Designing Capital-Ratio Triggers for Contingent Convertibles

Maxime Segal

Thesis of 180 ECTS credits submitted to the Department of Engineering at Reykjavík University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

October 27, 2023

Thesis Committee:

Sverrir Ólafsson, Supervisor
Professor, Reykjavik University, Iceland

Heiðar Ingvi Eyjólfsson, Co-Supervisor
Assistant Professor, Reykjavik University, Iceland

Matthieu Garcin, Member of the Committee
Professor, École Supérieure d'Ingénieurs Léonard de Vinci, Member of the Committee

Hersir Sigurgeirsson, Member of the Committee
Professor, University of Iceland, Iceland

Roman Goncharenko, Examiner
Assistant Professor, KU Leuven, Belgium

ISBN 978-9935-539-26-7
ISBN (electronic version) 978-9935-539-27-4
ORCID iD 0000-0003-3560-347X

I dedicate this work to my family, friends, former teachers and colleagues that always supported me, and without whom I would not have been able to do it.

To my students, in the hope that my enthusiasm for financial engineering has helped to spark a passion and vocation in them.

Contents

List of Figures

List of Tables

List of Symbols

Symbol	Description
$PV(.)$	Present Value
$D(.)$	Discount Factor
$\mathbb{E}_t[.]$	Expected Value
$e^{(.)}$	Exponential Function
$N(.)$	Standard Normal Cumulative Distribution Function
$N'(.)$	Probability Density Function of the Normal Distribution
$N_2(.)$	Bivariate Standard Normal Cumulative Distribution Function
$\mathbb{P}(.)$	Probability
$\mathbb{1}(.)$	Indicator function

Designing Capital-Ratio Triggers for Contingent Convertibles

Maxime Segal

October 13, 2023

Abstract

Contingent Convertible (CoCo) bonds represent a novel category of debt financial instruments, recently introduced into the financial landscape. Their primary role is to bolster financial stability by maintaining healthy capital levels for the issuing entity. This is achieved by converting the bond principal into equity or writing it down once the minimum capital ratios are violated. CoCos aim to recapitalize the bank before it is on the brink of collapse, to avoid state bailout at a huge cost to the taxpayer. Under normal circumstances, CoCo bonds operate as ordinary coupon-paying bonds, which only in case of insufficient capital ratios are converted into equity of the issuer.

However, the CoCo market has struggled to expand over the years, and the recent tumult involving Credit Suisse and its enforced CoCo write-off has underscored these challenges. The focus of this research work is on the first hand to understand the reasons for this failure, and, on the other hand, to modify its underlying design in order to restore its intended purpose: to act as a liquidity buffer, strengthening the capital structure of the issuing firm.

The cornerstone of the proposed work is the design of a self-adaptive model for leverage. This model features an automatic conversion that doesn't hinge on the judgement of regulatory authorities. Notably, it allows the issuer's debt-to-assets ratio to remain within predetermined boundaries, where the likelihood of default on outstanding liabilities remains minimal. The pricing of the proposed instruments is difficult as the conversion is dynamic. We view CoCos essentially as a portfolio of different financial instruments. This treatment makes it easier to analyse their response to different market events that may or may not trigger their conversion to equity.

We provide evidence of the models effectiveness and discuss the implications of its implementation, in light of the regulatory environment and best market practices.

Keywords: Contingent Convertible Bonds, Hybrid instrument, Banking Stability, Financial Engineering, Capital-Buffer

Hönnun fjárhlutfallskveikja fyrir skilgreind breytanleg skuldabréf

Maxime Segal

13 október 2023

Útdráttur

Skilyrt breytanleg (e. Contingent Convertible, skammstafað CoCo) skuldabréf eru nýstárleg gerð af fjármálagerningum sem nýlega komu fram á sjónarsvið fjármálamarkaða. Helsta hlutverk þeirra er að efla fjármálastöðugleika með því að viðhalda hæfilegum eiginfjárgrunni fyrir útgefendur þeirra. Þetta er gert með því að umbreyta höfuðstól skuldabréfs í hlutafé eða með því færa þau niður þegar krafa um eiginfjárhlutföll eru rofin. CoCo hefur það markmið að endurfjármagna bankann áður en hann fellur og þar með koma í veg fyrir björgunaraðgerðir af hálfu ríkisins, sem hefur í för með sér mikinn kostnað fyrir skattgreiðendur. Undir venjulegum kringumstæðum virka CoCo skuldabréf eins og hefðbundin arðgreiðslu- skuldabréf, sem einungis er breytt í hlutafé þegar eiginfjárhlutföll útgefanda þeirra eru ekki nægjanleg.

Eigi að síður hefur markaður fyrir CoCo átt erfitt uppdráttar í gegnum tíðina og hefur nýlegur titringur í kringum Credit Suisse og þvingaðar afskriftir þeirra á CoCo skuldabréfum ýtt enn frekar undir erfiðleikana. Helsti tilgangur þessarar rannsóknar er tvíþættur. Annars vegar er ætlunin að skilja hvers vegna CoCo hefur ekki átt meiri velgengni að fagna en raun ber vitni. Hins vegar er henni ætlað að breyta grundvallarhönnun CoCo í þeim tilgangi að endurheimta upprunalegan tilgang þeirra: sem er að vera stuðpúði lausafés sem styrkir fjármagnsskipan útgáfu fyrirtækisins.

Hornsteinn verkefnisins er hönnun á líkani með sjálfaðlögunarhæfni með tilliti til skuldsetningarhlutfalls. Líkanið býr yfir sjálfvirkri umbreytingu sem ræðst því ekki af reglum eftirlitsyfirvalda. Það gerir útgefanda því kleift að viðhalda hlutfalli skulda á móti eignum innan fyrirfram skilgreindra marka, þar sem líkur á vanskilum vegna útistandandi skuldbindinga haldast í lágmarki. Verðlagning gerninganna sem lagðir eru til í rannsókninni er þó vandasöm þar sem umbreytingin er dýnamísk. Í meginatriðum verður litið á CoCos sem safn ólíkra fjármálagerninga. Með þessari aðferð er hægt að greina viðbrögð þeirra við mismunandi markaðsatburðum sem geta mögulega hrint af stað umbreytingu yfir í hlutafé.

Sýnt verður fram á skilvirkni líkansins ásamt því að álykta um innleiðingu þess með tilliti til regluverks og bestu markaðsvenja.

Efnisorð: Skilyrt skilgreind skuldabréf, Blandaðir gerningar, Fjármálastöðugleiki banka, Fjármálaverkfræði, Eiginfjárauki

List of Abbreviations

Abbreviation	Description
Am	American Barrier
AT1	Additional Tier 1
AT1P	Analytically Tractable First-Passage Model
BBG	Bloomberg
bn	billion
C	Conversion (to equity)
CBR	Combined Buffer Requirement
CDS	Credit Default Swap
CET1	Core Equity Tier 1
CIR	Cox-Ingersoll-Ross
CoCo	Contingent Convertibles
CRD IV	Capital Requirements Directive IV
CRR	Capital Requirement Regulation
D&I	Down-and-In
D&O	Down-and-Out
DCL	Dynamic Control of Leverage
DS	Digital Share
DWBO	Digital Window Barrier Option
EBA	European Banking Authority
ECB	European Central Bank
EDM	Equity Derivatives Model
EPS	Earnings Per Share
ERN	Equity Recourse Note
ESG	Environnement Social Gouvernance
EU	European Union *or* European Barrier
EUR	Euro
FED	Federal Reserve
FIG	Financial Institutions Group
FINMA	Eidgenössische Finanzmarktaufsich (Swiss Financial Market Supervisory Authority)
FWD	Forward Contract
gBm	geometric Brownian motion
GBP	British Pound
GN	Glasserman & Nouri
HFI	Hybrid Financial Instrument

HV	Historical Volatility
IAS	International Accounting Standards
ISIN	International Securities Identification Number
ISK	Icelandic Kronur
ITM	In-the-Money
IV	Implied Volatility
MDA	Maximum Distributable Amount
MLE	Maximum Likelihood Estimator
MVP	Minimum Viable Product
NAD	Nondeferrable American Digitals
NVE	Non-Viability Event
OCR	Overall Capital Requirement
P2G	Pillar 2 Guidance
P2R	Pillar 2 Requirements
PE	Private Equity
PONV	Point of Non Viability
PT	Pennacchi & Tchistyi
RCD	Reverse Convertible Debentures
RT1	Restricted Tier 1
RWA	Risk-Weighted Assets
SAFE	Simple Agreement for Future Equity
SEC	U.S. Securities and Exchange Commission
SEK	Swedish krona
S&P	Standard & Poor's
SRB	Single Resolution Board
SREP	Supervisory Review and Evaluation Process
SW	Sundaresan & Wang
T2	Tier 2
USD	U.S Dollar
VC	Venture Capital
WD	Write-Down
WU	Write-Up
ZC(B)	Zero Coupon (Bond)

Publications

This dissertation is original work by the author, Maxime Segal. Portions of the text are published as intermediary articles. It includes:

- M. Segal and S. Ólafsson, "Design of a self-adaptive model for leverage". Finance Research Letters, Volume 54, June 2023.
  doi: https://doi.org/10.1016/j.frl.2023.103721.
- M. Segal and S. Ólafsson, "Overview of an alternative trigger for DCL". Finance Research Letters, Volume 58, Part A, December 2023. doi: https://doi.org/10.1016/j.frl.2023.104281
- M. Segal and S. Ólafsson, "Dynamic Control of Leverage". Pre-print submitted to Financial Innovation, Springer. (*Revision stage*)
- M. Segal and S. Ólafsson, "Semi-Analytical framework for the pricing of DCL instruments". *Working paper*. Pre-print submitted to Quarterly Review of Economics and Finance, Elsevier. (*Initial submission*)

Acknowledgments

> I have fantasies of going to Iceland, never to return.
>
> *Edward Gorey.*

I first came to Iceland as a tourist in 2012, and I promised myself to be back. When five years later, my French engineering school (ESILV) offered me the chance to do an academic exchange (Erasmus programme) at Reykjavik University (RU). I realized that it was not only the opportunity to be back but also to live there.

