

The Implementation of the 'New Math' in Iceland: Comparison to its Neighbouring Countries

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Abstract

The New Math reform movement reached Iceland in the mid-1960s through several channels, mainly from Denmark. In this paper, the implementation and development are compared to those in the neighbouring countries, Norway, Denmark and England, where similar factors were influential: enthusiastic mathematics teachers, believing that the New Math would facilitate understanding, and official bodies, supported by the OECD paradigm of scientific and technological education enhancing social and economic progress. In all the countries the reforms were implemented within a framework of general school reforms, advancing education for all, including mathematics for all, and a dialogue was created between teachers of different cultures. The projects in all the countries were carried out by outstanding individuals. Contrary to the other countries, the experiments burst out of control in Iceland, while a channel for teachers' initiative and creativity was opened up.

Keywords: New Math, school mathematics reform, OECD paradigm, social and economic progress.

Introduction

Several factors made the Icelandic school system unique and distinguished it from other countries from the 1920s until the 1960s: the vestiges of a two-century-old tradition of home-education, a single dominating state high school (out of a total of two or three from 1950), a sole Teacher Training College (which did not support mathematics education and did not supplement it through continuing education), and nearly non-existing university mathematics until the 1970s. The tradition of home- and self-education achieved general literacy, while mathematical literacy was beyond the reach of that system.

The New Math reform movement hit Iceland at the time of a domestic crisis in education, resulting from a rapid transformation of the society from an agricultural tradition to an industrialized one and an internal disquiet associated with urbanization. In addition there were economic adolescence problems of a newly independent nation. Due to the influence of the OECD in the 1960s, it was widely believed that a progressive general mathematical education would transform Iceland into a prosperous society. The OECD paradigm had similar influence in neighbouring countries, but in hardly any of them was society so devoid of mathematical education in the traditional sense.

The implementation of the New Math in Iceland marked a landmark in its history of mathematics education. It was perceived as an intrusion of foreign ideas and values into a relatively isolated society, where traditional ideas about mathematics were basic arithmetic for the general public, high school mathematics for the educational elite, and university mathematics confined to the few preparing to become engineers. This raises the question:

To what extent did the implementation of the New Math in the 1960s develop similarly or differently in Iceland from that in its neighbouring countries, and what explanations can be offered for this?

In the following we shall briefly explain the evolution of the New Math in Iceland compared to England, Norway and Denmark, the countries in the geographical and cultural neighbourhood of Iceland. Denmark has a special position in this respect as Iceland was a part of the Danish realm until 1944, and Icelanders' traditional centre for specialized education remained in Denmark for a long time after its political independence. This piece doesn't attempt to analyze the processes in these countries in full. A great deal of literature is available on those processes. Only some sides, which throw more light on the history of Iceland, are discussed here.

The research was a part of a larger study conducted by the author of this paper (Bjarnadóttir, 2006). The history of the New Math in Iceland is told within the frameworks of the international history of mathematics education, of the history of education in Iceland and of its general history, traced through scholars' published works, reports and documents preserved in official archives. Where applicable, events are explored by referring to contemporary articles in journals. Supplementary knowledge was acquired through interviews with persons involved, and from publications such as textbooks, in addition to some personal experiences.

Origin of the New Math

In the aftermath of WWII, many countries considered reforms of their mathematics and science teaching. An outstanding arena was the

Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques, CIEAEM, founded in 1950. Among its members were the Swiss psychologist Jean Piaget, the mathematicians Hans Freudenthal from the Netherlands, Jean Dieudonné and Gustave Choquet from France, and outstanding secondary school teachers Emma Castelnuovo from Italy, Lucienne Felix from France and Willy Servais from Belgium. The main concern of the CIEAEM was a growing attention to the student and the process of teaching, the relevance of psychology in mathematics education, the key role of concrete materials and active pedagogy, and Piaget's research of the relation between mental and mathematical structures as introduced by the French Bourbaki group of mathematicians, including Dieudonné, called *Mathématique Moderne*, Modern Mathematics (Furinghetti, Menghini, Arzarello and Giacardi, 2008).

