NEW-MATH INFLUENCES IN ICELAND

Selective Entrance Examinations into High Schools

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ABSTRACT

The New Math was implemented in Iceland with the intention to facilitate understanding in the midst of increased demands for education for all. The article contains an analysis and comparison of typical papers in a selective entrance examination into high schools before and after the implementation period of the New Math during 1966–1968, and a discussion on the understanding it was expected to facilitate.

1 Introduction

An entrance examination to high schools in Iceland was established in 1946, intended to provide equal opportunities for education. By the mid-1960s it became considered a hindrance on young people's path to prepare for life. In the midst of increased demands for education for all, New Math was implemented, expected to facilitate understanding. The article contains an analysis and comparison of typical examination papers before and after the implementation period of New Math during 1966–1968.

The questions that arise concern educational expectations that can be gleaned from the examination papers: what content and performance expectations were considered optimal preparation for further studies; what changes did the implementation of the New Math bring, and did they promote better understanding?

2 Background

2.1 The history

Iceland belonged to the Danish realm from late 14th century until 1944 when the Republic of Iceland was established. From around 1800, one secondary level school was run in Iceland under the Royal Danish Directorate of the University and the Learned Schools. When Danish learned schools were split into language-history and mathematics-physics streams in 1877, Icelanders chose a language-history stream for their sole learned school. A mathematics-physics stream was established in 1919 and the teaching of Euclidean geometry was restored. In 1903, the Danish school system was split into a lower and upper secondary level, while this system alteration was not implemented in Iceland until 1946. The upper level will from now on be called high school. In the 1920s, educational opportunities in Iceland consisted of the sole six year school with a selective entrance examination, and several local technical schools. In the early 1930s, another high school and a number of lower secondary schools were established in towns and rural areas, providing general education such as arithmetic and languages, but without pathways to the high-school level.

In 1946, the new-born Republic of Iceland issued new education legislation (Law no. 22/1946). Its goal was to create a uniform educational system with eight-year compulsory education and equal access to high school education. A clear route from primary level to high-school level was created with a national high school entrance examination in eight main subjects. The examination was run in lower secondary schools all around the country. The high-school authorities were dissatisfied that their former six-year program was reduced to four years, and that they were deprived of selecting their students. As a compromise, regulations (no. 3/1937) for the former six-year schools' second year were chosen as a basis for that official examination.

2.2 The conditions of the national entrance examination to high schools

The goal of the examination was not officially defined at the outset in 1946, but was later analysed to be as follows: to ensure a certain and standardized minimum knowledge in a number of subjects; the selection of the fittest with respect to certain attributes, considered necessary for those who wanted to study in a high school and a university or other higher education; to offer all students and their relatives a certain assurance of an impartial assessment; and identical examinations for all students (Pálsson and Ólafsson, 1961). Assurance of an assessment by impartial persons was a reaction to rumours that the selection procedure had not always been on grounds of abilities.

The examination papers were official documents. The papers for the years 1946–1962 (Vilhjálmsson 1952; 1959; 1963) were published, first as an official report, while the latter two publications were made available for individual purchase. Later papers were copied in the schools and are available in the National Archives of Iceland.

2.3 The mathematics examination before New Math

For the first 20 years, the mathematics examination was divided into two parts with equal weight: seen problems and unseen problems, tested two days in a row (Vilhjálmsson, 1952; 1959; 1963; National Archives of Iceland). The seen problems were problems that students had previously solved in class under the supervision and assistance of their teacher. The content of the unseen part of the examination was typically 6–8 problems; 4–6 story problems on area, volume and proportions, some to be solved by setting up equations; and two rather complicated fraction problems with algebraic expressions in denominators. The story problems either described situations in contemporary daily life, or were versions of old problems, even from Leonardo Pisano's *Liber Abaci*, such as the problem of a fox chasing a dog (Sigler, 2003, p. 276). In the national examination's first year it became clear that examining all over the country in Euclidean geometry as prescribed in the former high-school regulations (no. 3/1937) did not work and was dropped thereafter.