After six months, I succeeded to extend my stay to 1 year. And although I had to be back in Paris to do my ultimate year before graduation, I was already in contact with Prof. Sverrir Ólafsson, one of my former teachers at RU, to become my supervisor during this four-year journey.

I will **always** be grateful to him for the trust and guidance, as well as the confidence associated with the five times assistant teaching role[1] and twice teaching position[2].

During the time of my Ph.D., I also assumed the role of reviewer, three times for *Finance Research Letters* (2023 Impact Factor 10.4) and once for *Financial Innovation* (2023 Impact Factor 8.4). I felt honoured by these invitations from journals that ranks respectively #1 and #3 out of 111 in the Business & Finance category, to review researches that might have a great impact on the world of finance.

I had also the opportunity to present some parts of this thesis to the 2022 European Researchers' Night / Science Week (Vísindavaka).

In particular, I should like to thank Sverrir Ólafsson, Matthieu Garcin and Hersir Sigurgeirsson for their continuous support and help. To Daniel Aidan also, who

[1] Derivatives & Risk Management (MSc), Fixed Income and Interest Rate Modelling (MSc), Financial Engineering of the Firm (MSc), Finance X (BSc), Modern Trends in Financial Engineering (MSc).

[2] Derivatives (BSc) in 2021 & 2022

introduced me to CoCo bonds. This discovery, fueled by his passion, triggered my decision to write a thesis on this subject.

This work was awarded and funded by the Reykjavik University Research Fund in 2021 and 2022.

This work was awarded by Rannís and funded through the Icelandic Research Fund (*Rannsóknasjóður*) in 2023.

Additional equipment was generously provided by Reykjavik University.

Preface

> The Bank never "goes broke." If the bank runs out of money, the Banker may issue as much more as may be needed by merely writing on any ordinary paper.
>
> *Rule issued from the Monopoly.*

Folk culture, along with a rule from the renowned property trading game quoted above [1], might lead many to believe that banks cannot default if they run out of money.

This belief is patently incorrect. Certain crisis periods underscore the necessity for banks to be adequately funded. Depending on their significance in the global financial system, a lack of funds can lead to a *Too Big to Fail* (TBTF) scenario.

The term TBTF was first used by Stewart McKinney, a member of the United States House of Representatives, during a 1984 Congressional hearing discussing the seizure of the Chicago bank, Continental Illinois, by the Federal Deposit Insurance Corporation [2]. He was referencing a new category of financial institutions whose failure could cause a significant economic shock due to their size and interconnectedness. Such institutions often require government intervention to prevent catastrophic outcomes.

Despite regulatory adjustments, the banking industry continued to expand, at times becoming so large that it reached a point described as *Too Big to Save*. In such cases, even a government might be unable to rescue a failing bank. A notable example is Iceland during the 2008 financial crisis. The country's three main banks, Glitnir, Landsbanki, and Kaupthing, had debts amounting to five times the nation's GDP[3]. Given the banking sector's size relative to the Icelandic economy, the

[3] In March 2008, *Statistics Iceland* reported a GDP of ISK 1.279 trillion for 2007, while the three Icelandic banks reported borrowings of ISK 6.345 trillion in Q2 2008 [3, 4, 5, 6].

Central Bank of Iceland (*Seðlabanki*) was unable to prevent the financial system's collapse.

Such outcomes arise from operating in a growing and auspicious economic environment that encourages risk-taking and performance-driven behaviours. This era of financial stability, spanning from 1985 to 2008, is referred to in literature as The Great Moderation. It was marked by controlled inflation, low uncertainty, and steady yet positive growth accompanied by rising asset prices [7].

This aligns with financial instability theories, notably posited by Hyman Minsky in 1992. These theories discuss how extended periods of economic prosperity can lead to increased risk-taking and bubble formation, driven by banks' inherent nature as yield-seeking entities [8].

This thesis comprises nine chapters. The first introduces the origins of Contingent Convertibles, aiming to better understand their benefits and limitations. It also includes a literature review, drawing on extensive research to examine how CoCos interact with firm structures, their valuation, and their legal implementation. The second chapter uses a real bond from the Icelandic market to spotlight potential flaws in this financial instrument and discusses conversion probability modelling. The third chapter establishes a framework for identifying arbitrage opportunities across various CoCo bonds, providing two numerical examples. The fourth chapter introduces a novel form of contingent convertible, the *Dynamic Control of Leverage* (DCL), addressing some existing criticisms. The fifth chapter delves into the current regulations surrounding Contingent Convertibles and their integration into a firm's capital structure. Chapter 6 evaluates the efficiency of the previously introduced DCL instruments. Chapter 7 discusses stability and equilibrium considerations for DCLs. Chapters 8 and 9, respectively, explore an alternative trigger for this new CoCo bond type and a pricing framework using exotic derivative products.

Chapter 1

Introduction

"Bankers are merchants of debt who strive to innovate in the assets they acquire and the liabilities they market."

Hyman Minsky, [8].

1.1 What are Contingent Convertibles?

As a result of the 2007/08 financial crisis, financial authorities in the G20 realised the importance of imposing financial regulation on banks [9] to reduce the likelihood of financial disasters and facilitate refinancing measures for bankruptcies that do not require capital-expensive Government intervention [10]. One way to achieve these goals is to allow banks to issue specially-structured bond instruments that, under certain circumstances, convert to equity or are written down. In recent years, a particular instrument known as contingent convertible (CoCo) bonds [11], has emerged as an interesting candidate for this approach.

CoCo bonds are similar to conventional convertibles, as they are hybrid securities that function essentially as coupon-paying bonds under normal conditions but can be converted to equity [12] or written down when the banks capital or liquidity falls below a specified level. In this way, CoCo bonds serve as a loss-absorbing instrument that can maintain required capital and liquidity ratios. An interesting characteristic of CoCo bonds is the large number of features that can be built into them, providing a potentially flexible funding impact for the issuing bank while simultaneously necessitating quite challenging pricing methods [13].

A detailed discussion of the main research focus is introduced in the subsequent sections to delineate the scope of this thesis.

The triggering mechanism

Debate exists in the field on the triggering mechanism for CoCo bonds, to define under what circumstances the bond would convert into equity and at what conversion ratio. The triggering mechanism would, in all cases, be related to certain capital or liquidity ratio values for the issuing bank or directly linked to the bank's share price. Where these ratios are set determines the probability that a trigger is registered, which impacts the bonds price and its effectiveness in maintaining the required capital ratios. As the primary role of CoCo bonds is to provide improved capital or liquidity ratios in difficult times, the trigger should be set at such a level that conversion to equity only occurs when the banks capital position is becoming critical.

For additional consideration, conversion to equity leads to dilution of the pre-existing equity. This important result is not in the best interest of existing equity holders, who typically prefer a write-off or conversion only during difficult times when ratio improvements may be essential to avoid bankruptcy, followed by a total loss of equity.

However, some researchers agree on the ineffectiveness of current CoCo bonds, leading to the inconvertibility of the instrument [14]. No CoCo would have been converted during the 2008 crisis, as shown by [15], suggesting a failure to fulfil its role. Such limitations of CoCos were also observed in 2023 when their forced conversion failed to prevent the bailout of Credit Suisse.

Driven by these observed concerns and the inefficiency of CoCos, the core incentive of this thesis is to design a new trigger mechanism. While the intrinsic mechanism of debt conversion provides a potential capital buffer to the firm in exchange for a higher risk premium to the investor remains innovative, these techniques must be employed on a more suitable trigger [16]. In Chapter 4, we construct a dynamic trigger mechanism that drip-feeds the bank with debt-to-equity conversion sufficient to maintain required capital ratios.

The triggering event

Two scenarios can result from a trigger. First, a permanent or temporary, full or partial write-down may be applied to the nominal value of the CoCo. Second, the bond may convert its value into equity, determined by a certain factor (i.e., the conversion ratio) for how much equity is received for the nominal value. Additional complexity arises when the write-down is temporary or partial [17] because the bond could regain a fraction of its original value if capital or liquidity ratios improve.

CoCo bonds can typically be priced as conventional coupon-paying bonds with the following features: 1) the possibility of conversion, 2) a probability of conversion events being triggered, and in the case of a trigger, 3) the bond may permanently convert to equity or be written down by a fraction, either permanently or for a set period determined by the evolution of specific capital or liquidity values. Pricing CoCo bonds under these complex unfolding events requires an understanding of the historical probabilities of fixed capital and liquidity ratios attained. In principle, these probabilities can be estimated by applying the structural models of Black, Scholes, and Merton. If established, then our working hypothesis is that pricing can be performed with a structural model approach or an equity derivatives method that relies on barrier options [18]. The outcome must then be empirically verified and the model calibrated.

The work presented in this thesis pursues traditional CoCo bonds and our proposed dynamic model, respectively, in Chapters 2 and 3, and in 4 and 9.

CoCo specifications impact bank funding

This research aims to understand the impact multiple variables have on the bonds price, such as in the case of the Greeks vanilla options [19], which is essential for constructing hedging strategies involving CoCo bonds. For example, do CoCo bonds stabilise the banking system, or can they unwittingly create new risks? As described above, CoCo bonds function as a kind of capital buffer or stabiliser. When a banks financial ratios (capital and leverage) are assumed healthy, the bond is settled as a conventional coupon-paying bond. On the other hand, CoCos are only triggered when these financial ratios are signalling the bank may be in a financially critical situation. Following a trigger event, the bond is not settled with principal payments. However, the principal, or some fraction of it, is converted into equity according to predetermined ratio values. This approach releases the bank from making principal or coupon payments while its equity position is improved, which reduces the probability of default. Quantifying the stabilising effect CoCo bonds have on the banking system is a challenging research question.