By the end of the 1950s, widening discontinuities between the mathematics taught at universities and that taught in lower schools were beginning to give rise to curriculum reform projects in various countries that collectively became known as the New Math. Concerns about school mathematics curriculum triggered research studies as people sought evidence to substantiate some of their proposals for development. Mathematicians and psychologists were brought together in curriculum development projects and studies were undertaken that drew upon both perspectives. A revival of interests in issues such as learning by discovery, readiness for learning, processes of learning and aptitude for learning, helped people from different disciplines to see some common ground (Kilpatrick, 1992).

The actions by CIEAEM and the New Math movement had roots in common with the Bourbaki School: set theory, functions, relations and logic were to find their places in the new curricula, supported by the methodology of discovery. A conference was held in Woods Hole, Massachusetts, United States, in September 1959, where university professors in mathematics and natural sciences met professors of psychology and pedagogy to discuss development of reform projects in mathematics and natural sciences. Among the participants were Bärbel Inhelder, Piaget's collaborator, Edward Begle, the director of the extensive School Mathematics Study Group project, SMSG, and Carl Allendoerfer. Jerome Bruner gave the reforms a psychological and learning-theoretical basis in his book *The Process of Education* (Bruner, 1960).

The various reform movements gathered at a seminar on school mathematics reform in November 1959, held by the Organisation for European Economic Co-operation, OEEC, at Royaumont, France. The member countries and the United States, Canada and Yugoslavia were invited to send three delegates: an outstanding mathematician, a mathematics educator or person in charge of mathematics in the ministry of education, and an outstanding secondary school teacher of mathematics. The seminar was attended by representatives

from all the invited countries except Portugal, Spain and Iceland (OEEC, 1961, pp. 7, 213–219). Among its guest speakers were Dieudonné, Choquet, Felix and Servais from the CIEAEM, Begle from the United States, and the Danish professor Svend Bundgaard. Three Nordic countries were represented there, Norway by Kay Piene and two others; Sweden had two representatives and Ole Rindung represented Denmark. The Royaumont seminar may be considered a continuation of the Woods Hole meeting as several persons attended and/or led both meetings (Bruner, 1960; OEEC, 1961). One of its final recommendations was that the member countries proceeded to reform mathematics teaching according to their needs, and it was recommended to establish as much cooperation as possible (OEEC, 1961, p. 125).

The Nordic participants at the seminar agreed upon organizing a Nordic cooperation on the reform of mathematics teaching. The ideas were presented to governmental bodies and the issue was taken up in the Nordic Council, which decided to set up a committee under its Culture Commission, Nordiska komittén för Modernisering af matematikundervisningen, or The Nordic Committee for Modernizing Mathematics Teaching, abbreviated as NKMM (Bjarnadóttir, 2006, pp. 243–268). Each of the four countries – Denmark, Finland, Norway and Sweden – appointed four persons to the committee, which dominated mathematics instruction in the Nordic countries for most of the 1960s (Gjone, vol. 2, p. 78). The Icelandic university and high-school mathematics teacher Guðmundur Arnlaugsson studied mathematics and worked as high school teacher in Copenhagen during 1933–36 and 1939–45 while Iceland was still within the Danish realm. Arnlaugsson knew Bundgaard from his Copenhagen years. Bundgaard was therefore Iceland's natural contact to the Royaumont Seminar and the NKMM where it was not represented.

The Nordic committee issued a report, (Nordisk råd, 1967a, 1967b), where the goals were written by professor Bent Christiansen from Denmark. The program for the Nordic reform was to analyse the situation within mathematics education in each country, to work out curriculum plans, and to write experimental texts. The committee appointed teams of writers. The focus was on the mathematical content, and the teaching of grades 7 to 12 was its main object. However, mathematics courses throughout the school were to be handled, and the committee contacted for that purpose experts for grades 1 to 6. The NKMM primary-level material was written by primary teacher Agnete Bundgaard, sister of Svend Bundgaard, and her collaborator. Persons from each country would translate and adapt the joint publications to each language. This was only implemented for grades 7 to 12. Norway, for instance, introduced translated series from the School Mathematics Study Group, SMSG, for grades 4 to 6 at primary level (Gjone, 1983, vol. 2, pp. 78–80).

