report stated that the proportion of certified teachers was not connected with student performance and that students in private schools had better results than students in public schools (Skolverket, 2005). According to the present study the first holds true also to the Swedish PISA 2003 results; however, the latter association was not found. Interestingly, in Finland the proportion of fully certified teachers was positively connected with mathematical literacy and students in public schools had better results than students in private schools, when all the other variables were controlled for. Obviously, the roles of teacher certification and private education may be different in the Finnish and Swedish educational systems.

One always has to bear in mind that hierarchical linear models do not show the direction of the relationships. An example of this is the negative connection between performance and the time spent on homework that constantly comes up in studies such as the present one (tables 1 and 2). This does not imply that spending more time on homework leads to worse performance. On the contrary, the negative coefficient indicates that lower performing students have to spend more time on their mathematics homework. Furthermore, the present study showed that this relation was stronger in Sweden than in Finland.

The present study mainly used the international PISA 2003 database and found that many of the readymade indices were useful in explaining mathematical literacy in the Finnish and Swedish educational contexts. However, the Finnish analyses also utilised some additional national variables (NUTS 2 large areas and textbook information) and three of them remained in the final model as school-level variables. This can be seen as encouraging the inclusion of some nationally meaningful optional items in the international assessment questionnaires.

International assessments offer rich data that can be used in secondary analyses in order to deepen the picture presented by the first results of the assessments. The present study shows an example of how multilevel modelling can be used to compare different educational systems and thereby try to better understand the functioning of the educational systems in question. Comparisons between models obviously help us to recognise results characteristic of particular educational systems. However, the models do not explain why the results are characteristic of a specific system.

Explaining these features often requires deep insight into the educational system in question, and in many cases also further research on the results.

REFERENCES

Howie, S.: 2006, 'Multi-level factors affecting the performance of South African pupils in mathematics', in S. Howie and T. Plomp (eds.), *Contexts of learning mathematics and science,* Routledge, London, pp. 157-176.

Kupari, P.: 2006, 'Student and school factors affecting Finnish mathematics achievement: Results from TIMSS 1999', in S. Howie and T. Plomp (eds.), *Contexts of learning mathematics and science,* Routledge, London, pp. 127-140.

Malin, A.: 2005, *School differences and inequities in educational outcomes. PISA 2000 results of reading literacy in Finland*, Institute for Educational Research, University of Jyväskylä, Jyväskylä, Finland.

Marks, G., Cresswell, J. and Ainley, J.: 2006, 'Explaining socioeconomic inequalities in student achievement: The role of home and school factors', *Educational Research and Evaluation* 12(2), 105-128.

OECD: 2004, *Learning for tomorrow's world - First results from PISA 2003*, Author, Paris.

OECD: 2005a, *PISA 2003 Data analysis manual*, Author, Paris.

OECD: 2005b, *PISA 2003 Technical report,* Author, Paris.

Park, C. and Park, D: 2006, 'Factors affecting Korean students' achievement in TIMSS 1999', in S. Howie and T. Plomp (eds.), *Contexts of learning mathematics and science,* Routledge, London, pp. 177-191.

Ramírez M. H.: 2006, 'Factors related to mathematics achievement in Chile', in S. Howie and T. Plomp (eds.), *Contexts of learning mathematics and science,* Routledge, London, pp. 97-112.

Thorpe, G.: 2006, 'Multilevel analysis of PISA 2000 reading results for the United Kingdom using pupil scale variables', *School Effectiveness and School Improvement* 17(1), 33-62.

Skolverket: 2005, 'Att mäta skolors relativa effektivitet - En modellanalys baserad på resurser och resultat' [Measuring the relative effectiveness of schools – A model analysis based on resources and results], the author, Stockholm, in Swedish.

Törnroos, J., Ingemansson, I., Pettersson, A. and Kupari, P: 2006, 'Affective factors and their relation to the mathematical literacy performance of students in the Nordic countries', in J. Mejding and A. Roe (eds.), *Northern light on PISA 2003 – a reflection from the Nordic countries,* Nordic Council of Ministers, Oslo, pp. 87-100.

A COMPARATIVE STUDY OF ASSESSMENT ACTIVITY INVOLVING 8 PRE-SERVICE TEACHERS: WHAT REFERENT FOR THE ASSESSOR?

Marc Vantourout - IUFM of Pays de la Loire

EDA Laboratory, SHS-Sorbonne Faculty, Paris 5 University

Member of the OPEN network (Teachers' Practices Observation)

This paper pertains to studies describing teachers' approach to build their judgment of assessment. More precisely, we are comparing the process of 8 pre-service teachers' activity in assessing productions of students in mathematics. This study leads us to interrogate the main way used nowadays to describe the assessment activity.

INTRODUCTION

Nowadays, in accordance with a cognitive approach, the assessment activity is generally described as a confrontation between a referent ("référent" in French) and a "référé" (in French). We will name this conception the "referent/référé" model :

- the referent is related to the legitimate expectation system, judged legitimate by the assessor, which orientates the reading of the object to be assessed (Hadji, 1999, pp.43-45). In Noizet and Caverni (1978), where it is, instead, a matter of a "model of reference", this component forms the subject of a more precise descripion and gathers several elements, among others "the norm product"[1] (p.69) and "the expected products"[2] (p.69) ;
- The référé indicates a "reduced model" of the assessed object, constructed by the assessor (idem).

This description derives from "the explanatory model of the assessment behaviour in the academic works"[3], introduced by Amigues and al. (1975). From this psychological approach, we retain that "the core of the assessment activity is a comparison between a model of reference on one side against a production (a copy) on the other side" (Noizet and Caverni, 1978, p.98) and that "the assessment task cannot be possible if the assessor does not have a model of reference established in his cognitive structures" (Amigues and al. 1975, p.794).

In brief, the referent/référé model is the subject of a wild consensus. Nevertheless, according to our knowledge, the explanatory model – and also the referent/refere model which derives from the former – have only been established on the basis of experimental situations of correcting copies achieved by teachers specialised in the subject to assess. This situation leads us to two questions. Do these two models allow describing:

- the assessment activity in situations other than in marking a copy?
- the teachers' assessment activity with no specialisation in the subject to assess?

In order to attempt to give some answers, we have developed an experimentation allowing to study the activity of some teachers during some assessment practices in mathematics (Vantourout, 2004).

AN INTRODUCTION OF THE EXPERIMENTATION: A FEW ELEMENTS OF THE THEORETICAL FRAME

Regarding the theoretical aspect, our references come from several origins: the professional didactic[4], the mathematics didactic and the field of assessment.

The professional didactic contributions

We have developed a research device by borrowing from the professional didactic one of its favourite ingeniery: a simulation (the purpose is to simulate a situation of assessment) followed by a debriefing. This device is only partly developed below because of its complexity[5].