What should the conversion ratio be?

A higher conversion ratio relates to a CoCo bond converted into more equity. This outcome is positive from the perspective of the Debt/Equity ratio, which reduces the probability of default on other obligations. On the other hand, the ownership of existing equity holders is diluted, which reduces their control rights. Such a compromise may be necessary for the improved financial health and stability of the bank in question. A CoCo could be interpreted as being designed to punish the initial shareholders for the excessive risk taken prior to conversion. Upon triggering, the risk-shifting is reduced as well as preventing excessive debt overhang recourse [20].

Finding a balance between these two objectives, i.e. dilution of existing equity value and financial stability of a bank is one of the thesis goals. We provide comprehensive insights into the influence of CoCos on a bank's funding mechanisms. However, we do not undertake a quantitative approach to derive a solution.

1.2 Literature Review

1.2.1 Origins

Fig. 1.1 presents the number of issuances per year and the nominal amount (in bn USD equivalent). The surge in CoCo bond issuance observed in 2019 may be linked to several factors:

(i) Following investor demand fueled by the prevailing low interest-rate environment, CoCo bonds that offered higher yields emerged as an attractive alternative. Notable, CoCo bonds outperformed equities during this period when comparing the *iBoxx Contingent Convertible Liquid Developed Europe AT1 TRI (EUR Hedged)* to the *MSCI Europe Banks Net Return EUR Index.*

(ii) The implementation of higher minimum capital requirements in the European regulatory landscape significantly contributed to the upswing in CoCo issuance. These requirements gradually increased since 2014 and reached a peak in 2019.

(iii) Some market-specific contributors, such as China, further amplified this observed trend. Chinese regulators enforced the global Basel III norms on bank capital adequacy with the goal of increasing the resilience of lenders against the financial risks resulting from a decade of rapid debt growth, which led to record defaults. Consequently, from 2019 to 2022, Chinese banks were urged to raise $260 billion in new capital, a fraction of which was met through CoCo bond issuance [21].

(iv) A potential cyclical effect existed due to the five-year, non-callable period constraint on CoCo bonds. The initial surge in CoCo bond issuance observed in 2014 corresponds to many of these bonds hitting their call dates in 2019. Then, another wave of issuance is seen as these instruments rolled over for a subsequent five-year period.

In the recent context of low-interest rates, primarily driven by the global economic policies responding to recession fears from Central Banks, contingent convertible instruments may appear to contain all the properties necessary to satisfy the loss-absorption goal of yield-seeker investors and financial regulators of issuers with stressed capital or liquidity. CoCo bonds were often criticised for their complexity but are now associated with stable issuance. Many factors have limited the expansion of this market, including jurisdiction discrepancies to the equity- or debt-like treatment and high discretionary trigger mechanism bringing uncertainty to the conversion event. Today, this asset class seems to be losing momentum and relies on alternative incentives instead of focusing on the core mechanism to increase investor interest. This challenge was especially observed when BBVA issued the world's first green CoCo bond (AT1) in July 2020[1].

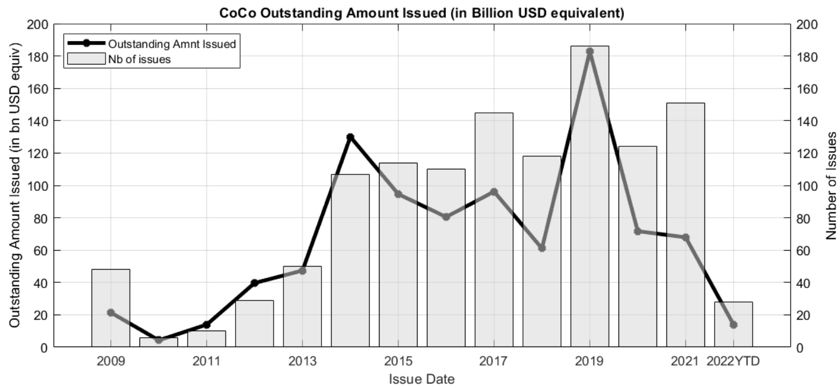


Figure 1.1: Overall CoCo market represented by the number of issuances and the associated outstanding amount (in bn USD equivalent). Data as of April 5, 2022. Source: Bloomberg.

The total CoCo market share is less than 1 trillion USD compared to the 100 trillion USD of the outstanding global bond market [22]. We explore the pricing aspect and the key issues related to Coco issuance, including factors that have limited the expansion of this market. CoCo issuance's regulatory framework is also considered as it plays a significant role. While CoCos encompass many enhancements, such as callability and floating coupons, they diverge from straight convertible bonds that embed options but convert to shares at the investor's discretion. As for the option for callability, the decision to turn the bond to equity or early redemption depends on if a conversion or callable price favours the investor

[1] ISIN: ES81321102

or issuer [23]. Assuming both parties involved acting in their best interest, a variant on this decision considers the optimal early redemption as a function of the firm's assets instead of the bond price [24].

Each section of this literature review deals with an aspect of the contingent convertible that a market participant should be aware of when investing or originating CoCos. The first section describes the regulatory environment, and the second explains valuation. Next, the curtailments around these hybrid securities are presented in the third section. The fourth section introduces the firm's capital structure notion, and the final section establishes the motivations intrinsic to pursuing this thesis.

1.2.2 Regulation environment

Although policies on contingent convertibles may vary widely depending on the region, Basel III is a standard in the industry that establishes new financial ratios and minimum thresholds with which banks must comply. Triggers for issued instruments are expressed as the ratio of the issuing company's Common Equity Tier 1 (CET1) over its Risk-Weighted Assets (RWA). The CET1 is expected to be the safest holding that is primarily composed of the bank's common shares and retained earnings, along with intangible assets and specific qualifying issues and adjustments. Basel III recommends this ratio be at least 4.5% of the RWA.

On top of the CET1 in the firm's capital structure comes the Additional Tier 1 (AT1), which acts as a security buffer for the financial stability of the bank. The summation of AT1 and CET1 must be at least 1.5% of the RWA to guarantee quality (and liquidity) in the assets held and includes preferred shares and high trigger[2] CoCos.

Finally, Tier 2 (T2) refers to other non-CoCo subordinated debt and low-trigger CoCo (i.e., below 5.125% of RWA). This layer, summed with the prior two, must account for 8% of the overall RWA. Under this regulation and considering the numerical constraints, infinite maturity is required to qualify as AT1. A discretionary trigger in the form of regulatory control is also needed if a firm attains a supposed Point of Non-Viability (PONV), regardless of the AT1 or T2 qualification [9].

Perpetual instruments are sometimes treated as equity paying dividends due to their unlimited and regular cash-flow transfers, which aligns with International Accounting Standards [25]. No specification or requirement is issued regarding the loss-absorption structure, meaning that a Conversion to equity (C), Principal Write-Down (WD) or Cash Principal Write-Down (CWD) can be used. The difference between WD and CWD lies in the remaining instrument held by the investor post-conversion. For example, assuming a write-down has a ratio below 100%, the CoCo converts to subordinated debt (WD) or directly to cash, an uncommon

[2] At least 5.125% to qualify a CoCo issue as AT1.

mechanism notably used by Rabobank as part of their two billion USD issuance in early 2011. As a privately-owned Dutch bank, no publicly-traded equity existed on the market, so the conversion could only be in cash [26].

Previous research has not unanimously favoured one approach over the others [14]. However, some investors might favour C instead of WD because of the limited reparation, while others could favour WD for its clarity. Conversion to equity leads to some dilution of pre-existing equity and is not in the best interest of the existing equity holder, who should prefer write-offs in more difficult times when ratio improvements may be essential to a total loss of equity. Nonetheless, some cases have demonstrated that the decrease in the EPS[3] expectation due to ownership dilution might be offset by the leverage decrease following the conversion to equity [27]. Empirically, C provides a yield 2.5% higher than non-CoCo subordinated, which must be compared with the 3.9% higher yield for WD [15]. Not being the only two options, an embedded Write-Up (WU) can also be considered, which allows for only a temporary write-down that is cancelled if the bank's situation improves. Regulators tend not to encourage WU features because it is in opposition to the only equity-increasing principle.

A multi-variate trigger is another possibility that appears in research papers and is already issued in existing cross-asset products. This solution is utilised by issuers with parent-holding companies or groups with a central entity. In this case, the first trigger relates to the CET1 ratio at the level of the bank and the other at the group level [28]. An alternative is a combination of macro and micro triggers, such as a global economic-related trigger that could indicate a threat to the financial system (macro) and a bank-related indicator (micro), which was evoked in [29].

As highlighted in [30], credit rating agencies face difficulties in assessing the risks CoCo devices represent. These obstacles are due to jurisdiction discrepancies, unforeseen triggering possibilities by regulators, or the likely violation of the absolute priority rule that intends to refund creditors (debt holders) before equity holders in the case of default. Even though the many features that can be built into CoCos make their potential funding impact for the issuing bank flexible, their pricing methods simultaneously become challenging, and comparison between two distinct bonds becomes difficult [13]. Such a lack of normalisation led the European Banking Authority (EBA) to release in October 2016 a standardised template for banks inquisitive in AT1 issuance, which provides details on the prudential terms and conditions relative to AT1 instruments.

The regulatory environment encompassing hybrid instruments and practical considerations is explored further in Chapter 5.