2.4 The New Math and the mathematics entrance examination

The mathematics examination was run in a similar form each year until 1966 when the New Math was first introduced at all school levels in Iceland. The draft to a curriculum document (Landsprófsnefnd, 1968, pp. 56–60), preparing for its introduction to the country-wide examination, stated that the aim was to base school mathematics on the basic concepts of the set theory, which simultaneously were simple and general, and increase emphasis on the meaning and nature of numbers and of number computations. Clearly, these changes required a different approach in the national examination classes, where the basis was laid for algebra. Symbolic language of set theory allowed ideas and their relations to be expressed in an exact and clear way. It was desirable to delay the algebra of numbers (i.e. the conventional algebra) until students acquired mastery of the relations of sets and the introduction to set theory.

The only textbook at that time fulfilling the requirements on basic concepts of the set theory and the meaning and nature of numbers and of number computations was *Tölur og mengi*, [*Numbers and Sets*] (Arnlaugsson, 1966; Bjarnadóttir, 2015), specifically written for this purpose. In his forewords, the author stated that the basic concepts of logic and set theory would facilitate understanding, even for small children. The textbook *Numbers and Sets* was used for preparing to the national entrance examination during 1967–1975 together with a conventional textbook on algebra by Ó. Daníelsson (1951), first published in 1927. Set theory had however to be taught for a while as an isolated topic, unrelated to other topics in school mathematics contrary to draft curriculum recommendations, as the basics of number algebra had to be taught as soon as possible in the only one short academic year intended to prepare for the entrance examination and thus for academic studies at high schools and universities.

2.5 Problems and critique of the entrance examination

Until 1960, a constant rate of 20% of the cohort attempted the examination and 13-14% reached high-school-admission minimum grade. By 1969 the rates had risen to 34% vs. 21% (Bjarnadóttir, 2006/2007, p. 421). From 1966 onwards, the examination time was shortened, the seen problems were replaced by small problems, only testing one item each, and the number of problems rose to 50 and later 100 small and often unrelated items, presumably to simplify grading, but also to help the less able students to show basic competences.

After 1960, the examination came under growing attack from prominent persons and researchers. A longitudinal research in 1967 by psychologist Dr. Matthías Jónasson showed a correlation between results on IQ tests and the national examination. Shortly after, Jónasson (1968) wrote that an entrance examination to higher education had for a long time had the role of filtering or selecting, which was neither painless nor infallible. This could be justified in nations with educational institutions in a constant funding crisis, where channelling only the fittest students into higher education might seem the preferable utilization of available educational provisions. However, the preparation time for the national examination was far too short. The teachers needed more time to learn to know the capacity and the diligence of their students and have more opportunity to give them guidance. Moreover, the studies could be more carefully planned. One year only led to too tight a time schedule, pressure and hurried work which a youngster in a formative period could not easily sustain.

Jónasson's opinion, gradually shared by many influential people, was also that the national examination would have to be changed from the ground up. The host of incoherent details that the students were expected to remember was horrifying. Would the answers to such questions be the correct measure of the capacity of youngsters for higher education? What about inventiveness, judgement, reasoning and creativity? Jónasson's critique about incoherent details concerned the examination in eight school subjects as a whole, but could as well be applied to the mathematics examination as it began to develop from 1966 onwards.

2.6. Results in the examination

The entrance conditions were strict. Students had to reach 60% average out of the eight subjects examined. The native language had double weight. During the periods 1952–1955 and 1962–1966, the mathematics average was always lower than the average of all eight subjects. The national total is not available but a survey from 5–8 schools indicates an average difference of 5% from the total average of all subjects. However, exchanging the seen problems in 1966 for shorter problems did not make a difference in this respect. Data from years 1958–1962 for one school with a number of classes indicate that grades for the seen problems were on average about 12% higher than the unseen problems. The total national average is only available during 1968–1973. The difference between total average and mathematics average reduced slightly from 1970, and in 1972, the national mathematics average was higher than the total average by 2% (Bjarnadóttir, 2006/2007, pp. 200–205, 286–288).

3 Exploration of the examination papers

The 30-year period 1946–1975 of national examination may be divided into sub-periods, with different characterizations: the experimental period 1946–1950; the period of traditional mathematics 1951–1965; the transition period 1966–1968; and the New Math period 1969–1975 which divides into two sub-periods, 1969–1972 with one syllabus, and 1973–1975.

