

# Next nearest neighbour coupling with spinor polariton condensates

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We report on experimental observation of next-nearest-neighbour coupling between ballistically expanding spinor exciton-polariton condensates in a planar semiconductor microcavity. All-optical control over the coupling strength between neighbouring condensates is demonstrated through distance-periodic pseudospin screening of their ballistic particle outflow due to the inherent splitting of the planar cavity transverse-electric (TE) and transverse-magnetic (TM) modes. By screening the nearest-neighbour coupling we overcome the conventional spatial coupling hierarchy between condensates. This offers a promising route towards creating unconventional non-planar many-body Hamiltonians using networks of ballistically expanding spinor exciton-polariton condensates.

Strongly correlated quantum many-body systems have attracted a lot of interest as a promising tool to engineer and explore phases of matter in extreme settings [1–3] and to simulate complex Hamiltonians [4, 5]. Such systems include ultracold atomic ensembles [4], trapped ions [6, 7], nuclear and electronic spins [8, 9], superconducting circuits [10, 11], and nonlinear photonic systems [12]. Of interest, recent milestone achievements in programmable connectivity in condensed matter using cold atomic gases [13] now permit construction of intriguing networks of coupled elements. However, in general, many lab systems are by their physical nature unable to form unconventional graph topologies. In the past decade, driven-dissipative Bose-Einstein condensates of exciton-polaritons (from here on, polaritons) in planar microcavities have substantially advanced in optical reprogrammability [14–21]. There, each condensate is driven by a focused non-resonant optical excitation beam forming a localized macroscopically coherent wavefunction [22]. The coupling strength between neighbouring condensates is roughly given by their mutual overlap with an exponential fall-off as a function of separation distance [23–25]. This means that nearest-neighbour (NN) coupling dominates over next-nearest-neighbour (NNN) coupling making polariton networks inherently planar in a graph topology sense. Overcoming this spatial coupling hierarchy can offer opportunities to observe spontaneous ordering and emergent polariton effects in non-planar graph topologies [26–32]. However, this is extremely challenging, requiring very fine control over the two-dimensional polariton potential landscape with limitations of its own [33].

In this Letter, we demonstrate that spin-orbit coupled (SOC) exciton-polariton condensates can overcome this challenge. Polaritons are quasiparticles exhibiting intermixed properties of excitons and photons, which appear when light and matter are brought to the strong coupling

regime [34]. As a consequence, the photon polarisation is explicitly connected to the polariton pseudospin (or just "spin" for short) with  $\hat{\sigma}_z = \pm 1$  spin-projections along the cavity growth axis representing  $\sigma^\pm$  circularly polarized light. Their two-component integer spin structure has led to deep exploration into nonequilibrium spinor quantum fluids [35]. Polaritons mostly decay through photons leaking out of the cavity containing all the information on the condensate such as energy, momentum, density, and spin. This salient feature allows direct, yet non-destructive, measurement of the condensate spin distribution using polarization resolved photoluminescence (PL) imaging.

Both the polariton condensate and the incoherent photoexcited background of excitons sustaining it adopt the circular polarisation of the nonresonant excitation [36, 37] due to the optical orientation effect of excitons [38, 39] and spin-preserving stimulated scattering of excitons into the condensate [40]. This permits excitation of a condensate of a well defined macroscopic  $S_z \sim \langle \hat{\sigma}_z \rangle$  spin projection [41–44]. Subsequently, the inherent TE-TM splitting of the microcavity [45] will start rotating the spin of any condensate polaritons which obtain finite wavevector and flow away from the pump spot [46, 47]. This is also referred to as the optical spin Hall effect [48, 49]. Namely, the splitting between TE and TM polarized cavity photon modes acts as a directionally dependent in-plane effective magnetic field [48, 50] (i.e., effective SOC [51]) causing the spins of outflowing condensate polaritons to start precessing [see Fig. 1(a) and Fig. 1(b)]. The strength of this effective SOC scales quadratically with the polariton momentum,  $\propto k^2$  and can even be electrically tuned [52, 53]. This makes so-called ballistic condensates ideal for enhanced SOC effects [46, 47] due to their extremely high kinetic energies obtained through repulsive Coulomb interactions with the locally pump-induced exciton reservoir. More-

over, because of their long-range coherent particle outflow, ballistic condensates can couple over macroscopic distances much greater than their respective full-width-at-half-maximum [24] while also preserving their spin information [44, 46, 47].