Meanwhile, conferences on school mathematics were held in England with the participation of mathematicians specializing in applied mathematics, delegates from private and national industry, and secondary school mathematics teachers. Discussions were initiated on issues such as a conceived teachers shortage, e.g. in the widely read *The Times*, and the solutions envisioned were on the one hand applied mathematics, especially "modelling", and on the other, "modern mathematics," emphasizing "structure". By late 1961, the need to reform school mathematics was seen as legitimate by actors in a number of arenas, including the industrial and political (Cooper, 1985, p. 230–231).

Several reform projects were established. One of them, the School Mathematics Project, SMP, a multi-school project, was conceived in 1961 to produce new textbooks. Its director was Bryan Thwaites, a professor at Southampton University. It was funded by industrial sources and educational trusts but not from the state. In an interview in 2008, one of its actors, Geoffrey Howson, said that serious emphasis was laid on geometry where they moved over to doing transformation geometry, such as reflections and rotations, and then placed in an algebraic background, by giving matrix representations. Coordinate geometry, which formerly had been an upper level topic, was introduced early. Algebra was, therefore, introduced earlier, providing tools to link the geometric transformations with the algebra and the matrices which were also introduced (Karp, 2009). Probability and statistics were also treated much earlier than usual, which turned out to be a successful modification.

Reform in Iceland

The reform in Iceland began on an experimental basis in 1964 by adopting an American New Math textbook by Allendoerfer and Oakley (1963) in the dominating Reykjavík High School. Arnlaugsson, its head mathematics teacher, had been on a sabbatical leave in the United States and Denmark in the autumn term of 1963 to learn about the New Math. Upon returning home, he was appointed as consultant in a half position to the Ministry of Education for the compulsory school. He made a survey for the ministry, which he interpreted as demonstrating poor standing of children and adolescents in mathematics. This was confirmed by a survey made by physicist S. Björnsson (1966), indicating that the lower secondary syllabus in mathematics, physics and chemistry was markedly behind that in Norway and Denmark (Lárusson, a personal communication, March, 2002; Bjarnadóttir, p. 251–252). Meanwhile the Reykjavík Education Office had been on the outlook for suitable material for the primary level. Its director, Jónas B. Jónsson, had written arithmetic textbooks himself for grades 1–3 of primary level and initiated the search. He sent a mathematics teacher, Kristján Sigtryggsson, to the United States in

1963–1964 to investigate New Math material there, e.g. the SMSG material. Upon his return, Sigtryggsson wrote:

We are fortunate enough to have good public education already and thus be counted among the world's culturally most advanced nations. But every glory is problematic. Now we have to protect our honour and interests in the cultural competition of today; be aware that the requirements are different from what they were. The isolation of the country no longer exists. Interaction with other nations increases every year and our educational institutions are weighed and evaluated in comparison with the best ones abroad (Sigtryggsson, 1964).

Arnlaugsson's and Björnsson's researches, as well as Sigtryggsson's remarks, indicate a concern that Iceland was behind other nations in education. The search for primary school teaching material ended in 1966, with Arnlaugsson presenting NKMM primary level material developed by Agnete Bundgaard, which was implemented on an experimental basis in seven groups in two schools in Reykjavík. It had been tested in grade 1 for two years in Frederiksberg, Denmark, and was being prepared in a final version. Very little translation was needed, as the text consisted mainly of diagrams made for pupils who had not yet learned to read. Texts for grades 2 and 3 were still being tested and the text for grades 4–6, where problems later emerged, did not exist as yet (Gíslason, 1978). Furthermore, Arnlaugsson (1966) wrote a textbook on numbers and sets for college-bound lower secondary school students.

All these activities on primary and lower and upper secondary levels were thus on the initiative and advice of high-school and university teacher Arnlaugsson, who was informed about primary level mathematics material by his former schoolmate, Svend Bundgaard. Arnlaugsson arranged in-service courses on the New Math for primary and secondary level teachers together with a colleague, Björn Bjarnason, using Bent Christiansen's mathematics education textbooks. Thus, a dialogue was established between the elite high-school level mathematics teachers and their counterparts at lower levels (Bjarnadóttir, 2006, pp. 256–260).