3.1 The period 1946 to 1950

The examination paper in 1946 contained six questions:

1. Three merchants buy and sell boxes of oranges. The questions concern all versions of percentage computation.
2. Simplifying a complex algebraic expression, containing all operations and e.g. a need to factorize $a^3 - b^3$ and $a^3 + b^3$.
3. A cylindrical water container, finding its volume and time to fill it.
4. Information about sums, differences and proportions between three groups from which to set up an equation.
5. To draw a circumscribable quadrilateral with the following given: two adjacent sides, the angle between them and a diagonal from that angle.

6. A is an obtuse angle. On one of its arms B lies between A and D, and on the other arm C lies between A and E. Prove that DE > DC > BC.

The examination results were reasonably good in the two former six-year schools but worse in the rural and small-town lower-secondary schools around the country. Euclidean geometry such as presented in questions 5 and 6 disappeared altogether from the examination papers. In 1947, they were replaced by a question that could be solved by arithmetic only, and another complex algebraic expression for simplification. Euclidean geometry had only been taught at high school level from 1919 and not all high school teachers had received such training.

3.2 The period 1951–1965

The examination paper in 1951 also contained six questions.

1. An arithmetic problem similar to that of 1947.
2. A percentage problem as in 1946.
3. A double cylinder problem, somewhat more complex than that from 1946.
4. An algebra problem, solvable by division.
5. An equation to be set up, similarly to the 1946 problem.
6. A fairly complex algebraic equation with two known variables and two unknown and a complex insertion after simplification.

The examination continued until 1960 in a similar format with 6–8 large unseen problems; two or three of composite algebra, an algebraic equation was often one of them; one or two large arithmetic problems; one or two stories to set up equations from; and a composite measurement problem of volumes of pyramids, cylinders, cones or spheres, often applying the theorem of Pythagoras.

From 1961 the unseen problems were 10 in total, and from 1962 the examination was given in two sessions, run in the morning and the afternoon of the same day, in order to provide enough time. The seen problems were posed in a special session the day before as earlier. This was continued through 1965 after which the seen problems were dropped. During the period 1957–1965, the ratio of problems formulated in a story, so-called “word problems”, was 60-71% of the unseen problems but began to decrease from that time on (Bjarnadóttir, 2006/2007, pp. 426–427).

3.3 The period 1966–1968

This was a transition period, preparing for introducing the New Math for all undergoing this examination. In 1966, the seen problems were replaced by smaller problems, only testing one item each. In years 1967 and 1968, two different versions of the examination were presented, one based on former syllabus of arithmetic and algebra, and another one the New Math syllabus based on Arnlaugsson’s (1966) *Numbers and Sets* as a half part against Daníelsson’s (1951) algebra. The syllabus of the New Math contained elements from number theory and set theoretical concepts and corresponding operations. Also, in 1967, the proportion of the cohort attempting the examination reached 30%. One person had previously been external examiner and used intuitive evaluation methods of complex problems (H. Steinþórsson, personal communication, 2003) which had provided assurance of a uniform assessment. This was no longer possible due to the increased number of participants. This situation may have contributed to selection of a number of small problems with right/wrong answers. The ratio of story problems decreased also, especially in the New Math problems, 55% in 1967 and 45% in 1968 (Bjarnadóttir, 2006/2007, pp. 426–427).

3.4 The period 1969–1975

During 1969–1972, only one of the two syllabus options was offered: the combination of algebra and the New Math. The ratio of story problems dropped to 30–40%, and the problems were presented in 50 items. Comprehensive textbooks were introduced from 1973 to present the entire syllabus in one set-theoretical framework. During 1973–1974, one option out of two was the syllabus of 1969–1972, first presented in 1967, and the other option was based on a translated Swedish textbook series, composed on behalf of the Nordic Committee for Modernizing Mathematics Teaching, NKMM. In 1975, the third option was based on a domestic series by H. Lárusson (1974–1976), which remained in widespread use until around 1990. The problems were presented in 100 items, and the ratio of word problems was 20–34% (Bjarnadóttir, 2006/2007, pp. 426–427).

4 Analysis

4.1 Analysis according to the TIMSS framework

We shall analyse a selection of examination papers with unseen problems, as examples of the content and performance expectations in the national examination of its period. We choose 1953, when the examination had become established with traditional mathematics; 1966, right before the implementation of the New Math when the number of participants had grown considerably, and the seen problems had been removed; 1971 when the implementation of the New Math had been established; and one of the three examination versions of New Math from the final year 1975 (National Archives of Iceland).