Recently, it was theoretically predicted that ballistic condensates could invert their neighbour coupling hierarchy, making NNN stronger than NN, through a spin-screening effect made possible by the effective SOC stemming from TE-TM splitting [54]. Here, we provide experimental evidence of these recent predictions. We present a study of a spinor polariton dyad (two coupled condensates) and a triad [three coupled condensates, see schematic Fig. 1(c)] wherein each condensate ballistically emits a coherent pseudospin current which rapidly precesses as it propagates [46, 47]. We demonstrate control over the coupling strength between neighbouring condensates by changing the spatial distance between them (denoted  $d$ ) relative to the spatial precession period of the condensate pseudospin (denoted  $\xi$ ).

We briefly explain the idea of spin-screened polariton coupling. The three peaks in Fig. 1(c) represent the condensate centers excited by three co-localized Gaussian pump spots of equal intensity. The red-blue colour map shows the precession of the polariton pseudospin as it radially propagates in-plane away from each condensate center, with red representing  $S_z = +1$  (spin-up polaritons) and blue representing  $S_z = -1$  (spin-down polaritons). The height of the peaks represents the intensity of the condensate emission. The distance between the condensate centers relative to the spatial oscillations of the pseudospin modifies the coupling between them. In the non-screened state [Fig. 1(c)] NN condensates are excited at a distance equal to integer number of periods of pseudospin oscillations,  $d = n\xi$  where  $n = 1, 2, 3, \dots$ . This means that propagating condensate polaritons arrive at NNs with unchanged spin projection. On the contrary, in the screened state [Fig. 1(d)] NNs are separated by  $d = (n-1/2)\xi$  and polaritons arrive at their NNs with opposite spin-projection which reduces the condensate coupling, while coupling between NNNs is still maintained.

The microcavity used in this study consists of a  $5\lambda/2$  AlGaAs cavity surrounded by two distributed Bragg mirrors (DBR) of 35 and 32 pairs of AlGaAs/AlAs for the bottom and top DBR correspondingly with the 12 GaAs QWs separated into four sets of three QWs placed at the antinodes of electric field within the cavity. The cavity quality factor is around  $Q \sim 16000$  with the corresponding polariton lifetime  $\tau_p \approx 5$  ps and Rabi splitting of 9 meV. The measured TE-TM splitting is  $\approx 0.2$  meV at  $k = 3 \mu\text{m}^{-1}$  in-plane wavevector. Microcavity had the same detuning in all pumping spot positions providing equal values of the condensation threshold for each single isolated condensate. See section S1 in the Supplemental Material [55] for further experimental details.