During the "simulation", a teacher, facing a computer, discovers the experimental material: the wording of a problem and the pupils' copies. He has to answer the following question: "How these pupils' works are satisfactory or unsatisfactory?" The teacher's task consists of assessing the productions and the activity of "fictitious"[6] pupils who have solved a proportionality problem with the use of graphics. This task is rather difficult, because the to be examined pupils' works cannot have, a priori, a clear and immediate assessment.

Then the teacher gets involved in a debriefing leaded by the experimenter adapting to the method of the "explicitation" discussion (Vermersch, 1994).

The experimentation is organised around a mathematics problem "The cyclist"[7]. In this problem, the distance to be run by a cyclist on a certain time has to be defined. The questions are asked under a numerical frame (distance to be run in 12, 20 and 90 minutes), then under a graphical frame (distance to be run in 25, 37, 45 and 108 minutes). This problem is interesting because the situation is only partly a situation of proportionality[8], though the context does not change. "The most interesting question" is to define the distance to be run by the cyclist in 90 minutes. In fact, this "numerical" question applies to "the affine" or "non-proportional" part of the cyclist' ride.

The material includes – except the wording – several documents stored on a CD-ROM acting as a "simulator". The computer allows the teachers to access freely the task asked to the pupils, the development of their written works (their graphics in particular), "the results" they produced (one sheet where they have written their answers) and their discussions.

The pupils work in pair; among the productions to be assessed, there is one production made by David and Alexis (cf. annexe). David does not succeed in coming out from the situation of proportionality in order to find the distance to be run in 90 minutes: he alters the chart's wording by keeping the situation of proportionality for all the duration of the cyclist' ride. On the other hand, Alexis uses cleverly and implicitly the property of the affine functions called “property of proportional increase”. When these pupils start to draw the graphic of the ride, they use the information written in the chart’s wording and draw the graphic requested. David is aware of his mistake by observing the graphic requested and notices that Alexis has the right answer.

The use of such device – computer and CD-ROM – is mainly justified by the accurate information it provides about the course of events of the assessment activity during the simulation. Actually, an electronic file – “the journal of the documents displayed” – is automatically created during each use. It records the name of all the documents displayed, the sequence and the time of displays.

The links in the field of the mathematics didactic

They appear by their position in first place allowed to the mathematics and graphics contents involved in the problem (as shown above) and by two levels, briefly presented here (for more details, cf. Vantourout and Maury, 2006).

The first one refers to the very construction of the experimental material which involves a priori analyses, requiring some knowledge in didactic related to proportionality and graphics. These analyses concern the problem chosen (the task supposedly asked to the “fictitious” pupils) and also, the “invention” of the productions and of the behaviours allocated to these pupils.

The second one deals with some general aspects of the mathematics didactic developed in France. They consist of taking into consideration the mistake as an indicator of the functioning of knowledge and, so to try, from the observable, to identify and to analyse the procedures and their meaning based at the knowledge level. This works when we develop the “expected task”, in other words, the real content of our expectations: we expect from the teachers, as the designer of the task and the experimenter, to undertake a didactic analysis of the pupils’ work, which one would demonstrate what is relevant and irrelevant in their executions.

The link with the assessment field

Looking at the characteristics of the simulation situation (undeveloped here due to lack of space), we can say that the task proposed to the teachers is similar to an “under task” of a formative assessment and its realisation requires an observation activity by the teachers to achieve it (Vantourout and Maury, 2006) The simulation allows studying precisely this observation activity. Its role is acknowledged by many specialists as fundamental (Allal, 1998; Hadji, 1999), in most of the assessment types and in particular during formative assessments.

A comparative approach

The choice of a simulation situation through which it is possible to propose an identical task to different teachers, allows various comparisons, especially a comparison between secondary school teachers in mathematics (PLC, teaching in secondary school) specialized in the subject and “professeurs des écoles” (PE, teaching in primary school), unspecialized in the teaching of mathematics.

The problem chosen can be proposed either to CM2 pupils (10-11 years old) or 6^{e} pupils (11-12 years old). During the experimentation, pupils are assimilated as pupils in CM2, if the teacher is a PE teacher, or assimilated as pupils in 6^{e}, if the teacher is a PLC.

The population

8 teachers, finishing their initial training at the IUFM[9], are chosen and split into 3 categories: actually, in admitting that the link to the mathematical and graphical knowledge (“rapport au savoir“ in French; cf. Maury and Caillot, 2003) can have some effect in the assessment activity during the simulation, we distinguish, among the PE, those who have received a training that can be called “scientific”, as they have a scientific secondary school examination (A-Levels).The others are named “non-scientific”. The population is made up of 2 PLC (coded Pr1 and Pr2), 2 “scientific” PE (Pr3 and Pr4) and 4 “non-scientific” PE (from Pr5 to Pr8). Sometimes, we use the term “scientific” to name the teachers when we gather the PLC and the scientific PE.

A FEW RESULTS

The use of the wording and the solution to the problem by the teachers

In the experimentation, the answers to the questions of the problem are not given to the teachers; so, the use of the wording and the solution to the problem seem to be essential in order to let the teachers have a referent.

The study of the use of the wording is based on all has been said and done about it by the teachers. For each teacher, we analyse; 1) his journal of the documents displayed (information about his way of referring to the wording); 2) his rough papers, used during the simulation (in order to see if the problem was solved and how, or unsolved); 3) the transcript of his debriefing.

The results we found put forward:

- the position more or less important of the wording granted by the teachers; we base our judgment here on the time of displays rising up to five times (cf. chart 1); the journals of the documents displayed show us that the 8 teachers start their work by looking at the wording, but, the main difference is that some will spend time on it and others will move quickly to something else;
- the fact that 1 teacher in 2 does not solve the problem before assessing the pupils’ works (cf. chart 1); in other words, only 4 teachers try to solve the problem. Among them, Pr5 and Pr8 have only one part of the answers to the questions, on the opposite, Pr2 and Pr4 have all the answers. Pr4 by drawing his graphic as asked in the wording, is the only one to solve the problem.

		Pr1	Pr2	Pr3	Pr4	Pr5	Pr6	Pr7	Pr8
Wording display : • Duration of the display at the beginning of the assessment • Total duration of the display during the assessment • % of the total duration of the assessment		 2 min 30 3 min 11%	 8 min 12 min 20 28%	 3 min 30 3 min 50 7%	 6 min 30 18 min 30 34%	 7 min 20 7 min 20 13%	 3 min 3 min 50 7,5%	 5min 30 5 min 30 13%	 2 min 10 3 min 20 7%
Full solution of the problem		no	no	no	**yes**	no	no	no	no
Solution to the problem	12 and 20 min. (numerical)		yes		yes	yes			yes
	90 min. (numerical, "affine")		yes		yes	yes			yes**
	25, 37 and 45 min. (graphical)		yes*		yes				
	108 min. (graphical, "affine")		yes*		yes				
	Expected graphic				yes				

Chart 1 : Wording display and solution to the problem by the different teachers
yes* means that the answer only results of an algebraic solution
yes** means that 2 answers (one of which being false) have been written and kept

We can notice the results on chart 1 – durations of displays and solution to the problem – do not allow distinguishing exactly the teachers' categories. Nevertheless we can observe individual differences very easily.