Arnlaugsson (1966; 1967; 1971) expressed in his writings expectations that the new concepts would facilitate deeper understanding of arithmetic and mathematics in general, and recommended readings by psychologist J.S. Bruner to teachers (Bjarnadóttir, 2006, p. 266–267). In the foreword to his textbook, Arnlaugsson stated an echo from the Piagetian theories:

The emphasis on skills and mechanical ways of work has moved aside for demands for increased understanding. This development has pushed several basic concepts from logic, set theory and algebra down to primary level. The experience from many places indicates that children – even very young children – can easily adopt these concepts, which previously were

only introduced at university level, and enjoy them. Furthermore, they seem to be conducive to increased clarity and exactness in thinking and arithmetic (Arnlaugsson, 1966, pp. 4–5).

The primary level reform experiment was initially established on behalf of Reykjavík Educational Office with consent of the Iceland Ministry of Education. The experiment in 1966–1967 went well; the teachers were supported by meetings with supervisors, and parents were regularly informed. In 1967, all primary schools in Reykjavík were invited to participate in the experiment and, indeed, many (probably too many) of them did opt to participate. Soon it reached nearly all pupils in Reykjavík and many outside the capital city (Bjarnadóttir, 2006, pp. 267–269).

Figure 1 shows the proportion of the year cohorts that completed the six-year primary level New Math program. Many reverted to the traditional program after three years (Bjarnadóttir, 2006, pp. 295–96).

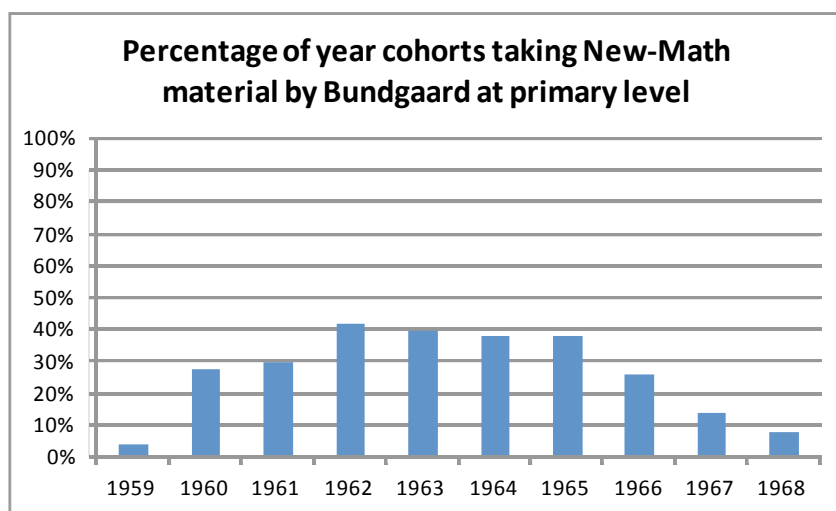


Figure 1. Percentage of year cohorts studying Bundgaard material for six years (Menntamálaráðuneytið, 1977).

The latter part of the Bundgaard-series turned out to be highly theoretical (Høyrup, 1979, p. 59). The commutative and associative laws, Roman numerals and place-value notation to the base five, prime numbers, permutation of three digits and the transverse sum together with its relation to the nine times table were introduced in the third grade. Set theory with pairing, subsets, intersection and union, more place-value systems and geometry in a set-theoretical framework were added in fourth grade. Last but not least, there were algorithms that were different from those Icelanders were accustomed to, especially the multiplication algorithm (Bjarnadóttir, 2006, pp. 293–295).

Bent Christiansen, the author of the NKMM goals, was professor in mathematics at the Royal Danish School of Educational Studies. Icelandic primary teachers had no domestic choice for further education, so a handful

of teachers attended the Royal Danish School with the aim to guide other teachers, among them Anna Kristjánsdóttir, who was supported to study there in 1969–1972 and subsequently to lead the reform at the Reykjavík primary schools. One of her first tasks was to translate the NKMM lower secondary level geometry textbook (Bergendal, Hemer and Sander, 1970).

The reform soon became a part of a larger scheme of redefining the Icelandic school system by a new School Research Department, SRD, in the Ministry of Education. Minister of Education, Gylfi Th. Gíslason, ensured financial support to the reform activities. Gíslason was Minister of Commerce as well. He was aware of theories, advocated by the Organisation for Economic Co-operation and Development (OECD), that science and mathematics education would enhance social and economic progress. For example, a representative from the OECD visited Gíslason's ministry in 1965 to present these ideas (Efnahagsstofnunin, 1965). The SRD decided in 1968 to extend the reform to a national entrance examination to the high schools by prescribing the textbook by Arnlaugsson (1966) as the basis for the examination. A mathematics teacher, Hörður Lárusson, was supported to study in the United States, to lead the reform at the SRD from 1969 and to write textbooks for grades 7 to 9 to follow up the Bundgaard material (Lárusson, a personal communication, March, 2002).