The normalized Stokes parameters of the cavity emis-

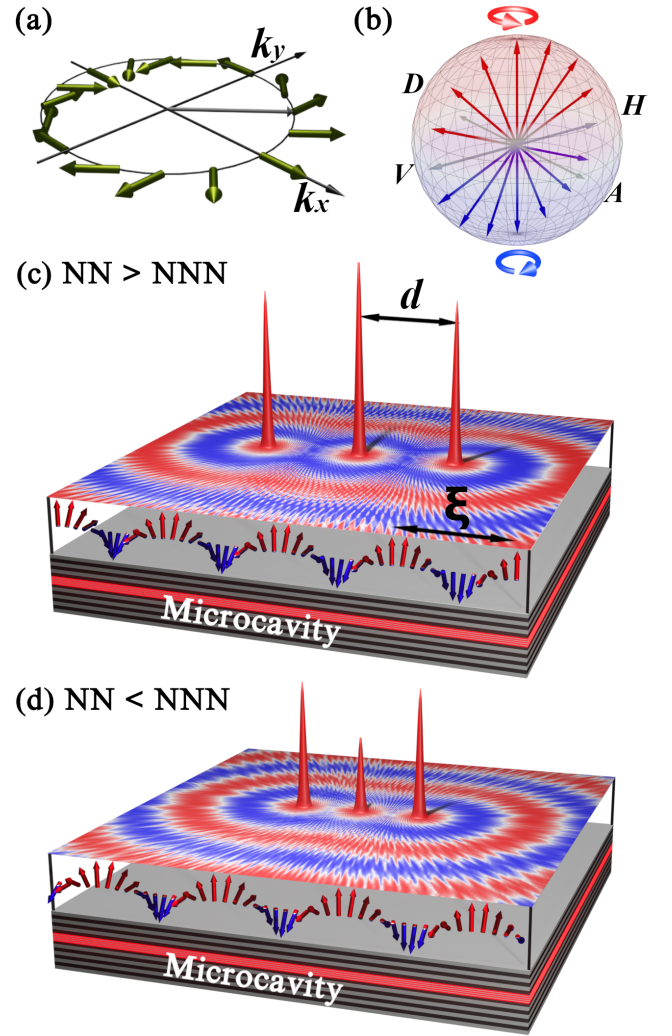


Figure 1. (a) Schematic of the effective SOC magnetic field distribution (dark olive arrows) from the TE-TM splitting on a momentum-space circle. (b) Schematic of the Poincaré sphere showing example pseudospin precession as polaritons propagate (blue and red arrows). Schematic representing two pump geometries where the distance between the central and edge pump spots equals to (c) one full period of pseudospin oscillation (NN is stronger than NNN) and (d) half oscillation period (NN is weaker than NNN). The height of the peaks represents the intensity of the condensate emission, and the red, white, and blue colour map shows the precession of the polariton pseudospin propagating in the cavity plane, with red representing  $S_z = +1$  (spin-up polaritons) and blue representing  $S_z = -1$  (spin-down polaritons). Red and blue arrows show the pseudospin precession of the polaritons propagating from the edge condensates along the triad axis

sion are written,

$$S_{x,y,z}(\mathbf{r}) = \frac{I_{H,D,\sigma^+}(\mathbf{r}) - I_{V,A,\sigma^-}(\mathbf{r})}{I_{H,D,\sigma^+}(\mathbf{r}) + I_{V,A,\sigma^-}(\mathbf{r})}, \quad (1)$$

where  $\mathbf{r} = (x, y)$  is the in-plane coordinate and  $I_{H(V),D(A),\sigma^+(\sigma^-)}(\mathbf{r})$  corresponds to horizon-

tally(vertically), diagonally(antidiagonally), and right-circularly(left-circularly) polarized (RCP and LCP for short) PL, respectively. Formally, the Stokes parameters relate to the condensate pseudospin through  $\mathbf{S} = \langle \Psi^\dagger | \hat{\boldsymbol{\sigma}} | \Psi \rangle / \langle \Psi^\dagger | \Psi \rangle$  where  $\Psi = (\psi_+, \psi_-)^T$  is the condensate spinor order parameter and  $\hat{\boldsymbol{\sigma}}$  is the Pauli matrix-vector. The  $S_x(\mathbf{r})$  and  $S_y(\mathbf{r})$  components represent the degree of linear and diagonal polarisation but are not important in this study (also due to the predominant circular polarisation of the condensates used here). Experimental measurements were reproduced using a generalised two-dimensional Gross-Pitaevskii equation (2DGPE) (see section S2 in the Supplemental Material [55] with references [56, 57] therein).