The teachers' referent and the course of their assessment activity

The previous results and the analyses of the debriefings (see below), referring to the referent of the 8 teachers, lead us to divide these teachers in 4 groups introduced and illustrated here with extracts from debriefings.

- ***Pr2 and Pr4 (group 1): teachers who solve the problem and who get all the answers***

Pr2 and Pr4 are scientific, PLC and PE respectively. These 2 teachers spend on the wording the longest time; they have the exact answer at each question with an algebraic solution (we notice the algebraic procedures used are not the same as the procedures available for the CM2 or 6^{e} pupils). This is the necessity in having the right answers, to value the accuracy of pupils graphic's reading, that leads to this solution mode option. Pr4 is in a position to solve the entire problem. Then he compares his answers with the pupils' answers. For Pr2, the entire solution does not take place at the beginning of the assessment process. At first, he answers the early questions and tries to assess the productions without paying attention to the exact answers for the graphic questions. Nevertheless, his knowledge regarding pupil's inaccurate way of answering this type of questions drive him to solve the problem with an algebraic solution. Then the answers are compared with the pupils' answers.

We associate their right answers to the questions with the norm product. Pr4's referent might be the most complete; his graphic, compared with the algebraic solution, corresponds more to an expected product. The activity of assessment may be described here as a comparison between a referent and a "référé", in other words, by the "referent/référé" model.

- ***Pr5 and Pr8 (group 2): teachers who undertake to solve the problem but who do not succeed in solving the entire problem and, eventually, who do not have all the problem's answers***

These 2 teachers are non-scientific PE. Pr5 spends, at the beginning of his activity, 7 minutes to the solution to the problem. He proceeds that way as he finds the problem difficult. When he reads the wording he is immediately aware of some proportional and non-proportional parts. He tries to answer the questions and finds easily the answer for the first 2 questions. Regarding the "affine" and "numerical" (90 minutes) question, he finds "*the same result as the children*"[10]. He finds this answer "*intuitively*" correct and he admits to be unable to explain it.

Pr8 spends no more than 2 minutes on the wording at the beginning of his activity. He identifies a problem of proportionality because of the chart. He answers the first 2 questions (we notice he has written and kept 2 answers for 90 minutes, one of which is false). He neglects the graphical questions as he does not read the wording thoroughly. When he discovers them, he displays the wording again and tries to answer the questions. He admits to be able to find the distance run in 25 and 37 minutes, but to be unable to establish the distance run in 108 minutes[11] (affine part of the ride); he recognises, more than once, "*to trust the pupils*" for the answers.

The description of the beginning of the assessment activity – when the teachers have the answers to the questions – can refer to the referent/référé model: actually, we can assume there is a referent at that particular moment. But, is it the same when Pr5 and Pr8 assess a question they do not know the answer or do they trust the pupils' answers? We will discuss it after the introduction of group 4.

- ***Pr1 and Pr3 (group 3): teachers who do not get involved in the solution to the problem but who have the "solution structure"***

Pr1 and Pr3 are scientific PLC and PE respectively. To illustrate, we will be using only Pr1's activity. Pr1 spends only 2 minutes 30 on the wording at the beginning of the assessment (and 3 minutes in total). He says, more than once, he does not try to find the solution to the problem. Nevertheless he succeeds, in a very short space of time, in getting the necessary elements to assess. Though he does not read thoroughly the wording about the numerical values; he does not neglect the main points: he notices the cyclist's stop in the chart and recognises roughly the steps of the cyclist's ride. He also explains he paid attention to the questions and related them immediately to their "goals". Furthermore, the pupils' answers drive him (he is the only one among the 8 teachers) to question about the determination of the moment the cyclist stops. The indication or non-indication of this element is an important didactic variable. At last, he admits to have a graphic image "*obviously*" made up by "*two slopes and one level*". In a word, he conducts his assessment activity by "*following the pupils' progress*" without solving the problem but being aware of the main elements.

Some questions can be asked: can the assessment activity be described here with a referent/référé model? What builds Pr1's or referent? What about the existence of the norm product and its characteristics as Pr1 does not have any correct answers? To answer these questions, we will say Pr1 (like Pr3) owns the solution structure (structure de la solution in French) that replaces the referent function, such as the one introduced by Hadji (see above). This structure, based on the mathematics and graphics knowledge, introduced and identified elsewhere (cf. Vantourout and Maury, 2006), would allow Pr1 to receive and to analyse the pupils' works in confronting them in a frame of possibilities, at the same time opened to many eventualities and bordered by his own knowledge. In a word, if we refer to Hadji (*idem*), we think we can describe Pr1's activity as an orientated reading of the productions to be assessed, based on his expectations judged to be legitimate.

• Pr6 and Pr7 (group 4): teachers who do not get involved in the solution to the problem and who do not have any answer

Pr6 and Pr7 are 2 non-scientific PE. To illustrate, we will be using only Pr7's activity. There is no answer on the Pr7's rough papers: the 5 minutes used on the wording at the beginning of the assessment allows him only to copy the questions and the chart. At his first reading, he thinks there is no situation of proportionality in the different ride stages: he thinks "*the cyclist never has a regular pace"*. He will become aware of his misjudgement with David and Alexis who discuss about proportionality in their dialogues. He tries to solve the problem but he doesn't have enough time. He defers the solution thinking he will obtain the solution *"automatically"* by assessing the pupils' answers. He has on mind a curve completely different from the one expected: *"this is not a straight line but a curve rising higher and higher at the end"*. Once more, he recognises that Alexis and David's work allows him to correct his mistake and to obtain the right answer. He acknowledges that the fact of having a wrong idea about the expected answer *"is a bit disturbing"*.

The questions are the same as the previous ones. On the other hand, our answer is different (this one involves also a part of the teachers' activity in the group 2). We can assume there are no norm product and no expected product in Pr7's referent as he is unable to modelise properly the situation and does not have any answers. We think we have reached here the limit of the field of validity of the referent/référé model. In fact, this is not a comparison activity between the referent and the référé, but an activity taking some information, or even some knowledge, from the productions to be assessed in order to obtain possibly correct answers. The assessment activity is no longer a reading orientated by a referent, but on the contrary it becomes mainly, and sometimes only, a reading orientated by the pupils' works and answers, in other words orientated by some elements referring more to a référé than a referent.