Indeed, the decision process was a rather informal one. The Ministry of Education was actually only a small office and Arnlaugsson in his part-time post was at first its only specialist in mathematics teaching. There were no boards or committees to offer advice, have discussions, or make decisions. The Reykjavík High School and Arnlaugsson were highly respected and his advice was followed, both by the Reykjavík Education Office and the ministry, but only a few more people were involved. Arnlaugsson went on to lead a new project of a modern high school in Reykjavík and was no longer a consultant after 1966. The work was later continued by Lárusson and Kristjánsdóttir.

The reform in Iceland slowed down gradually during the 1970s. When the primary school reform had reached a large proportion of the year cohort, the leaders of the reform were not able to guide the teachers who generally did not have specialized education as mathematics teachers. Arnlaugsson (1967) warned teachers that the New Math was not about one new method in addition to the old ones. Parents, the public, and even the teachers, did not understand the point of the New Math and the SDR began in 1971 to prepare new material for the primary level upon the initiative of primary school teachers and, from 1976, for the lower secondary level under the leadership of Kristjánsdóttir. The actors of the New Math were reverting from its orthodox form of set theory and structure, returning to a more child-centred form of emphasis on investigation and use of concrete material.

The three high schools that existed in the 1950s were in many respects autonomous in choosing their syllabus. The American New Math textbook by Allendoerfer and Oakley (1963) was soon discarded and experiments were made with products of the NKMM. For a while the English School Mathematics Project, SMP, was tested and partly translated for vocational streams of the upper secondary level. It contained novelties that were to last in Icelandic curriculum: probability, statistics, coordinate-geometry and geometric transformations.

In parallel with mathematics reform, the school system, administered by the SRD, was in flux until the 1980s. The high schools in Iceland had been organized within the Danish high school system and continued to be adapted to it until around 1970, while specialized education was sought in Denmark. There was an enormous increase in the number of pupils seeking education after compulsory school. The national entrance examinations into the high schools, which had been implemented in 1946 as an act of justice, began to be considered an obstacle to the education of young people. The high school level was opened up to all, first modeled after a higher preparation programme, Højere forberedelse, HF, established in Denmark in 1966 (Indriðadóttir, 2004; Branner-Jørgensen, 1981, p. 195). The choice of studies extended from college preparation to include a variety of technical and vocational studies. By 1988, the number of high schools had reached 23 in addition to several one and two year programs, acting as bridges in the rural areas from the compulsory schools to the high schools (Bjarnadóttir, 2006, pp. 322–326).

Textbooks in foreign languages, which had been the rule at that level, now became great obstacles, and young teachers at the extended high school level began to translate and adapt teaching material to Icelandic circumstances. Some of the NKMM textbooks, especially Danish books by Kristensen and Rindung (1962), were considered too theoretical, and were exchanged for Swedish textbooks (Bergendal, Håstad and Råde, 1970), which were later translated. Many experiments were made and the individual high schools each tried to adapt their syllabus to their students. Legislation on the new situation on that level was only issued in 1988. Mathematics reform, too, must be considered within this context. The process allowed space for initiative and creativity for teachers, at the upper secondary level as well as the compulsory level, and the SRD of the Ministry of Education created favourable conditions for such activity by providing funding (Bjarnadóttir, 2006, pp. 276–78, 318–331).

Reforms in the neighbouring countries

Geographically, one might argue that Norway, Denmark and England are not the closest neighbouring countries to Iceland. However, from the cultural point of view, they are. One might also argue that the United States of America should belong to that group. Direct influence from the US in this respect was confined to Allendoerfer and Oakley's (1963) book, so it is left out for simplicity.

Norway

The population of Norway is about 20 times that of Iceland. As in Iceland, the population is spread over a large area, and living conditions are similar in many areas. However, the Norwegians were more self-sufficient than the Icelanders in the post-war period; they had a university with an established mathematics research since 1811, several technical colleges and a well-developed mining and technological industry based on access to inexpensive electricity.