In Fig. 2 we present results for two polariton condensates separated by  $d \approx \xi/2$ . Data for a single isolated condensate gives a  $S_z$  period around  $\xi \approx 90 \mu\text{m}$  (see section S1 in the Supplemental Material [55]). Figures 2(a) and 2(b) show the measured and simulated spatial distribution of the  $S_z$  component with spatial pseudospin oscillations clearly visible due to the SOC rotating the spin of the outflowing polaritons. Note that unavoidable dephasing of polaritons in experiment results in lowered  $S_z$  values compared to simulations as indicated on the colorbars. Smaller ripple-like modulations are also visible due to the standing wave interference between the two phase-locked condensates as reported before [24, 44, 54]. These ripples are characterized by a small-scale period  $\lambda = 2\pi/\langle k_c \rangle \approx 3 \mu\text{m}$ , where  $\langle k_c \rangle$  is the average outflow momentum of polaritons from their condensates. In contrast, the large-scale  $S_z$  period is given by  $\xi = 2\pi/\Delta_k \gg \lambda$  where  $\hbar\Delta_k/\sqrt{2\varepsilon_c} = |\sqrt{m_{\text{TE}}} - \sqrt{m_{\text{TM}}}|$  and  $\varepsilon_c \approx 3 \text{ meV}$  is the condensate energy (measured from  $k = 0$  at the dispersion) and  $m_{\text{TE},\text{TM}}$  are the effective masses of TE and TM polarized polaritons [45].

The spin screening effect can be observed as periodic extrema in the integrated PL intensity, which represents the condensate occupation, as a function of separation distance  $d$  in Fig. 2(c). At the maxima the coupling is unscreened and NN coupling is strong. At the minima the coupling is screened and NN coupling is weak. Black dots and black solid curve denote experimental measurements and calculations, respectively. In the absence of SOC one would observe monotonically decreasing emission intensity with only short variations (order of  $\lambda$ ) corresponding to in-phase and anti-phase flip-flop transitions between the synchronized condensates [24]. Instead, we observe strong non-monotonic behaviour with clearly visible maxima around 67 and 154  $\mu\text{m}$ , and minima around 56 and 135  $\mu\text{m}$ . Notice that the distance between the two maxima and the two minima correlates with the measured  $\xi \approx 90 \mu\text{m}$  period of  $S_z$  oscillations.

The discrepancy between the absolute locations of the minima and maxima with the predicted critical distances for screened ( $\xi/2, 3\xi/2$ ) and unscreened ( $\xi, 2\xi$ ) coupling, respectively, can be understood as follows. Firstly, when

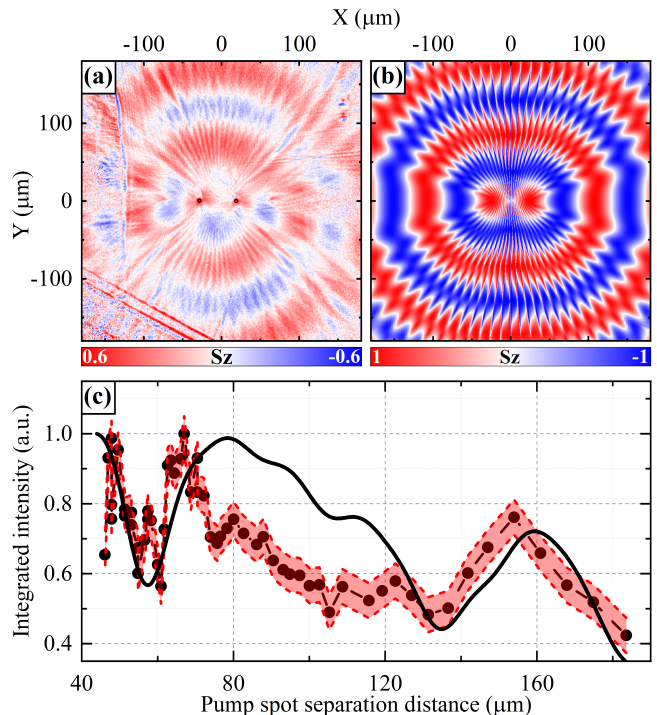


Figure 2. Two polariton condensates. (a) Experimentally measured and (b) simulated numerically real space  $S_z$  component of the Stokes vector of the polariton condensates emission. In panel (a) black circles show the position of pump spots. (c) Total integrated emission intensity dependence on the separation distance between two condensates pump spots. In panel (c) black dots shows the experimentally measured values with red region representing the error of the total intensity value. Black curve shows the same dependence calculated numerically