The analysis is based on a curriculum framework for TIMSS (Robitaille, Mc Knight, Schmidt, Britton, Raizen and Nicol, 1993, pp. 61–66, 75–83). The examination papers differ in format and can only be compared qualitatively. The results of the analysis are presented in Tables 1 and 2. Numbers indicate the numeration of the problems posed in the examinations papers. A caveat is that a number of problems, especially from earlier dates, contain varied content and performance expectations, so they have been classified several times each.

In the paper of 1953 as a representative of the period 1951–1965, the main emphasis of the content was on fractions and decimals, including percentages; measurements, always including volume; and equations and formulas, which were the main examination topics. The content remained basically the same in the 1966 paper, of the transition period 1966–1968.

Table 1. Content of a selection of examination papers

	Year	1953	1966	1971	1975
1	**Content**	6 items	25 items	50 items	100 items
1.1.	**Numbers**				
1.1.1	**Whole numbers**				
1.1.1.1	Meaning (place value and numeration, ordering …)			9, 48-50	
1.1.1.2	Operations	1,3	16-17	24, 27-28	
1.1.2	**Fractions and decimals**				
1.1.2.1	Common fractions (meaning, representation, computations …)	2, 3, 4, 6	2	11, 12, 13	
1.1.2.2	Decimal fractions (meaning, representation, computations …)	3, 4, 6	11, 20-22	10	
1.1.2.3	Relationships of common and decimal fractions				
1.1.2.4	Percentages	3, 4, 5	4, 8, 10, 14, 23-25		1-2, 9-10, 49-52, 95-96
1.1.3	**Integer, rational and real numbers**				
1.1.3.1	Negative numbers, integers and their properties			17	
1.1.3.2	Rational numbers and their properties (terminating, recurring …)				5-6
1.1.4	**Other numbers and number concepts**				
1.1.4.1	Binary arithmetic and/or other number bases			21, 22, 23	
1.1.4.2	Exponents, roots, and radicals			15	
1.1.4.4	Number theory (primes and factorization, …)		1	16, 18-20	
1.1.5	**Estimation and number sense**				
1.1.5.2	Rounding and significant figures				3-4
1.2	**Measurement**				
1.2.1	Units	3	6, 12, 13		
1.2.2	Perimeter, area and volume	3, 4	6, 12, 13		65-68
1.3	**Geometry: position, visualization and shape**				
1.4	**Geometry: symmetry, congruence, and similarity**				
1.5	**Proportionality**				
1.5.2	Proportionality problems	1, 5	8, 20, 21, 22	44-47	7-8, 97-100
1.6.	**Functions, relations, and equations**				
1.6.1	Patterns, relations, and functions (number patters, operations on functions…			13, 24	11-12, 13-14, 17-18, 53-56
1.6.2	Equations and formulas (representation of numerical situations, …operations with expressions, factorization and simplification, linear equations …)	2, 4, 5, 6	3, 5, 7, 8, 9, 11, 14, 15, 16-17, 18-19, 20-22, 23-25	14, 15-20, 25-26, 27-28, 29-30, 31-33, 34-35, 36-38, 39-41, 42-43, 44-47, 48-50	15-16, 19-20, 21-24, 25-26, 27-30, 31-34, 35-40, 43-44, 57-60, 61-64, 65-68, 69-72, 73-76, 77-80, 81-84, 85-88
1.7	**Data representation, probability, and statistics**				
1.7.1	Data representation and analysis				45-48
1.7.2	Uncertainty and probability				89-94
1.9	**Validation and structure**				
1.9.2	Structuring and abstracting (sets, set notation)			1-8, 14, 15-20	11-12, 13-14, 17-18

The content changed considerably with the New Math in the paper of 1971, representing the period 1969–1972. It laid emphasis on numbers and number-theoretical items: whole numbers, negative number, recurring decimals, but no percentages, binary arithmetic and arithmetic in other bases, in addition to number patterns, and sets and set notation.

The content changed in 1975, too, with less emphasis on whole numbers and number bases. Area, volume, and percentages appeared again. Rounding numbers appeared. Number patters and sets and set notation continued, while statistics and probability were new topics.

The ratio between the categories does not emerge through the above categorization. It may however be seen that category 1.6.2., equations and formulas, described in details as many kinds of algebraic activities (Robitaille et al., 1993, pp. 78–79), are presented in more than 50% of all problems in all the examination papers.