CONCLUSION

Now, several elements enable us to answer our two questions:

- the referent/référé model (and also the explanatory model) can be used to describe the assessment activity in situations other than in situations of marking copies, in particular in the simulation situation used by us and similar to an under task of formative assessment;
- for the type of tasks mentioned above, this model, in this study, can be used to describe the assessing activity of teachers non-specialist in the subject to assess, scientific and non-scientific PE.

Nevertheless, we have to take precautions with these results as they refer to only one part of the teachers involved in the study. Actually, there are also teachers with no answer to the problem who would not structure a "real" or referent. But, here again, we have to be careful because among this half of the population, where the 3 categories are represented, we can distinguish:

- scientific teachers – PLC and scientific PE – who use the solution structure to assess the pupils' productions ; we consider this structure, at a functional level, capable to stand as a model of reference or as a referent;
- non-scientific PE who have to obtain information and knowledge from the pupils' works to assess them; we say here that we reach what we call the limit of the field of validity of the "classical" model.

In a word, the unsolved problem does not have the same effect as it depends on teachers, either scientific (PLC or scientific PE), or non-scientific PE. An additional analysis of the activity of these teachers shows that the evaluative behaviour of some non-scientific PE is related to a lack of mastering some knowledge in the mathematical and graphical fields (cf. Vantourout and Maury, 2006).

Now, there is one question to ask: would the limit of the validity of the referent/référé model only appear when the assessor is a non-specialist of the subject-matter to assess, in other words, a subject-matter he would insufficiantly master? A study recently published, (Nabbout, 2006) involving Lebanese titular teachers in mathematics, shows that some of these teachers, when assessing pupils' works in probabilities (the stochastic independence is the concept involved), have a behaviour close to the ones we have identified in some non-scientific PE. In a word, the model of referent/référé would not be suitable to describe all the assessment activities of pupils' productions.

---

[1] For example, a dictation without mistake.

[2] The expected products are selectioned "among all the possible products which means among the products that can be observed" (Noizet and Caverni, 1978, p.98).

[3] All the quotes are our translation.

[4] The professional didactic ("la didactique professionnelle" in French) aims at "analysing the acquisition and communication of professional skills in order to improve them" (Pastré, 1995, p.404). To do so, it mobilizes a "professional didactic ingeniery at the interface, and in the historical prolongation, of the cognitive ergonomics, on one side, and of the didactic of scientific subjects, on the other side" (Ibid).

[5] For a detailed introduction and a critical analysis of the device, cf. Vantourout (2004).

[6] The teachers have to assess works assigned to absent pupils and they do not know them. These works are developed for the experimentation needs, after observing "real" pupils working in tandem and asked to solve the same problem.

[7] This wording is inspired by a situation presented in the "Petit x" review (Galai and al., 1989).

[8] The cyclist's ride is broken down into 3 stages; each of them is characterized by a different average speed. The first stage (speed S_1 for the durations from 0 to 40 minutes) can be modelled by a linear function, the second stage by a constant function (speed 0, from 40 to 60 minutes) and the third stage by an affine function (speed S_2 (is equal to the double of S_1), from 60 to 100 minutes).

[9] In France, the IUFM – Institut Universitaire de Formation des Maîtres – in coordination with the universities and the different local partners, provide the first professional training of the primary and secondary school teachers.

[10] Italics indicate the expressions used by the teacher during the debriefing.

[11] Do not forget it is possible to solve this problem with graphical procedures.

REFERENCES

Allal, L. (1998). *Vers une pratique de l'évaluation formative*. Bruxelles: De Boeck.

Amigues, R., Bonniol, J.-J., Caverni, J.-P., Fabre, J.-M. and Noizet, G. (1975). Le comportement d'évaluation de productions scolaires : à la recherche d'un modèle explicatif, *Bulletin de psychologie, 28 n°318*, (793-799).

Galai, M.-C., Gerente, M., Grenier, D. and Rivoire, R. (1990). Analyse de deux situations problèmes autour de la proportionnalité, *Petit x 22*, (5-22).

Hadji, C. (1999). *L'évaluation démystifiée*. Paris: ESF.

Maury, S. and Caillot, M. (2003). *Rapport au savoir et didactiques*. Paris: Fabert.

Nabbout, M. (2006). Enseignement des probabilités en terminale au Liban : études des représentations et des pratiques dans des situations aménagées - Thèse. Paris: Université René-Descartes - Paris 5 Sorbonne.

Noizet, G. and Caverni, J.-P. (1978). *Psychologie de l'évaluation scolaire*. Paris : PUF.

Pastré, P. (1995). L'ingénierie didactique professionnelle. In P. Caspard (éd.), *Traité de formation*, (pp.403-417). Paris : Dunod.

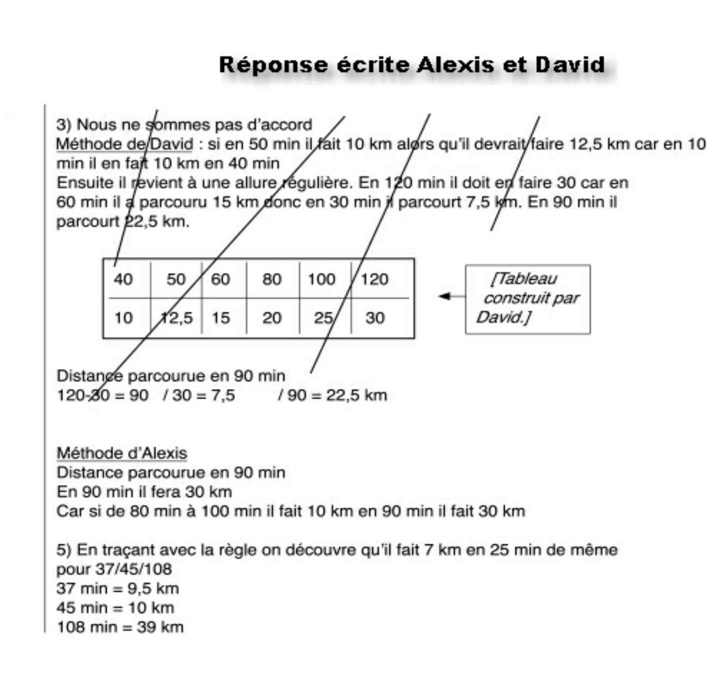
Vantourout, M. (2004). Etude de l'activité et des compétences de professeurs des écoles et de professeurs de mathématiques dans des situations « simulées » d'évaluation à visée formative en mathématiques - Thèse. Paris: Université René-Descartes - Paris 5 Sorbonne.