[3] Earnings Per Share

1.2.3 Valuation

All authors cited in this section agree on the four components that cause drifting in CoCo pricing, including the coupon payment, the nominal redeemed at maturity, the dividend paid in case of conversion, and the final share value at maturity. Implicitly, investing in a CoCo is equivalent to holding a claim on these components. Because of its structure and conditional payoff, the pricing of CoCo bonds is a challenging problem that could be tackled in three ways. These valuation approaches include structural modelling, credit modelling, and equity derivatives. In the latter, the payoff of CoCo is approximated by a portfolio of a 0-coupon bond, a synthetic knock-in forward (calculated with the sum of the Down & In Call plus Down & In Put), and the sum of a set of binary Down & In options to represent the coupons missed due to conversion [18, 31]). Erismann [31] also considers jump-diffusion processes to translate an inefficient market.

One of the credit derivative approaches is the J.P Morgan model, where the CoCo is the sum of zero recovery and conversion to equity. Other methods involving credit derivatives are more widely used, such as intensity models where the conversion time is modelled by the first passage of a non-homogeneous Poisson process. This model has the advantage of being invariant from the value of assets. However, economic rationality is more difficult to exhibit. Finally, the structural approach is founded on Merton's work that explores the default event only at maturity. This default event is triggered if the firm's asset stochastic process falls below the liabilities in a scenario where the bank's assets are driven by the sum of equity and debt with a face value B and a single maturity T [24]. More accurate models have since been created, such as:

- By considering short-term deposits, bonds (that are possibly convertible), and equity [32].
- By allowing for additional risk considerations, such as coupon cancellation eventuality or extension risk[4], which is the Analytically Tractable First-Passage Model (AT1P), assuming a deterministic floating conversion barrier with time-dependent volatility. [33].

Before these models, other widely-used techniques followed the same bankruptcy condition as established by Merton [23] and extended it to a default that could occur if the asset value falls below a fraction of the initially promised payment defined in the bond covenant **during the bond lifetime** [34] (and not only at maturity).

[4]When the bond is not called by the issuer at the first callable date, which goes against the standards in the industry.

Modelling the distribution of the CET1 ratio is complex because it relies on opaque calculations for the risk weighting factors and is an accounting measure usually disclosed quarterly by banks. These conditions do not enable continuous monitoring of the ratio.

Regardless of the approach, the triggering condition on the CET1 must be replicated by a proxy value in the CoCo valuation process. When using derivatives, the underlying stock price level as a benchmark is the most reasonable, especially if the conversion price C_p corresponds to a fixed number of shares or the triggering event instead of being defined at issuance. This value is then easily calibrated from the available market data during issuance. Conversely, the ratio equity-to-assets might be favoured when employing structural techniques. Contrary to the hypothesis of complete conversion featured in the prior pricing approaches, a partial conversion of the instrument may provide the firm with sufficient capital to maintain the required metrics above the minimum threshold. This assumption highlights that the original shareholder owns a fraction of equity that is solely a function of the minimum asset value achieved by the stochastic process V_t that tracks the book value of the firm's assets [17]. According to this study, protecting the original shareholder can be accomplished by setting a high trigger for the hybrid security. The firm should also consider increasing the portion of this instrument in its overall balance making it less likely to default and, by extension, decreasing the yield on its senior debt. Caution must be heeded in some cases[5] because the elasticity of junior bonds may stretch above 1 (and even to infinity), meaning that the debt considered safer than the firm's assets is not the case [34].

Moreover, in some cases, a coupon rate below the risk-free rate is not impossible if the market environment incentivises the investors to benefit from the equity conversion.

Finally, the papers that consider CoCo valuation observe a double equilibrium problem that occurs when the trigger varies due to the convexity of the price curve [31, 35, 36]. Table 1.1 summarises the effects related to the conversion price.

[5]Conditional on the firm's value.

Conversion price below share value ($C_p < S_\tau$)	Equivalent to obtain new shares at discount: transfer of wealth from the initial shareholders to the CoCo investors.	SW (2015) argue that more than one equilibrium price might exist, raising a double equilibrium problem.
Conversion price at par ($C_p = S_\tau$)	Equivalent to obtain new shares at par: no transfer of wealth.	No equilibrium problem manifestation.
Conversion price above share value ($C_p > S_\tau$)	Equivalent to obtain new shares at a premium: transfer of wealth from the CoCo holders to the initial shareholder.	SW (2015) argue that potentially, no equilibrium price would exist.

Table 1.1: Effects of having the conversion price set below, equal, or above the share value at the time of the conversion on the transfer of wealth, resulting in a multi-equilibrium problem dealt with by Sundaresan and Wang (SW in the table) [35].

The research on CoCo is not static, and some papers suggested a single solution even when the CoCo conversion penalises the initial shareholder. For example, Glasserman and Nouri showed that by having an instrument that is continuously traded and a conversion price C_p low enough, the existence and uniqueness of a share value can be found, despite the high dilution factor [37].

Later, Pennacchi and Tchistyi (hereafter PT) [20] extended the scope of Glasserman and Nouri by identifying an error that invalidates Theorem 1 from Sundaresan and Wang [35]. Compared to Glasserman and Nouri, PT derived closed-form equations for the price of the stock (unique equilibrium) and the contingent convertible security. Then, the requirement from Sundaresan and Wang was shown to be too strict, making it possible to have a single and stable solution even if $C_p \neq S_\tau$. This restriction is required only at the conversion time and not previously, as was stated by SW. Nonetheless, multiple equilibrium behaviours are observed to still manifest in deterministic models but not when asset modelling is continuous.

Concerning this multiple equilibrium problem, Glasserman and Nouri found its origin in the discrete-time design of CoCos, where prices that could not adapt progressively would only shift value at conversion time [37]. This eventuality is obviated with continuous-time models. Yet, Equity Recourse Notes (ERNs) introduced by Bulow and Klemperer solve this issue, although the conversion occurs at scheduled times [38].

1.2.4 Curtailments

The intrinsic complexity surrounding CoCos could be a factor in making the market relatively illiquid [39]. Additionally, some papers consider the ascertainment that Contingent Convertibles are breaking away from their core motivation, underlined by no CoCo based on accounting triggers would have been converted, even during the latest financial crisis, suggesting a failure to fulfil its role [15, 40].

Another critique is made on low-trigger CoCos at a level of non-viability where the issuer defaults before conversion [14]. This concern fuels all arguments favouring forward-looking triggers, such as those that are market-based[6] and not accounting or discretionary, which tend to be inefficient because they are backwards-looking. The naysayer may still argue that this solution is subject to the share price or CDS spread manipulation at a trading level close to conversion [41], even though a high trigger is expected to guarantee market-based benchmark reliability [37].

Finally, the lack of transparency in calculating the CET1 level and the compulsory discretionary trigger imposed by the regulation provide concerns that could lead to distortion. Such observations are made in Chapter 3. Ideally, the triggering ratio should be objective, transparent, fixed, with no jurisdiction disruption, publicly disclosed, and with an at-least quarterly frequency release [31].

The two features that a CoCo bond bears in opposition to traditional convertible debt are the trigger and the loss absorption mechanism (write-down, write-off, or conversion to equity). The effectiveness of this asset class is often discussed essentially by spotlighting the trigger [7]. Based on the research summarized above, focusing on the triggering system should help make CoCos better, such as designing a new capital ratio trigger that optimises the conversion timing and the converted amount. This trigger should represent the financial health of a firm entering a grey zone while not being too close to a bail-in configuration nor well ahead of a dangerous financial state. The next section studies optimal capital structure theories for this purpose.

1.2.5 Capital structure of the firm

The capital structure of a firm refers to how a business or corporation finances its operations. The three financing approaches are issuing common stocks (equity), bonds (debt), or using retained earnings. While the latter option appears to be the cheapest and offers more flexibility, leveraging retained earnings for financing is not always possible.

From the mid-twentieth century, the first theory of investment that considered the cost of capital for a firm[7] drew conclusions opposing standard industry beliefs at that time. Modigliani and Miller[8] showed that the market value of any firm and its associated cost of capital is independent of its structure, leading to how the corporation chooses to finance its operations. This conjecture holds only in a perfectly efficient market, where no taxes nor bankruptcy costs exist [27]. However,

[6] Equity prices playing a solid indicator of being distressed.
[7] Firms were assumed to perform similarly within the same class of operations (e.g., utilities or oil).
[8] Awarded the Nobel Prize for Economy in 1985 and 1990.

this outcome expressed by these authors being valid in a theoretical and perfect environment has been reviewed in response to tax benefits consideration, which slightly decreases capital cost when increasing the debt leverage.

The literature reflects that opinions differ on optimising capital structure. Ultimately, investment decisions should maximise profits and market value. Because of diverging views on how to generate profits and multiple personal judgments that typically exist on a firm's board, the market value allows synthesising the plethora of individual stakeholder beliefs with their attitudes on the market[9] [27]. Nevertheless, this opinion is contrasted by real capital structures, suggesting that management also emphasises securing their position by avoiding company default[10].

Thus, due to interest tax savings, a firm benefits more from debt issuance at the cost of increasing its default probability [23]. This observation must be contrasted as bondholders can sometimes benefit from early bankruptcy because a higher bankruptcy threshold constitutes safer debt[11] [34].

While Merton did not suppose any tax rebate in its model, he observed that when the probability of default is significantly reduced, the market value of the debt approaches the present value of the high-risk debt. On the other hand, if the default probability increases, then the corporate debt will be valued closer to the firm value[12]. The demonstration from Modigliani & Miller that the possibility to undo or replicate the leverage from any financial structure with a mixed portfolio of equity and riskless debt is consistent with the most modern introduced model [24]. In fact, to find the optimal leverage ratio, Brennan & Schwartz used the factor between the levered and unlevered firm value as a reference function to maximise.

Leland achieved a similar result to Brennan & Schwarz by focusing only on maximising the firm value of a levered corporation when tax benefits exist. Evidence was provided for how the tax shield's gain might be counterbalanced by the cost of debt [42]. Because the possible bankruptcy cost is considered in this model, a lower tax benefit is derived compared to the one expressed by Modigliani & Miller. Also, the debt valuation formula from Black & Cox (1976) could be extended to show that the equity is not equivalent to a vanilla call option. Here, bankruptcy is determined endogenously because of the absence of maturity and principal redemption when the firm cannot afford the coupon disbursement by

[9] Assuming they are free to execute their willing to buy or sell a stock on the market.