The similarities between the Icelandic and Norwegian education contexts lay in their common centrally organized structure. Textbooks and curricula for the compulsory level were centrally administered in Norway as in Iceland. Also, in Norway, the developmental work was within the frame of extending compulsory education to the age of 16, removing the separation of college-bound students at an earlier age. Iceland and Norway had in common the influences from the OECD policy that mathematics and science education would enhance social and economic progress. The implementation of the New Math reform in Norway was thus influenced by the view that Norway could not stay outside the development going on in Europe and the USA (Gjone, vol. 8, p. 8). Furthermore, the initial work in Norway was driven by individuals. Kay Piene, who attended the Royaumont seminar, was leader of the mathematics experiment committee and principal of the pedagogical seminar of the University of Oslo. With his premature death in 1968 the movement in Norway lost its main spokesman (Gjone, 1983, vol. 3, p. 25).

The differences in the reactions to the foreign educational currents lay in the decision process. In Norway, the implementation of the New Math underwent a process of controlled experiments and discussions about proposals for curricula in boards and committees, in addition to official discussions in parliament and public newspapers, but it never reached a stage of implementation in the general educational system. Compared to Norway, important steps in the implementation process in Iceland were missing. Only few were involved in the decision-making procedure. Far too many teachers and classes were let into the primary school experiment already in its second year. Its leaders could not provide the necessary guidance and control, and no national curriculum guide included the New Math until a preliminary one was produced at about the time that the experiment was coming to an end. The process in itself, however, created more knowledgeable personnel for leading further developmental work (Bjarnadóttir, 2006, pp. 388–9).

Denmark

Comparing the process of implementing New Math in Iceland to that in Denmark, the influences are obvious: three sets of Danish textbooks had been introduced in Iceland, Christiansen's textbooks for teacher training were being widely used in the country, the Bundgaard primary school material was presented in schools in Reykjavik, and the Kristensen & Rindung high school series was also being widely implemented. Arnlaugsson was in personal contact with the prime reform promoter in Denmark, Bundgaard, who recommended his sister's textbook series. Primary teachers and Kristjánsdóttir received advanced teacher training at the Royal Danish School of Educational Studies with Christiansen, the author of the goals for the Nordic NKMM project. Christiansen even came up to Iceland in February 1969 to discuss teachers' in-service training and publication of his textbooks (National Archives, 1989/S-56; Bjarnadóttir, 2006, p. 389).

However, the context was different, and the similarities are doubtful. In Denmark, a new handbook for teachers, the Blue Report/*Den Blå Betænkning*, prepared by school-level educators, had been published recently, tailored to the Danish system of primary and lower secondary education. Its implementation was disrupted by the New Math reform wave (Høyrup, 1979). In Iceland there was nothing to disrupt. No one was considered qualified to write teaching material from scratch, and thus people were looking for suitable material to translate. The Bundgaard material was translated and deployed with minimal modifications in its Icelandic adaptation. It was hardly a choice built on a debate or a broad unanimity among a number of people.

During the first few years, Icelandic classical high schools were "saturated" with the New Math, as were Danish high schools (Mogens Niss, a personal communication, October, 2004). While Arnlaugsson and Bjarnason were the leading mathematics teachers, there were direct Danish influences, but also some Anglo-Saxon. The orthodox series by Kristensen and Rindung was quickly abandoned for a more moderate approach, except with the most gifted college-bound pupils, and the Danish influences gradually dwindled.

Similar problems were dealt with at the secondary level in both countries, more due to changes in social settings than to direct influence. Primary and lower secondary education were being unified, thus separating lower secondary education from high school and its immediate influences. In both countries, ways and means were established to cope with mounting attendance at the upper secondary level, and for both youth and adults to bypass the traditional high school in order to acquire higher education and enter vocational training.

In Denmark there was a long-standing tradition of high schools to build upon. The explosive increase in attendance at the high school level was handled there in a formal way with legislation acts, regulations and curriculum guides, for example by establishing the higher preparation programme, HF, by legislation in 1966 (Branner-Jørgensen, 1981). In contrast,

a series of provisional laws, regulations and curricula were passed in Iceland, and the upper secondary level was legally in an experimental state until 1988; while experience and various influences were shaping it, a formal policy was not stated,. No Icelandic mathematics textbook series won the market, and translated Swedish and later Norwegian textbooks prevailed (Bjarnadóttir, 2006, pp. 389–390).