Vantourout, M. and Maury, S. (2006). Quelques résultats relatifs aux connaissances disciplinaires de professeurs stagiaires dans des situations simulées d'évaluation de productions d'élèves en mathématiques, *Revue des Sciences de l'Education (Montréal)*, *32/*3 (delayed publication).

Vermersch, P. (1994). *L'entretien d'explicitation en formation initiale et en formation continue*. Paris: ESF.

ANNEXE

PUPILS' PRODUCTIONS (SAMPLES OF DAVID AND ALEXIS' WORK)



Réponse écrite Alexis et David

3) Nous ne sommes pas d'accord
Méthode de David : si en 50 min il fait 10 km alors qu'il devrait faire 12,5 km car en 10 min il en fait 10 km en 40 min
Ensuite il revient à une allure régulière. En 120 min il doit en faire 30 car en 60 min il a parcouru 15 km donc en 30 min il parcourt 7,5 km. En 90 min il parcourt 22,5 km.

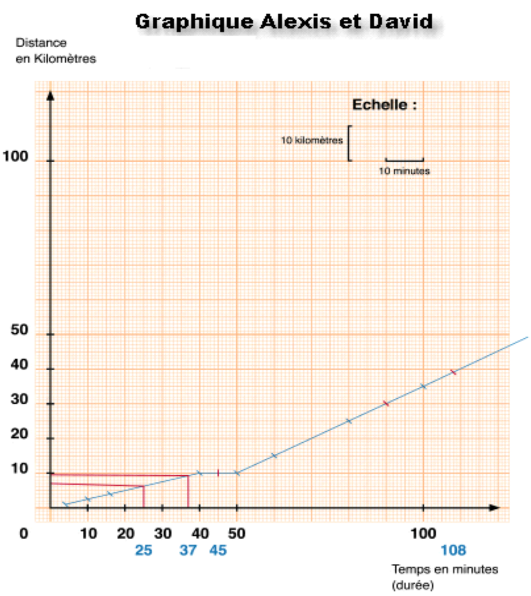
40	50	60	80	100	120
10	12,5	15	20	25	30

[Tableau construit par David.]

Distance parcourue en 90 min
120-30 = 90 / 30 = 7,5 / 90 = 22,5 km

Méthode d'Alexis
Distance parcourue en 90 min
En 90 min il fera 30 km
Car si de 80 min à 100 min il fait 10 km en 90 min il fait 30 km

5) En traçant avec la règle on découvre qu'il fait 7 km en 25 min de même pour 37/45/108
37 min = 9,5 km
45 min = 10 km
108 min = 39 km



THE CONSTRUCTION OF PERSONAL MEANING – A COMPARATIVE CASE STUDY IN HONG KONG AND GERMANY

Maike Vollstedt

Graduate Research Group on Educational Experience and Learner Development, University of Hamburg

Which kinds of personal meaning do students construct, and which goals do they pursue when they are dealing with mathematics? Are these types of personal meaning universal, or is it rather the case that different types of personal meaning are constructed in different learning cultures? These questions are investigated in the qualitative comparative study presented in this paper, which was conducted in Germany and Hong Kong. As the terms *personal meaning* as well as *meaning* in general are often used without a clear definition, the comprehension of these terms as used in this study are elaborated on at first. The concept of personal meaning is therefore related to different concepts from mathematics didactics, *Bildungsgang*-didactics, as well as educational psychology. A special emphasis is put on the relation between personal meaning, context and (learning) culture. Subsequently, the dominant personal meaning constructed by a 15-year-old student from Hong Kong is presented as exemplification of the concept.

INTRODUCTION

According to the German philosopher Blumenberg (1999), human beings show a desire for meaning in reality. Gebhard (2003) further specifies that it is a specific human desideratum to provide the world with meaning and sense so as to interpret it as being meaningful. In this way, the world can be understood by human beings. This attitude does not stop outside classroom doors but meaning is also – if not especially – sought inside the classroom when students deal with learning contents.

When we follow Brousseau's (1997) idea of a didactical contract between the teacher and the students, the latter assume that the objects they have to deal with in class are neither absurd nor do they lack any sense nor meaning. On the contrary, students assume that there is some kind of meaning in dealing with the problems given in the lesson. Therefore, they engage in the search for this meaning and need to construct their own personally relevant meaning with relation to the problem dealt with. In other words: the process of the construction of a personal meaning takes place.

The ongoing study presented in this paper is embedded in the Graduate Research Group on Educational Experience and Learner Development, which focuses its research on the learners' perspective on their educational process. Therefore, personal meaning (German: *Sinnkonstruktion*) is one of the exploratory foci in the Graduate Research Group (see also Vorhölter, in press). In the context of educational experience and learner development (German: *Bildungsgangforschung*[1]), the concept of personal meaning with relation to mathematics is embedded in a relational framework consisting of concepts from mathematics didactics, *Bildungsgang*-didactics, and educational psychology (see below). The study presented here seeks to find different kinds of personal meaning which students construct when they are dealing with mathematics. As the context in which learning takes place is highly influential for the construction of meaning (see below), the study is carried out in two very different learning environments: Germany and Hong Kong. This is done to be able to contrast typical personal meanings from the two very different learning cultures.

THE CONSTRUCTION OF MEANING

As shown above, students are in the need of meaning when dealing with learning objects in school. This means that they need to decide to what extent a certain task is *personally meaningful* to them, i.e. what its relevance for the respective student is.

Personal meaning vs. objective meaning

There is a discussion about meaning in mathematics education, which gets to the point in a collection of articles edited by Kilpatrick et al. (2005). To open up the field, they write:

> Some students find it pointless to do their mathematics homework; some like to do trigonometry, or enjoy discussions about mathematics in their classrooms; some students' families think that mathematics is useless outside school; other students are told that because of their weakness in mathematics they cannot join the academic stream. All these raise questions of meaning in mathematics education. (Kilpatrick et al., 2005b, p. 9)

One can see that there are very different kinds of meaning dealt with in this passage. On the one hand, meaning is used in a rather personal sense of the student "relating to relevance and personal significance (e.g., 'What is the point of this for me?')" (Howson, 2005, p. 18). On the other hand, meaning can also be used in a rather objective way when describing "an agreed, common meaning within a community" (Kilpatrick et al., 2005b, p. 9). It is important to keep this distinction. Therefore, in this paper, the terms *personal meaning* and *objective meaning* are to be used to describe these different aspects of meaning.