[10] While default and bankruptcy appear to be anxiety-inducing words, Black and Cox soften their real effect in the conclusion of their paper by considering that they result only in a transfer of ownership between the equity- and bond-holders. If neither of these two is willing to steer the corporation, then the previous managers would be awarded an opportunity to remain.

[11] The two authors examine the bond price as a convex function of the threshold, making the debt without risk if the bankruptcy occurs when the asset value falls below the present value (PV) of the future promised value, i.e., the PV of the principal.

[12] The risk level related to the debt shows a propensity toward that of the equity.

issuing further equity. In a later version, Leland considered finite maturity eventuality and demonstrated the continuity between the two models for an unlimited time horizon [43].

Further scrutinising the opposing findings is interesting. Merton asserted that the relative risk pertaining to the debt might decline even if the bond time horizon or corporation operational risk increases. However, this result is independent of the optimal leverage as Leland & Toft and Brennan & Schwarz showed that an increase in either the time horizon[13] or firm's asset standard deviation lowers the optimal debt-to-asset ratio. When supposing the coexistence of both junior and senior debt within the structure, investor motivations might conflict regarding the *ideal* firm's overall risk as the price of the two securities could move in opposition [34]. Also, even if the bankruptcy cost is zero, the optimal leverage remains below 100%.

The resilience of a financial institution is eventually addressed by the capital structure chosen by the firm. The introduction of hybrid securities, such as CoCos, in the balance sheet of a company reduces its expected bankruptcy cost and impacts the ideal capital structure. If compared with a company that only issues straight fixed-income bonds, the existence of a CoCo will increase the optimal leverage ratio, leaving more room for corporate operations while exhibiting the usefulness of the instrument for a business with substantial asset volatility [44]. When the volatility increases, risk transfers from the CoCo investor to the straight bondholder, resulting in a higher probability of conversion.

1.2.6 Motivations

Driven by the concerns related to the inefficiency of CoCos described above, the primary incentive of the thesis is to design a new triggering ratio. The intrinsic mechanism of how debt conversion provides a potential equity capital buffer to the firm in exchange for a higher risk premium to the investor remains brilliant and must be employed on a more suitable trigger. As observed so far, only two contingent convertible instruments suffer an "induced conversion". Shortly after the ECB declared the Spanish bank Banco Popular likely to fail in June 2017, the central resolution authority for the European Union, called the Single Resolution Board (SRB), decided on the merger between Banco Popular and Banco Santander, the latter of which buying back all shares of the failing bank for a symbolic one euro. The merge occurred before the conversion, so the CoCo failed to prevent

[13]The time maturity is considered differently in the calculations of Brennan & Schwartz (1978) and Leland & Toft (1996). On the one hand, the latter authors consider a rolling debt (issued and redeemed continuously). On the other hand, the former studied the cumulative effect of a single debt with results approximating the open-ended maturity from Modigliani & Miller (1958).

such a move from the SRB, leaving the investors without any claim on the bank [45].

The mission of the SRB is to resolve the condition of failing banks at the lowest price possible on the real economy so as to minimise the direct impacts on the financial system and the public finances of EU members. In 2023, the Swiss Financial Market Supervisory Authority (FINMA) generated controversy when they decided to write off the $17 billion Credit Suisse AT1 instruments. While shareholders could recover partially from their investments, this scenario was a clear violation of the absolute priority rule. This sparked a contagion effect on other CoCo instruments as investors realised that under a Point of Non-Viability (PONV) clause, the AT1 might be junior to Equity in the hierarchy of restitution.

For regulatory reasons, the conversion trigger is often expressed as a minimum ratio of the Core Equity Tier 1 that must be maintained. However, considering the problem another way, a bankruptcy visualisation is more easily conceivable and relevant when placed on a firms value/debt leverage plan.

From the critical achievements performed by the authors reviewed in Section 1.2.5, Fig. 1.2 suggests the existence of a leverage sweet spot for every corporation given current corporate taxes and bankruptcy costs. This debt-to-asset (reciprocally, debt-to-equity) ratio is the theoretically perfect value. However, every firms management decision cannot be guaranteed to always lead to this value. If used as a benchmark for CoCo regulatory requirements, this optimal conversion leverage must be calibrated from real market data. Ideally, it should lie between the leverage maximising the firms value (likely too early) and the leverage maximising the debts value (likely too late). Additional work is necessary to prove that this level is more conservative than the 5.125% CET1 requirement to qualify the issue as AT1. Leaving the triggering requirement as a managerial choice in the bond covenant provides nothing to prevent spreading it from the optimal, calibrated value[14].

[14] Leaving the option to otherwise qualify as T2, if we soften the requirement or incorporate more conservatism.

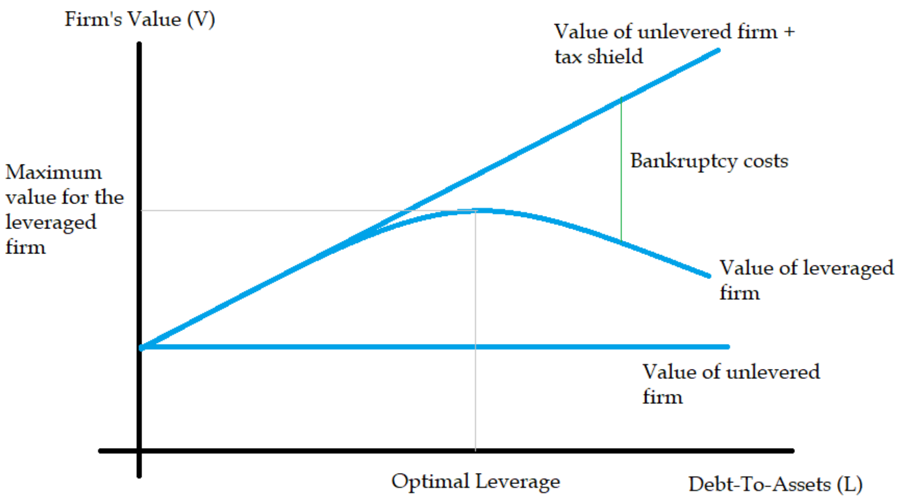


Figure 1.2: Firm's value as a function of the leverage for different corporate structures. Adapted from Chapter 60, pp. 935 of [46]

CoCos appear as a load-bearing instrument, where the leverage is defined by the debt-to-asset ratio

$$L = \frac{D}{A} = \frac{D}{E + D} \tag{1.1}$$

where A represents the assets, D the debt, and E the equity.

In the case of conversion, the debt is turned to equity, reducing the leverage ratio of the issuing firm. The probability of default PD is an increasing function of the leverage L. By mechanically reducing this leverage, the probability of default is also reduced. Because rating agencies rely on the probability of default to grade issuers, a lower PD level implies a better rating, enabling the issuing entity to access capital at a lower cost (i.e., better funding). This outcome is related to the risk-reward concept. Flannery extends our view on the importance of a bank operating with leverage that is not too high, asserting that a low credit rating limits the banks options on dealing with FX or initiating OTC transactions with counterparties. Other constraints could include difficulties in accepting additional credit lines at the request of clients [47]. CoCos should still be considered a primary candidate to avoid the repetition of the disastrous consequences of the previous financial crisis.

Finally, existing research on CoCos tends to focus either on pricing and risk management assessments or evaluating the effects attributable to the issuance on the bank's capital structure [33]. This thesis reconciles both considerations while filling the gap between research and market operationality. For example, some research introduced new products that are not suitable for direct issuance and market interest, making them potentially valuable from a theoretical perspective while revealing difficulties in their practical implementation.

Considering these multi-faceted challenges, this thesis contributes providing:

- An example of CoCo subject to the existing limits in terms of pricing and conversion risk evaluation. Chapter 2 focuses on the 2020 AT1 issuance from Arion Banki and identifies the discrepancy between its price and intrinsic risk with respect to the bank's high CET1 ratio.

- A framework for the arbitrage of CoCo and a numerical example based on a real-world listed CoCo. Chapter 3 models the realisable profit from a portfolio made of a CoCo bond and put options. An "arbitrage surface" is demonstrated to exist where an upper bound for the price of the CoCo can guarantee risk-free profit, depending on the underlying share price and volatility.

- A model for dynamically adjusting a firm's leverage level. Chapter 4 introduces a new type of hybrid instrument, called Dynamic Control of Leverage (DCL), that keeps the probability of default within acceptable values. The resulting dynamics offer interesting mean-reverting properties.

- An overview of the legal framework surrounding CoCo bonds. Chapter 5 discusses regulatory insight into CoCo bonds, delving into the current context and practical considerations, and highlights prevalent market practices.

- An efficiency examination of the proposed DCL model. Chapter 6 adapts the famous Vasicek model to examine the efficiency of DCL in terms of it ability to bind a firm's leverage between two limits. This study compares different DCL designs to a benchmark company that does not incorporate this new type of hybrid instrument in its capital structure.

- A stability and equilibrium examination. Chapter 7 considers the stability and equilibrium question in the pricing of Contingent Convertibles. The observed effects are investigated for legacy CoCo instruments and a class of Contingent Convertibles that allows for better capital control through dynamic payment or conversion to equity.

- An alternative trigger for the model, as introduced above. Chapter 8 suggests continuous leverage monitoring for DCLs, offering the desired flexibility while allowing the conversion to be triggered ahead of the cash payment. This alternative design, building upon the DCL mechanism, further reduces the probability of default and preserves the desirable mean-reverting behaviour of the leverage dynamic. A discussion is provided for the implications of this implementation and evidence of the model's effectiveness.