two condensates are coupled their energy is redshifted on average [24] leading to smaller  $\varepsilon_c$  and thus larger  $\xi$  in the coupled system. Second, the finite width of the pump spots modulates the phase of polaritons and causes a shift in the  $S_z$  period. Third, the cavity here has higher levels of disorder than strain-compensated cavities [58] which can affect the spatial coupling. That's why the relative distances between the extrema are more meaningful than their absolute locations. This interpretation is verified in 2DGPE modeling which accurately reproduces the locations of the extrema. Note that the slight discrepancy between modeling and experiment in Fig. 2(c) between 70 and 120  $\mu\text{m}$  can possibly be attributed to the large parameter space of the 2DGPE making quantitative matching somewhat challenging or localized defects in the sample which scatter outflowing polaritons, consequently decreasing the coupling efficiency. Indeed, the integrated emission intensity of the coupled condensate system is proportional to their non-Hermitian coupling strength from their mutual overlap over the pumped areas, since it determines both the imaginary (and real) part of their complex energies [23]. However, large de-



fects are scarce in the sample [46]. In order to verify that the modulation we observe is dominantly coming from the spin screening effect we have carefully chosen a clean part of the sample, with the minimum amount of large defects. We also tested a few other relatively clean sample locations and found qualitatively the same modulation period in the integrated emission dictated by the period of the spin precession.

In order to demonstrate the NNN coupling using the all-optical spin screening effect we investigated the system containing a chain of three condensates similar to the system depicted schematically in Fig. 1. As in the previous experiment with two condensates, all condensates were excited using tightly focused RCP laser pump spots of equal intensity above threshold. Figures 3(a) and 3(b) show the measured and simulated spatial distribution of the three condensate  $S_z$  component with NN distance of  $d \approx \xi/2$ . As in the previous case of two condensates, the system forms a joint macroscopic coherent state resulting in an oscillating  $S_z$  pattern elongated along the horizontal axis with three RCP condensate circles of equal degree of polarisation in the centre. Amazingly, the intensity of the central condensate was suppressed relative to the outer ones, evidencing reduced NN coupling due to the spin screening effect, see in Fig. 3(c) measured (red diamonds) and simulated (black solid curve) intensity distribution along the triad axis.

To unambiguously demonstrate the spin screening effect in the triad, we measured (dots) and simulated (solid curve) the dependence of the central condensate intensity as a function of NN separation distance with results presented in Fig. 3(d). Both experiment and calculations show a clear dip around  $d = 52 \mu\text{m} \approx \xi/2$ , corresponding to spin-screened NN coupling, followed by a small peak around  $d = 80 \mu\text{m} \approx \xi$  where the NN coupling is restored. The observed suppression of the central condensate intensity provides strong evidence of spin-screened NN coupling mediated by the spin coherence of the system.

Moreover, we experimentally measured pump power dependence for each separation distance and extracted polariton condensation threshold values, which are shown in Fig. 3(e) (red circles). The horizontal dashed line is the threshold value of the isolated condensate. In the absence of the TE-TM splitting monotonic increase of the threshold value converging to the isolated condensate threshold is expected with the increase of the separation distance between the condensates. In our system we observe maximum threshold at the separation distance of  $52 \mu\text{m}$ , which precisely corresponded to the minimum of the central condensate intensity in Figs. 3(c) and 3(d). It confirms that the NN condensate interaction is effectively screened at this separation distance due to the TE-TM splitting. Around a separation distance close to the full period of  $S_z$  oscillation ( $d \approx \xi$ ) a decrease in the threshold power was observed, as expected with NN coupling