This difference between personal and objective meaning comes to the point when the difference between philosophical and non-philosophical interpretations of meaning are considered. Kilpatrick et al. state that "we may claim that an activity has meaning as part of the curriculum, while students might feel that the same activity is totally

---

[1] As the English term 'Educational Experience and Learner Development' is very long and awkward to use, the German term *Bildungsgang* will be used interchangeably to refer to it. Similarly, the didactics resulting from this research field will be referred to as *Bildungsgang* -didactics.

devoid of meaning" (Kilpatrick et al., 2005a, p. 2). One can, however, even go a step further by saying that although a student might think that a certain activity is totally devoid of objective meaning, she still sees a personal meaning in relation with the activity. This personal meaning, then, can be of different kinds. It may be the case that she might still work on the task so as to fulfil her teacher's or parents' expectations, because she might after all seek her teacher's meaning, because she wants to get good marks in the next class test, etc.

Characteristics of personal meaning and its construction

Some assumptions can be made concerning personal meaning so that it is characterised by different traits. To begin with, personal meaning is *subjective* and *individual*. This means that every person has to construct her or his own meaning with respect to a certain object. There is no given objective meaning which just has to be applied; meaning cannot just be endowed. Also, as the construction of meaning is not collective but individual, different students sitting in the same lesson can also construct different meanings. However, offerings of meaning can be assimilated. They may be provided in a lesson e.g. in the shape of modelling tasks given by the teacher (see Vorhölter, in press), or by the context of the learning task which for instance shows a relation to daily life. But still – the individual is involved in the process of constructing a meaning before a certain personal meaning is generated.

Meaning-making is also *context bound*. Context hereby means on the one hand the subject context as well as the situation in the classroom. On the other hand, it also embraces the personal context of the students. The relation between personal meaning, context and culture will be elaborated in more detail below (see the respective section).

Finally, meanings *can be reflected* on but normally *do not have to*. This means that the process of meaning-making can in some parts be dominant in the situation so that one is aware of what is going on; the meaning enters consciousness. An 'Aha-experience', for instance, is an example of a meaning which is dominant and conscious in the very situation. On the other hand, meaning does not have to be conscious but can be constructed implicitly so that it is there without being dominant in the situation. From a constructivist perspective, Kilpatrick et al. state that

> the problem of *construction* of meaning itself is not really tackled. This is an evasive problem: It is difficult to know what each partner [i.e. student and teacher, MV] thinks; we can only hypothesise this by interpreting what they do and say (Kilpatrick et al., 2005c, p. 137, my italics)

So, although it is not possible to directly ask for personal meaning, it is still possible to indirectly ask for it and aspects/concepts related to it. Therefore, it is possible to reconstruct personal meaning from interview data. This, then, makes personal meaning researchable with qualitative methods.

RELATIONAL FRAMEWORK OF DIDACTICS AND EDUCATIONAL PSYCHOLOGY

To illustrate the concept of personal meaning in more detail, it will be put it in a relational framework of different concepts from mathematics didactics, *Bildungsgang*-didactics, and educational psychology which are assumed to have an impact on the construction of personal meaning (see Figure 1).

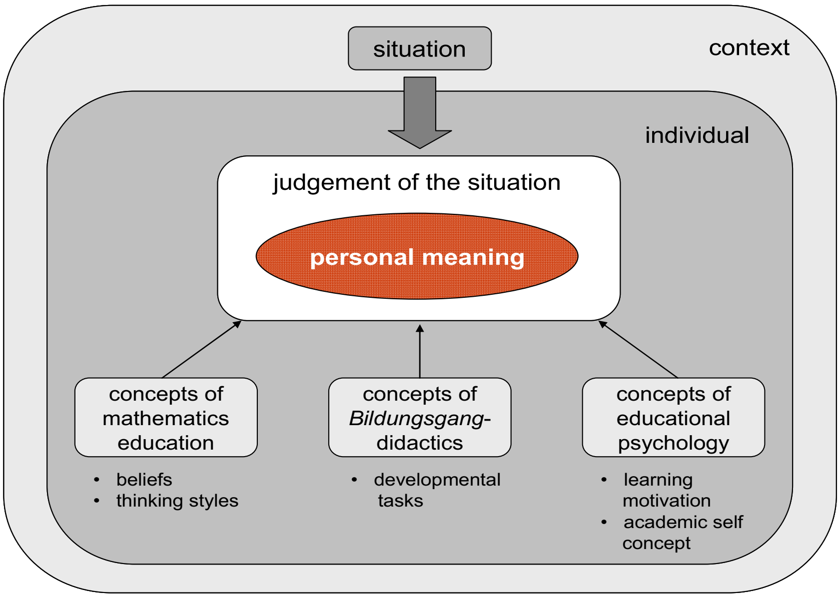


Figure 1: Relational Framework of Personal Meaning

Let us assume that we have a context in which an individual is dealing with a certain situation. In school we might have a certain learning context in which a student deals with a mathematical problem in class. This student judges the situation implicitly with respect to her personal attributes and her goals. She implicitly answers the question whether dealing with the situation does make sense for her, i.e. whether it is personally relevant or not. She also considers in which way an action which could possibly follow might affect her personal goals.

Different concepts from mathematics didactics, *Bildungsgang*-didactics, and educational psychology are of relevance for this judgement of the situation. The student might for instance judge differently depending e.g. on her mathematical thinking style (Borromeo Ferri, 2004) and her mathematical beliefs (Grigutsch, 1996; Maaß, 2004). It is also influenced by her interpretation of the developmental tasks she is dealing with at that certain point of time (see Havighurst, 1972; Trautmann, 2004). In addition, the judgement is influenced by different aspects of learning motivation (see Wild/Hofer/Pekrun 2001). Learning motivation here functions as a kind of cover term to summarise concepts which are relevant for learning processes. These include personal and situational interest, achievement motivation, intrinsic and extrinsic motivation, social motivation, and goal orientation. Also, the academic self concept, i.e. the students' own judgement of her abilities in mathematics and her perception of them (Möller/Köller, 2004), is a relevant concept for the process of constructing a personal meaning. The construction of a personal meaning therefore can be seen as a complex phenomenon which is embedded in a relational framework with concepts from mathematics didactics, *Bildungsgang*-didactics and educational psychology.

PERSONAL MEANING, CONTEXT AND CULTURE

It has already been mentioned that there are different kinds of context which are important for personal meaning. It is now to be specified further how the term *context* is understood in this study and how there is a relation to (learning) *culture*.

The context of a learning situation does not only consist of the plain subject context as also both, the situation in the classroom and the students' prior knowledge and experience are relevant for the construction of personal as well as objective meaning. (Kilpatrick et al., 2005b) These experiences and knowledge as well as the expectations students have of the learning situation are also part of its context.

> What counts as context for learners […] is *whatever they consider relevant*. Pupils accomplish educational activities by using what they know to make sense of what they are asked to do. As best they can, they create a meaningful context for an activity, and the context they create consists of whatever knowledge they invoke to make sense of the task situation. (Mercer, 1993, pp. 31-32, italics in original)

Therefore the students decide which information and experiences are relevant for them to deal with the task posed. These are, however, object to cultural influence as culture has a strong impact on the way how learning takes place in any learning situation.