- A semi-analytical framework for the pricing of this new type of hybrid instrument. Chapter 9 relies on known exotic derivatives to replicate the cash flows delivered by DCL and the values from both design alternatives considered in this thesis.

Chapter 2

Implied Trigger Calibration: The Arion Banki CoCo bond

"Everybody has some information. The function of the markets is to aggregate that information, evaluate it, and get it incorporated into prices."

Merton Miller [48].

Disclosure The author does not hold any position, regardless of long or short, on Arion Bank (ISIN: IS0000028157), Arion Bank SDB (ISIN: SE0010413567), nor the CoCo Bond dealt in this Chapter (ISIN: XS2125141445). This chapter does not reflect any opinion on the bank or financial operations; also it does not constitute a solicitation to purchase or subscribe to shares or other securities of the bank.

2.1 Abstract

Twelve years after the financial crisis in Iceland and its resulting disastrous consequences, Arion Banki was the first Icelandic bank to have issued a CoCo bond on the market. The issuance had been oversubscribed, with five times more bids than offers[1], showing significant interest from investors. This chapter describes the framework for the calibration of the market-implied trigger requirement for traditional CoCo bonds featuring an accounting-based trigger that was issued by Arion

[1] The demand from 90 investors amounted to 500 million dollars, whereas the issuance was only limited to 100 million dollars.

Banki in February 2020. After its introduction, the pricing model is calibrated for this instrument based on the data on the day of the issuance. This model exhibits the implied trigger that is intended to proxy the conversion threshold. One-year historical data is then compared to the theoretical pricing to provide key hindsight on the model's accuracy that is applied globally by fixed-income desks.

2.2 Introduction

CoCos were created after the crisis of 2008 and the failure of banks to absorb losses correctly. To ensure a safe level of capital reserve, the nominal amount of the CoCo turns into equity or suffers a write-down when a certain fixed threshold is violated by the issuing entity. In addition to the mechanic conversion, based on accounting parameters, the financial regulator can decide to convert the bond on a discretionary basis when it is estimated that a Point of Non-Viability (PONV) is reached. More specifically, an accounting-based trigger relies on the current level of the firm's Core Equity Tier 1 ratio (CET1) of the bank. As the CET1 is made of the safest holdings from the capital structure (including common shares and retained earnings), it is thought as a reliable indicator for the bank's solvability and should then fulfil its role, as this asset class effectiveness is discussed mostly by setting the spotlight on the trigger [7]. That trigger should represent a firm having its financial health entering a grey zone.

Low-trigger CoCos (i.e. when the trigger is set below 5.125% of the Risk-Weighted Assets - RWA) and PONV obligation in the bond covenants are said to be leading to a debt-induced collapse, where the issuer might default before the conversion [15]. Although CoCos designed with accounting-based triggers are often criticized, both because of the lack of transparency hidden by the calculation of the CET1 level and due to the conversion relying on a backwards-looking argument; the main issue in pricing this design of CoCo lies in the non-continuously observable control variable for the conversion.

Then, with this kind of implementation, the triggering condition needs to be replicated by a proxy value in the CoCo valuation process. The implied trigger is using the underlying stock price level as a benchmark. Indeed, the share price might be considered a forward-looking indicator of the firm's financial health, encompassing all information known about the company. Additionally, it is objective, transparent, fixed, and is a continuously as well as publicly disclosed metric [41].

The chapter is structured as follows. Section 2.3 introduces a framework for modelling the conversion probability. One method relies on an intensity-based approach, also known as the Credit Derivatives Method (CDM). A second is based on the Black-Cox first-passage equation, which will be used with respect to the underlying share price (i.e., the equity approach). In Section 2.4, we present the Arion Banki CoCo issuance event. Then, we calibrate our model to obtain the implied triggering share price in Section 2.5 based on the historical data from the issuance day. The last section (2.6) interprets these results, followed by a conclusion on the appropriateness of implied-triggers models and the question of using accounting triggers in CoCos.

2.3 Modelling of the conversion probability

2.3.1 An intensity-based approach

The Credit Derivatives model for the pricing of CoCos is an intensity-based approach that relies on the credit triangle formula [31],

$$s = \lambda(1 - R) \tag{2.1}$$

The original purpose of 2.1 was to link the credit spread on an issuance to a recovery rate R in case of default through an intensity factor λ that translates the default probability. Assuming a conversion event within the CoCo scope is intensity-based, the credit triangle can be adapted in the following way. The spread that the CoCo should pay in excess of the risk-free rate r_f is represented as s, and λ is the conversion intensity that reflects the probability of conversion between phases t and $t + dt$. R is the CoCo intrinsic recovery rate defined by the market value of the shares issued through the conversion (written as I^*) divided by the nominal investment in the CoCo (written as N),

$$R = \frac{I^*}{N}$$

where I^* is the market value of the newly issued shares. This value is obtained by multiplying the value of one share at conversion time (S_c) by the conversion ratio (C_R), which is the number of shares resulting from the conversion. After conversion, the investor is entitled to receive a number $C_R = \frac{N}{C_P}$, where C_P is the conversion price. Assuming they sell the shares at conversion time, the value I^* of his investment is

$$I^* = C_R S_c \tag{2.2}$$

Then, the recovery rate R in the case of the conversion is

$$R = \frac{S_c}{C_P} \tag{2.3}$$

Assuming no delay follows conversion, the total market value of the newly issued shares received by the investor can be approximated by S_c, the price per share at conversion time.

To relate the conversion intensity λ to the effective conversion probability $P(\tau \leq T)$ and model the waiting time, we assume the conversion, such as the default event, to be exponentially distributed. This assumption is consistent with the need for a continuous memoryless distribution, where the past history of the process does not affect its future behaviour. Then, we can write

$$P(\tau \leq T) = 1 - \exp(-\lambda T) \tag{2.4}$$

we know that the credit spread of CoCos can be linked to the triggering probability in an intensity-based approach by inserting Eq. 2.3 into Eq. 2.1, resulting in the following (after isolating λ),

$$P(\tau \leq T) = 1 - \exp\left(-\frac{s}{\left(1 - \frac{S_c}{C_p}\right)} T \right) \tag{2.5}$$

Assuming S_c and C_P are provided and based on the hybrid security credit spread (accessible from the market), Eq. 2.5 reflects the market belief in terms of a triggering probability at any time t before the horizon T.

2.3.2 An equity-based model

By its discrete nature, the CET1 is an indicator impossible to model practically. For this reason, the conversion condition on the CET1 is replaced by a market-value equivalent condition. Let τ be the time of the conversion event, i.e., the instant when the share price S_t drops below its minimum required threshold S_c to equivalently keep the CET1 ratio above the conversion level between the issuance t_0 and bond maturity T. Then,

$$\tau = \left\{ \min_{[0;T]} t \quad / \quad S_t < S_c \right\}$$

Such parallelism is often conducted by researchers in the CoCo space to translate various discontinuously disclosed trigger indicators, as in the following examples:

- De Spiegeleer and Schoutens [49] also replaced the CET1 condition with a requirement on the minimum share price when working on the Lloyds

Enhanced Capital Notes, the Buffer Capital Notes from Credit Suisse, and the Senior Contingent Notes from Rabobank[2]

- Albul et al. [50] modelled a CoCo where the conversion is dependent on the minimum asset value reached by a firm. These researchers assert their theorem on the existence of a unique equilibrium in prices that is suitable for an implementation based on the stock value.

Next, We determine $P(\tau \leq T)$, i.e., the probability the CoCo will be triggered at any time between $t = 0$ and its maturity T. This approach refers to the first passage equation for a geometric Brownian motion (gBm) that was derived by Black & Cox [34]. We assume that the CoCo conversion is triggered when the stock value drops below an implied level S_c. Thus, by letting $m_t = \min_{[0;t]} S_u$, the minimum value achieved by the process S_t before maturity, there is equality between the relations,

$$P(m_t \leq S_c) = P(\min_{[0;t]} S_u \leq S_c) = P(t \geq \tau)$$

Finally, as noted by Chin, Ólafsson, and Nel [51], the cumulative distribution function (CDF) for the first-hitting time in the case of a gBm is

$$P(m_t \leq S_c) = \phi\left(\frac{\log\left(\frac{S_c}{S_0}\right) - \alpha T}{\sigma\sqrt{T}}\right) + \left(\frac{S_c}{S_0}\right)^{\frac{2\alpha}{\sigma^2}} \phi\left(\frac{\log\left(\frac{S_c}{S_0}\right) + \alpha T}{\sigma\sqrt{T}}\right) \tag{2.6}$$

where $\alpha = \mu - \frac{\sigma^2}{2}$, $\mu = r - \delta$, and ϕ is the standard normal cumulative distribution defined as

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy$$

This formula (2.6) gives the risk-neutral probability that the stock price S attains the threshold S_c at any moment during the period $[0; T]$ based on the information vector $\boldsymbol{I} = (S_0, S_c, \mu, \sigma, T)$.

If required, the dividend δ can be adjusted to the risk-neutral world from a foreign investor perspective f. The change of numeraire from the underlying equity currency (domestic d) to the foreign currency f [18] is

$$\delta^f = r^f - r + \delta + \rho_{E/FX} \cdot \sigma \cdot \sigma_{FX} \tag{2.7}$$

[2] In the Rabobank issuance, the accounting trigger is defined by the ratio of the Equity Capital divided by the RWA, and not directly the CET1 ratio.

where r^f is the risk-free rate on the foreign currency, $\rho_{E/FX}$ the correlation factor between the equity and the FX rate, and σ_{FX} the volatility on the FX pair.

Later in Section 2.5, Eq. 2.6 will be calibrated to find the implied trigger S_c. In this context, calibration means varying S_c within a plausible spectrum to identify the values that allow equality between the conversion probabilities obtained through the two models. As we apply real market data (specifically, the tradeable credit spread of the hybrid obligation) as inputs in Eq. 2.5 for the credit model, our calibration is equivalent to finding the market belief in terms of the implied triggering price.