While the Royal Danish School of Educational Studies was influential in implementing the New Math at the compulsory level in Denmark, the Teacher Training College in Iceland played a minimal role in the implementation of the New Math. One of the main differences in Iceland from Denmark was the small size of the discussion forums in Iceland, which relied greatly on the individuals leading them. There is an Icelandic saying which goes “a person replaces a person,” but Icelandic society was very vulnerable in that respect. Those who really adhered to the philosophy of the New Math, Arnlaugsson and Bjarnason, worked hard at teacher in-service courses for only a few years. Compared to Denmark, one could say that in a sense, New Math in its orthodox form never reached the heart of the Icelandic mathematics education community. A few persons were enthusiastic, several tried to imitate their ideas, but the majority of ordinary teachers may have considered it much ado about nothing – the methods cumbersome, the explanations wordy, and the result causing a decline in computation skills – all the while disregarding the educational opportunities it brought to Icelandic education (Bjarnadóttir, 2006, p. 390).

England

The population of England is about 170 times that of Iceland. There are a great number of universities and secondary schools of varying prominence in England, influenced by a deep-rooted class-based division of society. Yet, there are some similarities. By the 1950s there were two broad traditions in England: selective and non-selective school mathematics. Two versions of mathematics were taught to two different categories of pupils, largely in different types of schools, by teachers who, broadly speaking, had been educated in two different types of post-school institutions: the university and the teacher training college (Cooper, 1985, p. 63). This picture was, however, becoming more complicated by an increased tendency during the 1950s in secondary modern schools of pupils to be enrolled for the GCE, General Certificate Examinations (Cooper, 1985, p. 42). Considerable class stratification existed in schools at the secondary level in Iceland as well, between high schools and their entrance preparation grade, and the general secondary school. This was demonstrated in the differently rigid syllabi and varying requirements for qualifications of their teachers. At the high schools and their entrance examination grades, university education was required of teachers and fulfilled if possible, while teacher training college education with minimal mathematics education was accepted of professionals at the general secondary level.

The New Math curriculum development in England was, contrary to the Nordic countries, not based on governmental organizing or support, but rather on the initiatives of university professors and mathematics teacher associations, and on support from various private actors in industry and at universities. One of the teacher associations was under the influence of the ideas promoted by the CIAEAM on the relationship between the New Math and the Piagetian theories (Cooper, 1985, pp. 87–95). Several main reform projects emerged – one of them the School Mathematics Project, or SMP. According to Barry Cooper (1985, p. 278), its success was conditioned by both the academic and social status of those involved and by their structurally determined access to resources.

The redefinition of Icelandic school mathematics originated at the dominating Reykjavík High School by Arnlaugsson and his collaborator. The success of Arnlaugsson in promoting the implementation of the New Math at all school levels in Iceland may be considered parallel to SMP's success: the teachers at the Reykjavík High School had the structurally conditioned status and academic legitimacy to redefine school mathematics. Arnlaugsson was able to promote the New Math in Iceland and ensure financial support for requisite resources, first from the Reykjavík Education Office and consequently from the Ministry of Education through its SRD.

The SMP curriculum was introduced in Iceland already in the 1960s in the dominating Reykjavík High School. The SMP textbooks were soon discarded at the Reykjavík High School as they were not considered capable of adequately preparing students for calculus and mathematical analysis at the engineering department of the University of Iceland. Parts and pieces of it were, it should be noted, later translated for the more vocational-oriented high schools.

Proposed changes in both countries were legitimized through a reference to the nation's need for scientific and technological manpower. There was no pressure in Iceland from any industry, but the hope in common to both countries was economic gain in line with the OECD's paradigm of science and mathematics education as an economic resource. There was also the fear of being left behind unless the New Math was implemented, as stated in a quotation from the editorial of the British journal *Mathematics Teaching* in April 1958 (Cooper, 1985, p. 76; Bjarnadóttir, 2006, p. 250). The quotation focuses attention on Piagetian theories as had been echoed by Arnlaugsson:

... much of the psychological work of Piaget suggests that many of the essential notions of modern algebra (which are regarded as a university study) have to form in the pupil's mind before he is even ready to undertake the study of number ... Such topics as the algebra of sets or relations might be taught with a profit not merely in the sixth form but lower down the school as well. In other countries they are learning how to do this, and unless we learn too we shall be left behind (Cooper, 1985, p. 76).