According to Mercer, learning in the classroom depends both on culture and context as it is, "(a) culturally saturated in both its content and structure; and (b) accomplished through dialogue which is heavily dependent on an implicit context constructed by participants from current and past shared experience." (Mercer, 1993, p. 43) Both, culture and context of a learning situation are very different in the East Asian and the Western traditions as they are based on Chinese/Confucian and Greek/Latin/Christian traditions respectively (Leung, 2001). Leung (2006) examined a number of different characteristics of the Chinese/Confucian culture to see whether they can provide an explanation for the great differences in student achievement shown in large scale comparative studies where students from East Asia outperformed Western students (Fan/Zhu, 2004). Leung shows that

> there are indeed different cultural values pertinent to education that may explain the differences. This is of course no proof that differences in student achievement are caused by cultural differences. But in the absence of clues from variables at other levels, it is probable that culture does matters [sic]. (Leung 2006: 44)

Therefore, it can be stressed that "the impact of cultural tradition is highly relevant to mathematics learning" (Leung et al., 2006a, p. 7).

The differences between East Asia and the West may be based in their different ideas of education. According to Zheng (2006), China is chiefly social-oriented with a rather prescriptive nature. The West on the other hand focuses rather on the personal development of the students so that the educational system is mainly individual-oriented (Zheng, 2006). Based on these different ideas of education, the great differences between Oriental/East Asian and Western countries can be summed up along the following lines:

> high vs. low pressure of examination, teacher-centredness vs. student-centredness, emphasizing exercise vs. emphasizing understanding, over-loaded vs. less homework, formal deduction vs. informal reasoning, stressing imitation vs. stressing innovation, working hard for reducing individual differences vs. polarization, and so on. (Zheng, 2006, p. 385)

Along these lines, great differences in education can be found in Germany and Hong Kong. Therefore, also the learning contexts of mathematics are supposed to be very different. This finally influences the students' processes of constructing personal meaning which, in turn, may presumably result in different types of personal meaning the students construct with relation to mathematics. The aim of this study is therefore to contrast different types of personal meaning made in Germany and Hong Kong.

METHODOLOGY

As there is a research gap concerning personal meaning, it is at near hand to work with qualitative methods to enlighten the concept. To get the data needed for the study, three classes of grade 9/10 were visited. In Germany, these were classes of the *Gymnasium*, the highest achieving school type in the tripartite school system; in Hong Kong, the collaboration was done with EMI-schools (English as Medium of Instruction). Therefore, it was possible to interview the Hong Kong students in English.

The classes were visited for one week. Every mathematics lesson the students had in this time was videotaped with a two-camera-design: one camera was fixed in front of the class focussing the students, the other one was also in front of the class but moving, i.e. following the interaction in the classroom. After every lesson a sequence of five to ten minutes in which the students learnt something new was cut from the material. These situations were chosen as new processes of construction of personal meaning are very likely to occur.

During free lessons, lunch break or after school, volunteering students were interviewed for 45 minutes in average. In total, 16 interviews were done in Germany, 17 in Hong Kong. The interviews always started with the video sequence of the last lesson. The students were asked to tell what was in their minds when they were sitting in class and what came to their minds when they were watching the sequence. After this stimulated recall (Gass/Mackey, 2000), a guided interview was done, which was structured by the interviewee. This means that the guide of questions was not followed in strict order but the order of questions was varied depending on the answers of the respective student. The guide contains questions with relation to the different concepts from the relational framework discussed above as well as, among others, connotations to mathematics and mathematics lessons, and the affective components of learning mathematics.

The data is being evaluated with the help of Grounded Theory (Glaser/Strauss, 2005; Strauss/Corbin, 1996). Similar to Tiefel (2005), the coding paradigm was adapted to fit this study.

The aim of this study is to reconstruct different types of meaning from authentic students' expressions. Ideal types of personal meaning will be developed with the theory of Kelle and Kluge (1999). As these will be idealised types, the single student will presumably not exactly fit into them but the types of personal meaning will as pillars put up a field of personal meaning. The students will exemplarily be located to illustrate the field.

The following exemplification is based on first preliminary results of the study. The dominant personal meaning of one Hong Kong student is presented to illustrate the concept of personal meaning.

EXEMPLIFICATION

William is 15 years old and attends a private EMI-school in Hong Kong. He is very good in mathematics and likes the subject very much. According to his own judgement, he primarily does mathematics as it is a subject at school. Had it not been a subject, he would not have come into contact with it. Therefore, he acknowledges the school's and curriculum's great importance for learning mathematics.

William's dominant personal meaning constructed in the context of learning mathematics can be described as *perception of his own competence*. His own achievement, e.g. being the 'faster one to finish' as he puts it, is very important for him. It is, however, astonishing that he hardly speaks of competition although competition is implicitly and explicitly a very important factor in Hong Kong lessons. It is therefore probable that it is merely important for William to experience his own competence rather than to experience that he is better than his classmates. This may also result from the fact that William is a high-achieving student who knows his position among his classmates. Comparison with them may therefore take a back seat. When asked when he is pleased with himself in a mathematics lesson, he therefore answers among other things: 'answering a questions from my classmate eh … because they have difficulties and I can explained to them'.

Due to his high achievements in mathematics and his desire for experiencing his competence, William is looking for challenges in the lessons. He for instance wants to find out the relation between mathematics and daily life on his own. He says: 'I don't want they [the teachers, MV] told us. Because ehm I'm I'm th- I think that …

they should think it by m- ourselves. This can increase our thinking logic thinking ability.' He, however, understands that the teacher has to show this relation for lower achieving students to enable them to participate in the lessons. William also provides himself with another challenge with his general refusal to use the calculator. The calculator enables him to quickly come to the results needed but denies the feeling of success which is so desperately sought by William. He says: 'I don't like using calculator eh because ehm using calculator is ... although it's fast but eh it's not ... success ... eh not there is not a feeling of successful so I like calculating by myself.'

Similarly, William wants to deeply understand the content taught in the lessons. It is not enough for him to memorize formulae or facts as it is done in subjects like history or geography. He says: 'Doing the … formula, solving the f- the formula eh is … make me feel … confidence. … Eh increase myself on this', and further: 'Mathematic lesson: no need to […] to remember all the things eh is ... just calculating and … observation to to the graph eh ... and is more eas- I think is more easier eh but ... interesting.' He wants to understand mathematics on a deeper level so that, when he does, the subject seems to be comparatively easy for him. This is also the case why dealing with formulae and reading information from graphs gives him more trust in his own mathematical skills. It also shows him how interesting mathematics can be and how good it is to train the ability to think logically. This is why William has an apparently good feeling after a mathematics lesson: 'after the mathematics ... lesson I go out to the corridor I feel very ... happy and ... ehm (2 sec) I have ... confidence. Yes, because ... eh maybe the logic thinking is ... eh I can do for the questions.'