We next provide an overview of the micro- and macro-environments of the CoCo bond treated in our study.

2.4 The Arion Banki CoCo

Arion Banki, as a part of their Medium Term Note Programme, was the first Icelandic institution to issue Additional Tier 1 Convertible Notes. According to the Information Memorandum, the USD 100 Million issue bears an annual fixed 6.25% interest rate for a five-year period beginning from issuance (February 26, 2020) to the first-callable date. Following that, the coupon is reset at a fixed 4.842%, plus the CMT Rate [52].

The issue was perpetual with a non-callability period. In this case, "NC5.5," meaning that the issuer can redeem the note *at par* after five years (February 26, 2025) and at any time for a period of six months from then[3].

[3] Other optional redemption times existed but are not discussed in this scope, e.g., at any interest payment following the first reset date or upon a Withholding Tax Event, Tax Deductibility Event, or a Capital Disqualification Event. In all cases, optional redemption is subject to supervisory approval (as required for AT1 eligibility).

Arion AT1 CoCo	
Issuer	Arion Banki Hf.
Currency	USD
Coupon	6.25% (fix-to-variable)
Maturity	Perpetual
Issue Date	Feb 26, 2020
First Call Date	Feb 26, 2025
Amount Issued	USD 100,000,000
ISIN	XS2125141445
Bloomberg ID	ARION 6 1/4 PERP
S&P Rating[4]	BB

Table 2.1: Information regarding the Arion Banki AT1 CoCo. Source: Bloomberg, Arion Banki [52]

The conversion to equity would occur if the consolidated Core Equity Tier 1 ratio of the bank falls below 5.125%, in addition to a PONV discretionary conversion eventuality, called a Non-Viability Event (NVE) by the bank. For simplicity, we assume the face value to be USD $1,000$[5]. The bond was denominated in U.S Dollars (USD, foreign currency) and the Arion Banki shares were denominated in Icelandic Kronur (ISK, domestic currency).

In the case of a triggering event, the conversion price of the CoCo is not deterministic but defined as the highest between the following values:

- The ISK equivalent of USD 0.473 at the time of the conversion (first floor).
- The par-value of one Arion Banki share, i.e., ISK 1.0 (second floor).
- The price at the time of conversion, i.e. S_c, which ensures avoiding the case $S_c > C_P$ where the CoCo holder benefits from the conversion and may be tempted to drive the share price down by shorting the stock or buying put options (as the recovery would be higher than one).

The second floor acts as a regulatory or safety net to take the lead on the first floor if the USD/ISK FX rate falls to 2.11. We exclude this event because we assume its probability is negligible. Otherwise, it would involve a 98% drop in the USD valuation compared to the ISK.

Arion Banki's AT1 instrument was not the only CoCo to feature such a conversion price C_P. The first CoCo issued by Credit Suisse in 2011 also encompassed a conversion price defined as the maximum between the share price at conversion and a pre-set floor price [49].

[5] In reality, the denomination is USD 200,000 with USD 1,000 increments

Now, in the Arion Banki case, the conversion price can be written as

$$C_P = \max(S_c^f, 0.473) \quad (2.8)$$

The floor is designed to limit the dilution risk for the initial shareholder by avoiding the eventuality of an infinite issuance of new shares.

We also exclude a conversion occurring at a share price above 0.473 USD (under the foreign numeraire) as it would imply a recovery rate equal to 100% (see Eqs. 2.3 and 2.8).

Reviewing the historical data, on the close of February 26, 2020, the issuance had a mid yield-to-call at 6.165%, close to the coupon as the CoCo traded around the par value. Considerations about the yield-to-call are justified in Section 2.5. Additionally, the underlying stock price traded at ISK 81.0 per share. We introduce additional historical inputs required for the numerical application in Table 2.2.

Additional Inputs	
Prevailing market conditions date	February 26, 2020
Mid Yield-To-Call, y_{market}	6.165%
$FX_{USD/ISK}$	127.87
Share price (domestic) S_0	81.0
Share Price (foreign) S_0^f	0.6335
Dividend Yield δ	6.6%
Observed spread (market) s	$s = y_{market} - r^f = 503$ bps
Reference rate - US Gov 5Y Yield (foreign) r^f	1.133%
Icelandic Gov 5-Year Yield r	2.862%
Equity Volatility σ	26.09%
FX Volatility σ_{FX}	9.62%
Equity/FX Correlation factor $\rho_{E/FX}$	−0.0151

Table 2.2: Historical micro- and macro-environment drivers for Arion banki. Source: Reuters, Arion Banki [52]

Arion Banki was listed on the market in June 2018, so the volatility and correlation factors are computed on the period from June 15, 2018 to February 26, 2020. With these framework inputs at the specified issuance date, we next process the calibration of the implied market trigger S_c.

2.5 Model calibration

The price of the CoCo (observable on the market) contains information about the expectations of market operators. More specifically, it allows deriving the share price S_c that would "equivalently" violate the requirement on the CET1 ratio.

The issue being perpetual, we assume that the maturity is equal to the distance to the first callable date, as is used in the industry to call perpetual issuance at the first possible time. However, this assumption neglects the so-called "extension risk." In February 2019, the bank Santander showed that this hypothesis might be accompanied by tangible consequences. At this time, the Spanish bank created a controversy after skipping the first callable opportunity on their €1.5 billion CoCo issue [6]. A similar event occurred on a Deutsche Bank CoCo bond in 2020. Some models, such as the Analytically-Tractable First Passage (AT1P), are more reliable as they consider the extension risk and the coupon cancellation eventuality [33]. However, these are unusable in the case of Arion Banki because no Credit Default Swap (CDS) exists at that time for the Icelandic bank.

Given that the Arion Bank bond was paying semi-annual interests, the Yield-To-Call (YTC) can be linked to the market price of the issuance by

$$P = \left(\frac{C}{2}\right) \frac{1 - \left(1 + \frac{YTC}{2}\right)^{-2N}}{\frac{YTC}{2}} + \frac{CP}{\left(1 + \frac{YTC}{2}\right)^{2N}} \tag{2.9}$$

where C is the annual coupon value, CP the Call Price (equal to the face value of the bond, in this case), and N the number of years before the first call date. This is a direct derivation of the discounted future cash-flows formula by identifying a geometric sum of the common ratio $q = \frac{1}{1+YTC}$. This yield metric provides the investor with information about the expected return from the bond if held until the first callable date, which might differ from the yield to maturity that bears more uncertainty in the case of a CoCo.

Mathematically, the aim is to find the value(s) of S_c that set the left- and right-hand sides equal when setting the conversion probabilities obtained through the intensity model (Eq. 2.5) equal to the probability obtained through the equity model (Eq. 2.6). Graphically, the calibration results from the *intersection* of these two probability functions and relies on the available market information, i.e., the CoCo spread (in Eq. 2.5) and the share value characteristics (in Eq. 2.6), both observed on the day of issuance.

[6] ISIN: XS1043535092

All other parameters are set constant, so the intersection(s) between the two curves, projected on the x-axis, allows calculating the implied triggering price S_c in ISK that is required to set the conversion probability equal at inception. Because of the convexity of the curve, and depending on the configuration, either zero, one, or two S_c values can verify the equality between the two equations depending on the other market parameters.

The triggering share price S_c is obtained by the intersection between the equity-based and credit-based model. Fig. 2.1 plots the results of the calibration, given the conversion price C_P derived in Section 2.4.

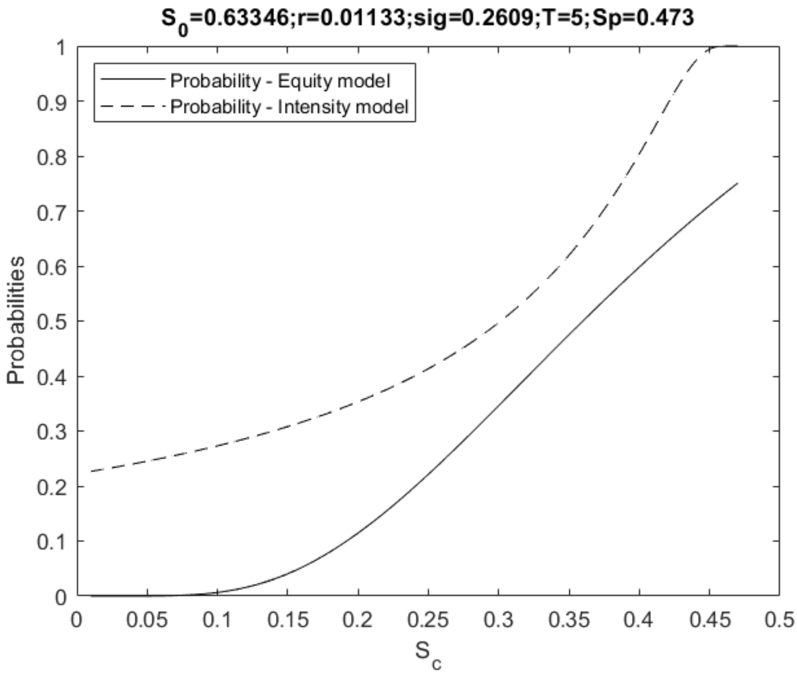


Figure 2.1: Calibration attempt of the implied trigger in terms of share value (USD). No intersection exists, so no conversion price S_c is obtained from the probability functions (Eqs. 2.5 and 2.6).

In the prevailing CoCo configuration at issuance, the conversion probability derived through the credit model (intensity approach, Eq. 2.5) never intersects with the conversion probability derived based on the equity model (Eq. 2.6). The approach using the market spread s results in a conversion probability higher than that obtained through the equity modelling, regardless of S_c.