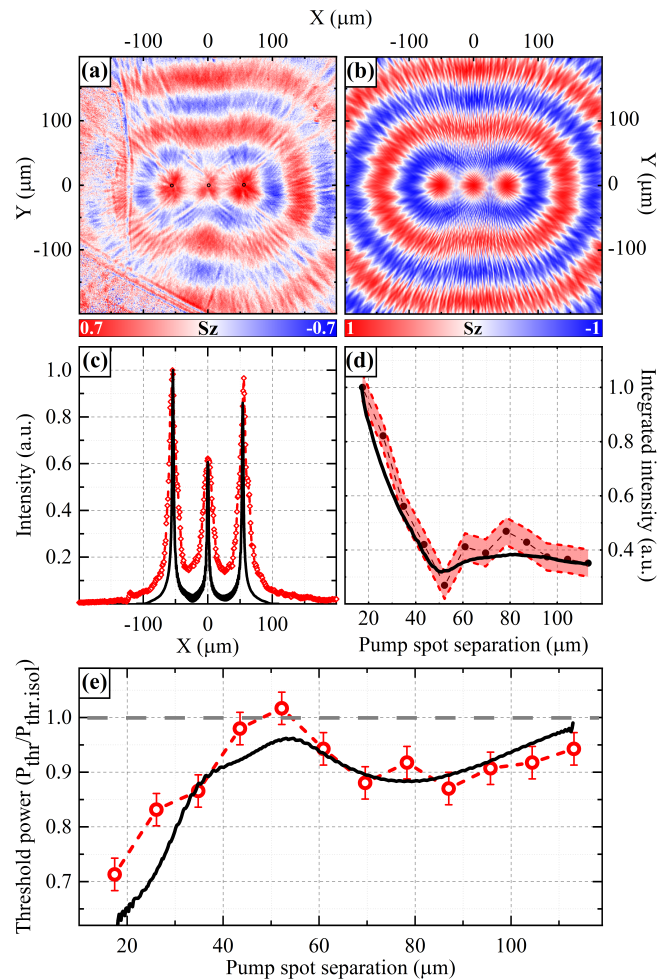


Figure 3. Three polariton condensates. (a) Experimentally measured and (b) simulated real space  $S_z$  component of the Stokes vector of the polariton condensates emission. In panel (a) black circles show the position of pump spots. (c) Measured experimentally (red diamonds) and calculated numerically (solid black curve) real space intensity distribution along the triad axis. (d) Dependence of the central condensate PL intensity on the separation distance between the condensates pump spots measured experimentally (black dots) and calculated numerically (solid black curve); red region represents the error of the total intensity value. The dashed curves are guides to the eye. (e) The system threshold power dependence on the separation distance between the condensates pump spots measured experimentally (red circles) and calculated numerically (solid black curve); red bars represent the error. Grey dashed line in panel (e) shows the threshold power for single isolated condensate.

restored. A simple linear coupled oscillator model [solid curve in Fig. 3(e)] is able to explain the behaviour of the threshold power (see section S3 in the Supplemental Material [55]).

In summary, we have experimentally demonstrated that next-nearest-neighbour coupling can be made stronger than nearest-neighbour coupling in ballistically expanding spinor exciton-polariton condensates which



was recently proposed in Ref. [54]. This unconventional near-inversion of the spatial coupling hierarchy between condensates stems from the combination of TE-TM splitting and the ballistic polariton flow from each condensate. Outflowing polaritons experience effective spin-orbit coupling which rotates their spin state as they propagate from one condensate to the next. Depending on distance, the overlap (coupling) between the condensates can become spin-screened depending on the polariton spin projection upon arrival at its neighbour. We believe that the demonstrated alteration of the conventional condensate coupling hierarchy could pave the way towards all-optical simulation of many-body ballistic systems belonging to non-planar graph topologies using networks of spinor polariton condensates. In particular, we have noticed a resemblance between the distance-dependent spinor polariton condensate coupling strength and the well known RKKY mechanism (see section S4 in the Supplemental Material [55] with reference [59] therein) which offers perspectives on polaritonic simulation on long-range coupled magnetic dipoles in conductive materials. Another, intriguing effect of making NN polariton coupling strength comparable to NNN couplings in square graphs is the potential to generate stable arrays vortices through XY energy minimization [21] (see section S5 in the Supplemental Material [55] with reference [32] therein).

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- distribution of  $S_z$  component of the stokes vector from a single isolated polariton condensate; two dimensional spinor polariton model; theory of the threshold behaviour in a spin screened condensate triad; comparison with RKKY mechanism and discussion on the potential generation of spinor polariton vortical array. The Supplemental Material also contains Refs. [56,57,59].
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