The results of analysis of performance expectations are presented in Table 2. Numbers indicate the numeration of the problems posed in the examinations papers.

Table 2. Performance expectation in a selection of examination papers

	Year	**1953**	**1966**	**1971**	**1975**
2	**Performance expectations**	6 items	25 items	50 items	100 items
2.1	**Knowing**				
2.1.1	Representing	2, 4	1	9, 10, 11, 12	11-12, 13-14, 15-16, 17-18
2.1.2	Recognizing equivalents	2	2	8, 21, 22	
2.1.3	Recalling mathematical objects and properties	3, 4	6		
2.2	**Using routine procedures**				
2.2.2	Performing routine procedures	2, 4, 6	10, 14	10, 11, 12, 13, 23, 31-33, 34-35	1-2, 3-4, 5-6, 7-8, 9-10, 19-20, 21-24, 25-26, 27-30, 31-34, 35-40, 47-48, 49-52, 69-72, 73-76, 77-80, 89-90
2.2.3	Using more complex procedures	2, 5	3, 5, 6, 7, 10, 12-13	24, 25-26, 27-28, 29-30, 36-38, 39-41, 41-43	41-42, 43-44, 45-46, 53-56, 57-60, 61-64 81-84, 85-88, 91-94
2.3	**Investigation and problem solving**				
2.3.1	Formulating and clarifying problems and situations	1, 3, 4, 6	4, 5, 6, 8, 9	44-47, 48-50	65-68
2.3.2	Developing strategy	2, 3, 4, 5, 6	5, 6, 7, 8, 9, 11, 12-13, 15, 16-17	44-47, 48-50	95-96, 97-100
2.3.3	Solving	1, 2, 3, 4, 5, 6	5, 6, 7, 8, 9, 11, 12-13, 15, 16-17	14, 15-20, 44-47, 48-50	95-96, 97-100
2.3.4	Predicting				
2.3.5	Verifying	1, 3, 4, 6	15		
2.4	**Mathematical reasoning**				
2.4.1	Developing notation and vocabulary	1, 4, 6	20-22, 23-25		
2.4.2	Developing algorithms	1, 4, 6	20-22, 23-25		
2.5	**Communicating**				
2.5.1	Using vocabulary and notation			1–7	
2.5.2	Relating representations			9	

In all the examination papers, performance expectations were similar: knowing, performing routine procedures without using equipment, performing more complex procedures, and solving problems. Only in the two former papers were students expected to verify their solutions. Notably, what is classified as mathematical reasoning, i.e. developing notation and vocabulary, and developing algorithm, disappeared in the 1971 and 1975 papers. Students were assisted in problem solving situations in word problems by suggesting which concepts to choose as unknowns and to set up equations at a certain step in the solution process. This may have helped students as well as eased the grading process.

Generalizing, conjecturing, justifying, proving and axiomatizing were not yet included in the syllabus, and so not demanded at examination. Communicating was not emphasized either. Only using the New-Math vocabulary and notation was seen, while describing, discussing and critiquing were absent in examination papers.

4.2 Presentation of problems in examination papers

What may not be read from the above tables is how the topics, in particular the new topic of set-theoretical items, were examined. Comparison of the first pages of three examination papers reveals a difference:

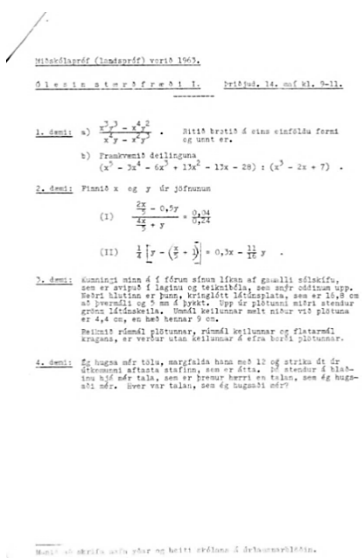

Figure 1. 1963 paper

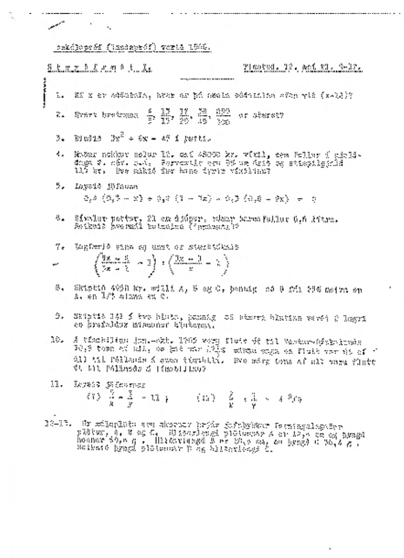

Figure 2. 1966 paper

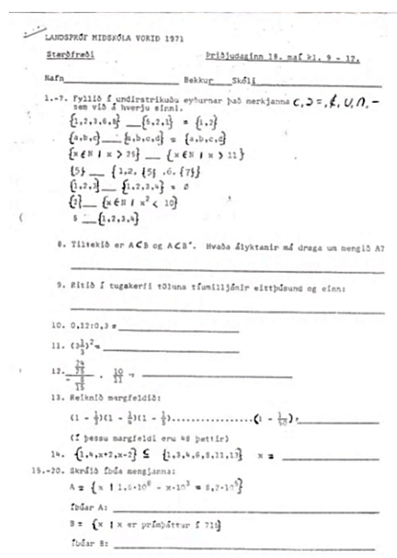

Figure 3. 1971 paper

The first page of the 1963 paper represents the first part of the unseen problems of the period 1946 –1965. All the problems contain either non-trivial algebra or story problems to be solved by setting up equations.

The content on the first page of the 1966 paper replaced the seen problems, and was considered straight forward: ordering fractions by size, finding interests, factorizing, solving an equation with decimal fractions, computing the diameter of a cylindrical pot, dividing two rational expressions, sharing two sums in different ways, solving two simultaneous equations with variables in the denominators, and computing the side and weight of a rectangular prism. The problems on this page count for half the grade of the examination.

Looking at the 1971 paper, the problems count for 16 out of 50 items of the paper. The first seven items, 14%, concern recognizing symbols: $\cup$, $\cap$, $\supset$. $\subset$, $\in$, $-$, $=$. In problems no. 14–20, set-theoretical notation is used to present seven open sentences, other 14%.

5 Understanding

Skemp (1978) distinguished between *instrumental understanding* and *relational understanding*. Instrumental understanding concerned knowing particular items without relating to previous knowledge, while in the case of relational understanding new concepts relate to a network of ideas and previous knowledge. There were also two kinds of mathematics: instrumental and relational. Within its own context, instrumental mathematics was usually easier to understand; the rewards were more immediate and more apparent; and one could often get the right answer more quickly. Relational mathematics, knowing not only what method works in a particular case but why, was however more adaptable to new tasks; easier to remember and relational knowledge could be effective as a goal itself, as the need for external rewards and punishments were greatly reduced. The difficulties in emphasizing relational mathematics and relational understanding lied e.g. in the backwash effect of examination, overburdened syllabi, and difficulty of assessment of whether a person understands relationally or instrumentally. The kind of learning which lead to instrumental mathematics consisted of the learning of an increasing number of fixed plans, by which students can find their way from a particular starting point (the data) to requiring finishing points (the answer to the question). In contrast, learning relational mathematics consisted of building up a conceptual structure (schema) from which its possessor could produce an unlimited number of plans for getting from any starting point within his schema to any finishing point. Skemp suspected that much of what was being taught under the description of ‘modern mathematic’ alias New Math, was being taught and learnt just as instrumentally as were the syllabi which have been replaced. This might happen due to mismatch between teachers whose conception of understanding is instrumental and aims implicit in the content.

Anna Sierpinska (1994, pp. 28–29) distinguished between *acts of understanding* and *processes of understanding*. An act of understanding was to relate mentally an *object of understanding* to another object. According to Sierpinska (1994, pp. 72–73), processes of understanding could be regarded as lattices of acts of understanding, linked by reasonings (explanations, validations). A relatively good understanding could be achieved if the process of understanding contained a certain number of especially significant acts, namely acts of overcoming obstacles specific to that mathematical situation (Sierpinska, 1994, p. xiv).

Another aspect of understanding mathematics was proposed by George Polya (1973) in his book *How to Solve It*. There, Polya suggested a four-step problem solving procedure: Understanding – Devising a plan – Carrying out the plan – Looking back. In this procedure, devising the plan is the hardest, and Polya suggested that one should try to think of a familiar problem having the same or similar unknown. Understanding consisted of realizing what was unknown, which data were available, and what was the condition.