FINAL REMARKS

As has been presented, the concept of personal meaning is quite complex and is related to concepts from mathematics didactics, *Bildungsgang*-didactics, and educational psychology. On the other hand, personal meaning is supposedly influenced by the different kinds of context and the (learning) culture in which it is constructed. Therefore, it is expected to find different kinds of personal meaning in Germany and Hong Kong.

As the analysis of the data with Grounded Theory is quite time-consuming, it is only possible to present first preliminary results of the study in this paper, i.e. the personal meaning of one student from Hong Kong. Therefore, nothing can yet be said about how different kinds of mathematical beliefs, mathematical thinking styles, or different ways how to deal with developmental tasks influence the construction of personal meaning. As William's dominant personal meaning can be described as *perception of own competence*, it is, however, obvious that the academic self concept can be highly influential for the construction of personal meaning.

William is a student who can also be found in Germany. He is high achieving in mathematics, interested in the subject, and engaged in the lessons. He especially likes mathematics lessons because there it is possible for him to have a feeling of success and other positive experiences by perceiving his competence. This dominant personal meaning, i.e. the perception of competence, is therefore at near hand for very good students and can also be expected to be found in Germany.

BIBLIOGRAPHY

Blumenberg, H.: 1999, *Die Lesbarkeit der Welt*, Suhrkamp, Frankfurt am Main.

Borromeo Ferri, R.: 2004, *Mathematische Denkstile*, Franzbecker, Hildesheim.

Brousseau, G.: 1997, *Theory of Didactical Situations in Mathematics*, Kluwer, Dordrecht.

Fan, L. and Zhu, Y.: 2004, 'How Have Chinese Students Performed in Mathematics? A Perspective from Large-Scale International Comparisons', in L. Fan, N.Y. Wong, J. Cai and S. Li. (eds.), *How Chinese Learn Mathematics: Perspectives From Insiders*, World Scientific Publishers, Singapore et al.

Gass, S.M. and Mackey, A.: 2000, *Stimulated Recall Methodology in Second Language Research*, Lawrence Erlbaum, Mahwah, NJ.

Gebhard, U.: 2003, 'Die Sinndimension im schulischen Lernen: Die Lesbarkeit der Welt', in B. Moschner, H. Kiper and U. Kattmann (eds.), *PISA 2000 als Herausforderung*, Schneider-Verlag Hohengehren, Baltmannsweiler, 205-223.

Glaser, B.G. and Strauss, A.L.: 2005, *Grounded Theory*, Huber, Bern.

Grigutsch, S.: 1996, *Mathematische Weltbilder von Schülern*, Dissertations-Druck Darmstadt, Darmstadt.

Havighurst, R.J.: 1972, *Developmental Tasks and Education*, Longman, New York, NY.

Howson, G.: 2005, ''Meaning' and School Mathematics', in J. Kilpatrick et al. (eds.), *Meaning in Mathematics Education*, Springer, New York, NY, 17-38.

Kelle, U. and Kluge, S.: 1999, *Vom Einzelfall zum Typus*, VS Verlag für Sozialwissenschaften, Wiesbaden.

Kilpatrick, J., Hoyles, C. and Skovsmose, O. (eds.): 2005, *Meaning in Mathematics Education*, Springer, New York, NY.

Kilpatrick, J., Hoyles, C. and Skovsmose, O.: 2005a, 'Introduction', in J. Kilpatrick et al. (eds.), *Meaning in Mathematics Education*, Springer, New York, NY, 1-8.

Kilpatrick, J., Hoyles, C. and Skovsmose, O.: 2005b, 'Meanings of 'Meaning of Mathematics'', in J. Kilpatrick et al. (eds.), *Meaning in Mathematics Education*, Springer, New York, NY, 9-16.

Kilpatrick, J., Hoyles, C. and Skovsmose, O.: 2005c, 'Communication and Construction of Meaning', in J. Kilpatrick et al. (eds.), *Meaning in Mathematics Education*, Springer, New York, NY, 129-137.

Leung, F.K.S.: 2006, 'Mathematics Education in East Asia and the West: Does Culture Matter?', in F.K.S. Leung, et al. (eds.), *Mathematics Education in Different Cultural Traditions*, Springer, New York, NY, 21-46.

Leung F.K.S., Graf, K.-D. and Lopez-Real, F.J. (eds.): 2006, *Mathematics Education in Different Cultural Traditions*, Springer, New York, NY.

Leung, F.K.S., Graf, K.-D. and Lopez-Real, F.J.: 2006a, 'Mathematics Education in Different Cultural Traditions', in F.K.S. Leung et al. (eds.), *Mathematics Education in Different Cultural Traditions*, Springer, New York, NY, 1-20.

Leung, F.K.S.: 2001, 'In Search of an East Asian Identity in Mathematics Education', *Educational Studies in Mathematics* 47 (1), 35-51.

Maaß, K.: 2004, *Mathematisches Modellieren im Unterricht: Ergebnisse einer empirischen Studie*, Franzbecker, Hildesheim.

Mercer, N.: 1993, 'Culture, Context and the Construction of Knowledge in the Classroom', in P. Light and G. Butterworth (eds.): *Context and Cognition*, L. Erlbaum Associates, Hillsdale, NJ, 28-46.

Möller, J. and Köller, O.: 2004, 'Die Genese akademischer Selbstkonzepte: Effekte dimensionaler und sozialer Vergleiche', *Psychologische Rundschau* 55 (1), 19-27.

Strauss, A.L. and Corbin, J.: 1996, *Grounded Theory*, Beltz, Weinheim.

Tiefel, S.: 2005, 'Kodierung nach der Grounded Theory lern- und bildungs-theoretisch modifiziert', *ZBBS* 1, 65-84.

Trautmann, M. (ed.): 2004, *Entwicklungsaufgaben im Bildungsgang*. VS Verlag, Wiesbaden.

Vorhölter, K.: in press, 'Personal Meaning in Relation to Modelling Problems', in *CERME 5: Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education 21 – 26 February 2007 Larnaca, Cyprus*.

Wild, E., Hofer, M. and Pekrun, R.: 2001, 'Psychologie des Lerners', in A. Krapp and B. Weidenmann (eds.), *Pädagogische Psychologie*, Beltz PVU, Weinheim, 207-270.

Zheng, Y.: 2006, 'Mathematics Education in China: From a Cultural Perspective', in F.K.S. Leung et al. (eds.), *Mathematics Education in Different Cultural Traditions*, Springer, New York, NY, 381-390.