In both countries, university faculty had most to say about the content, and after its implementation, they were also among the first to react negatively. In both countries, problems emerged when the redefinition was attempted at lower educational levels. The age level 11–13 was a common vulnerable area, as it created a conflict between the perspectives of the two types of teachers belonging to the two subcultures, trained at universities vs. teacher training colleges, where the latter were expected to implement a version of mathematics initiated by the former (Bjarnadóttir, 2006, p. 388). In many cases Icelandic primary school teachers, and probably others, missed the point of the reform, which they viewed as just another method to add to the old ones (Arnlaugsson, 1967).

Even if curriculum change innovations operate at many levels, and those involved are concerned with content, pedagogy and the attitudes established, the actual redefinition finally achieved by the actions of those involved in the movements in both countries has probably been primarily one of content. That is, quoting Cooper (1985, p. 281), mathematics teachers in both countries remained “transmission” oriented but new content was, in many cases, being transmitted. In both countries, the redefinition was permanent in the sense that the mathematics syllabi changed, even if the content and the pedagogy ran partly into conventional tracks. For example, the weight of probability and statistics and graphical representation increased.

The International Reform Movement – Discussion

In answering the research question about the extent to which the implementation of New Math developed similarly or differently in Iceland and in neighbouring countries, and the explanations that can be offered for these similarities and differences, the following may be remarked:

In comparing the implementation of the New Math in the above-mentioned four countries, Norway, Denmark, England and Iceland, several traits were detected. In all countries, OECD-theories on prospective social and economic progress were an underlying factor. There was also the factor of mathematicians’ beliefs that the New Math, supported by Piagetian theories on relations between mental and mathematical structures, would bring deeper understanding of mathematics to students. The OECD paradigm also triggered a fear that the nations would lag behind internationally unless implementing the New Math, as was explicitly expressed in Iceland, England and Norway. Moreover, in the three Nordic countries, the projects were carried out by enthusiastic individuals, all of them university or teacher college educators: Bundgaard, Christiansen, Piene, Arnlaugsson and Bjarnason. They mobilized the community around them and exerted influence on governmental bodies. Outstanding individuals also lead the SMP-project in England.

A conjecture also emerges that the original intentions of the New Math curriculum were egalitarian in nature. Coupled with improving national

economy in the post-war era there was a growing grass-roots wish for "education for all" which naturally implied "mathematics for all". A demand for different content in mathematics to serve an emerging technological society had also been created. These currents, to dissolve social stratification, to improve and extend public education, and to improve and alter mathematics education, were developing and amalgamating during the 1940s and into the 1960s. They were, for example, realized in a reformed upper secondary level in Iceland, through the GCE-examination in England and the HF-programme in Denmark, and in new nine-year compulsory school legislation acts in the Nordic countries in the 1960s and 1970s. Nor should the 1968 student uprisings and their social consequences be forgotten in this respect.

That mathematics reform coincided with school reforms was therefore only natural. The disturbing elements were the radical ideas of implementing university conceptions of a unification of the various branches of mathematics, through logic and set theory, in school mathematics. These ideas influenced or disturbed the internal development. In Denmark they disturbed the implementation of a modernized mathematics curriculum, prepared within the school culture. In Norway they went through a long decision process, which finally swayed the national curriculum away from its most orthodox form. In Iceland they caused disturbance, while they also gave rise to reconsideration and new creation during the reactive action. New Math caused, in all the countries in question, conflicts between different cultures within education: on one hand the educational culture of the universities, and on the other hand that prevalent in teacher training colleges. These conflicts were possibly harsher in mathematics than in other school subjects during this egalitarian process.

While the borders between these cultures may not yet be fully dissolved, the events in 1960s and 1970s contributed to a dialogue and communication flow. And in Iceland, where school mathematics had not had any attention since the early 1900s, the implementation of New Math during a meeting of different educational currents, however unfortunate in many respects, contributed to the creation of a long-needed channel for initiative and creativity on the part of the teachers belonging to both cultures.

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