6 Discussion

We have seen that the structure, content and performance expectations in the national high school entrance examination in mathematics developed drastically during the period 1966–1975, as did the population seeking admission. The examination paper in 1965 was similar to what had been conventional from next to the beginning in 1947. The reason was doubtlessly to ensure equality, not only from place to place, but also from year to year. The idea behind training students in seen problems may have been to provide an aid to industrious students to reach acceptable grade, but also to enhance problem solving skills, such as proposed by Polya (1973). The intention would then have been to aid the students in thinking of a familiar problem where a similar procedure could be used. It is also worth noting that most textbooks up through the 19th century did not contain exercises for the students to solve but published solutions attached to all examples for the students to learn from.

By the introduction of New Math, the content became more oriented towards whole numbers, number theoretical items and patterns, such as the draft to curriculum of 1968 suggested. Less emphasis was placed on large story problems demanding multiple approaches. The word problems became fewer and more abstract, and without a story. They were increasingly short, and the number of problems increased inversely with the shortness of the problems. The ratio of word problems to the total problems in the examination decreased markedly, i.e. from up to two-thirds of the examination to less than one-third, even as little as one-fourth. The word problems were replaced by short problems with one right solution, unrelated to each other. This leads to recalling Dr. Jónasson's remarks on the horrifying host of incoherent details that the students were expected to remember, and his doubts that answers to such questions, short of inventiveness, judgement, reasoning and creativity, were the correct measure of the capacity of youngsters for higher education.

Performance expectations became less oriented towards independent development of notation, vocabulary, and algorithms. The students were more often helped to choose variables in order to be able to form equations out of story problems. The seen problems were abandoned. The committee members may have expected that lower-ability students, presumably an increasingly large proportion of the examination candidates when a larger proportion of the year cohort attempted the examination, did better on the single item problems without stories. The needs of those students had previously been met by the seen problems in addition to generous grading for all first attempts at a problem, with increased demands when the scale came closer to full credit (H. Steinþórsson, personal communication, 2003).

The question if implementation of the New Math facilitated understanding is hard to answer. The role of set theory in the curriculum seems primarily have been to exercise notation in order to prepare the students for further studies. At this point it could only be used for minimum problem solving. Clearly there was not time in one academic year to postpone the introduction of algebra of numbers until the students had acquired mastery of the relations of sets as was proposed in the curriculum document of 1968. The role of set theory to increase clarity and facilitate understanding as Arnlaugsson (1966) had hoped was not relevant as yet.

Examination papers are not intended to be a learning material that could promote relational understanding as defined by Skemp, or Sierpinska's processes of understanding. The role of the national examination was to test if a certain and standardized minimum knowledge had been attained. The papers were, however, officially published in print until 1962, and in many schools, a great deal of the time in class during that only one preparation year was spent on reciting old examination problems. Jónasson remarked that the preparation time for the national examination was far too short. The teachers needed more time to learn to know the capacity and the diligence of their students and have more opportunity to offer guidance to their students. One year only led to overly tight a time schedule, pressure and hurried work. Skemp remarked that the backwash effect of examination, and overburdened syllabi promoted the more superficial instrumental understanding at the cost of relational understanding. Both Skemp's and Jónasson's reasoning point to conditions that could promote instrumental understanding. In this respect it is worth mentioning that the national entrance examination was considered of high importance, both for the students and for the individual schools, and the best qualified and most experienced teachers were chosen for instruction leading up to it. But even the best teachers' conception of understanding may have been instrumental and they may even have considered mathematics itself instrumental. As Skemp remarked, instrumental mathematics was usually easier to understand; the rewards were more immediate and more apparent; and one could often get the right answer more quickly.

One wonders if the long story problems from textbooks and previous examination papers could provide opportunity for teachers to delve deeply into composite problems together with their students and even create by them a lattice of acts of understanding, using Sierpinska's vocabulary. Possibly, obstacles specific to the mathematical situations presented in the examination papers were too difficult to overcome at that point of time in many youngsters' lives. Reality shows that the introduction of shorter problems moved the average of mathematics grades closer to the average grade in all school subjects. The mathematics examination thus ceased to be the blame for not reaching the desired goal, 60% average grade to be qualified for high school entrance. The question remains if this new trend of shorter problems with less concrete content and more diffused focus affected the students' attitude towards mathematics for the better or the worse.

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