Two conclusions are drawn from this result:

- **The market fails to estimate the conversion risk accurately, so it tends to overestimate it by pricing the CoCo below its fair price (resulting in a higher yield).**
- **The equity approach relying on an implied share price is not appropriate for the pricing of CoCo bonds.**

We develop these two aspects further in Section 2.6.

A different configuration could result in one or two possible solutions for the implied trigger S_c, which is attributable to the Black-Scholes model parameters used in assessing the conversion probability within the equity model. The Black-Scholes model, under the assumption of constant volatility, can yield different numbers of solutions for a given volatility value, depending on how the volatility σ compares with a certain threshold σ^*, including:

- Two solutions arise when $\sigma > \sigma^*$.
- One solution occurs at $\sigma = \sigma^*$.
- No solution might be found, as in our example, when $\sigma < \sigma^*$.

The aforementioned behaviour is not a novel discovery, as it has been handled in the financial literature by De Spiegeleer and Schoutens, who addressed this while calibrating the implied triggering price of a Credit Suisse CoCo [49].

When calibration results in two solutions for the implied stock price S_c, selecting one is usually not possible, as both are mathematically valid. However, one candidate S_c could be disqualified if its level is higher than the floor price FP, which would lead to a recovery rate equal to 100% for the CoCo investor. This outcome can be demonstrated by starting from the conversion price (C_P) definition of

$$C_P = \max(S_c, FP) \tag{2.10}$$

The conversion rate C_R can then be written as

$$C_R = \min\left(\frac{N}{S_c}, \frac{N}{FP}\right) \tag{2.11}$$

Upon conversion to equity and assuming the CoCo-holder can sell new shares immediately following conversion (at the price S_c), they are entitled to the recovery amount of

$$Re = C_R \,.\, S_c = \min\left(\frac{N}{S_c}, \frac{N}{FP}\right) . \, S_c \tag{2.12}$$

Here, if $S_c > FP$, then the recovery amount is equal to the CoCo nominal ($Re = N$).

2.6 Interpretation

The bond, having been issued on the market in 2020, already delivered key hindsight on the model viability, and by extension, on CoCos relying on accounting triggers. We plot in Fig. 2.2 Arion Banki's share price together with the CoCo value (upper panel) along with the quarterly disclosed bank's Core Equity Tier 1 ratio during 2020 (lower panel).

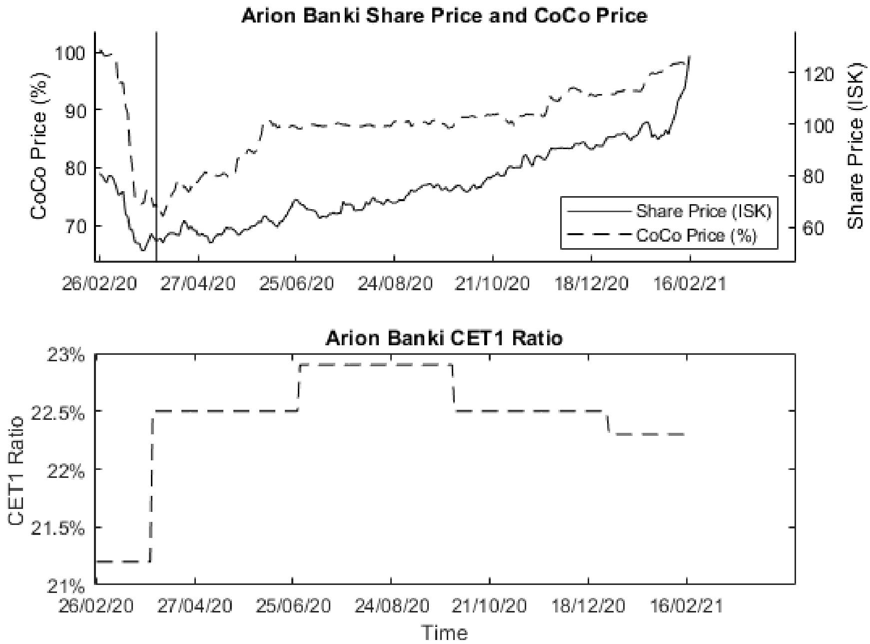


Figure 2.2: Comparison of the CoCo price (%) and share price (ISK) (upper panel) and the CET1 Ratio (lower panel) for Arion Banki. The straight line denotes the Q1 quarterly statement (March 31, 2020). Data: Bloomberg, 2020 Financial Statements [53].

The strong $\rho = 83.5\%$ positive correlation between the share value and the CoCo price is not unexpected, given the equity component of the hybrid security. Nonetheless, this result partly reflects how the market heavily relies on the underlying equity market to price the CoCo bond (as observed, when the share price drops, the CoCo price follows the same trajectory, and *vice versa*).

The plummet in market capitalisation during the first quarter of 2020 is due to the COVID-19 crisis. Under the assumption of efficient markets, the approximately 36% drop in the share price should reflect a deterioration in the bank's financial health. However, a few days following this all-time low, on March 31, 2020 (end of Q1 2020 related to Arion Banki's financial statement), the bank exhibited a robust 22.5% CET1 ratio, rising over the three preceding months, far from the real 5.125% applicable triggering level.

This market behaviour had an important repercussion. Following the release of the Q1 2020 bank's financial statement, the expectation would be that the CoCo trades at lower yields (equivalently, the price of the CoCo instrument increasing). If the hybrid instrument is split between an equity and a debt component, then the CoCo should be driven by a debt behaviour as the conversion eventuality is a distant concern. We emphasize again that the Q1 2020 report stated, underlying the solidity of Arion Banki, a 22.5% disclosed CET1 ratio, compared to the 5.125% level that would trigger the conversion to equity.

Instead of recovering from the heavy loss from March 2020, the bond continued being traded at a discount, even after the confirmation that it would not turn to equity in the near future. In other words, **these market price movements were not aligned with changes in the fundamental trigger measure.**

Because the CoCo conversion is driven by the issuer's CET1 ratio, an accounting measure of the firm's ability to absorb losses, the conversion time τ depends on the amount of capital a firm holds relative to its total risk-weighted assets. This measure does not directly reflect the market sentiment and should not be assumed to be inextricably linked with the underlying share price. More generally, we could expect the CET1 ratio and the share price to be two (highly) positively correlated variables. Here, the former is the proportion of the safest bank holdings with respect to the total risk-weighted assets in the balance sheet, and the latter is a market value mirroring the firm's financial health. As we highlighted, the empirical data show the inverse effect, with a share price moving in the opposite direction of the CET1 ratio. Therefore, because the CET1 and share price move with a ratio different than 1:1, the use of an implied share price is not appropriate for the purpose of pricing CoCo bonds, as we demonstrate it is not a reliable conversion indicator.

2.7 Conclusion

Contingent convertibles qualifying as AT1 today are mostly relying on the CET1 ratio measurement to trigger a conversion event that would turn the nominal of this debt into equity from the issuing company.[7]

Unfortunately, the CET1 ratio is only disclosed quarterly by the bank, leaving CoCo investors in unknown territory for extended periods, as they cannot accurately monitor the conversion risk. Instead, they attempt to map the requirement

[7] Excluding the Write-Down AT1 instruments that are simply writing-off partially or totally the AT1 nominal

on the CET1 that is set in the bond covenant and an equivalent minimum stock price threshold.

In this chapter, we introduced a case study to calibrate an implied trigger on a currently trading Contingent Convertible. After introducing two different models for estimating the conversion probability of CoCo instruments (equity- and credit-based), we attempted calibration of the model based on the known observable micro- and macro-economic data related to a historical issuance from the Icelandic bank Arion Banki.

We then demonstrated that in the early trading days of the hybrid security, the market seemed to extensively overestimate the triggering probability, ultimately trading the CoCo price at a steep discount. The share price was a forward-looking market value, but the "effective" conversion relied on a past-looking accounting measure. This illustrated the existence of discrepancies in the pricing, notably after the release of the Q1 2020 financial statement, exhibiting the robustness of the bank (in terms of Core Equity Tier 1) but still having a CoCo instrument trading far below par on the market. At this time, the CoCo should have behaved as a plain debt and not equity-like. Instead, it created an opportunity to benefit from the difference of information encompassed by market data on the fixed-income and equity segments.

As the efficient market assumption is an unreachable market paradigm, the need for a continuous and transparent observable control variable as a triggering mechanism for Contingent Convertible would reduce the uncertainty around conversion events for this type of hybrid security and ultimately benefit the AT1 industry.

Chapter 3

CoCo Arbitrage Framework and Numerical Example on Arion Banki

"The job of the Central Bank is to worry."

Alice Rivlin (1931-2019), former Vice Chairman of the Board of Governors of the FED.

3.1 Introduction and mechanisms

This chapter provides a generalized framework applicable to a wide class of Contingent Convertible bonds to evaluate if its current market environment allows benefiting from a potential pricing discrepancy compared to its associated real conversion risk. This framework is suitable for hybrid instruments designed in a similar way as the AT1 instrument from Arion Banki.

Our innovative approach establishes a condition on the CoCo trading price and share value that allows systematically taking arbitrage of the market by constituting if a portfolio Q pays a positive cash flow upon early conversion. We emphasise that the proposed actions do not require the conversion probability, which is an opaque measure because of the impossibility of practically modelling the backwards-looking triggering measurement (CET1 ratio) or interference with the PONV clause.

For the purpose of future calculations, we express the CoCo dirty price P_t (at time t) to be calculated as the sum of its clean price CP_t (market value quoting continuously) and the accrued interest AC_t, i.e., $P_t = CP_t + ACt$. The accrued interest is a function of the coupon rate c, the frequency of payment f, and the time before the next interest payment t_p, such that $AC_t = N\frac{c}{f}\left(\frac{t}{t_p}\right)$. Also, r is the risk-free rate in the CoCo-denominated currency. For simplicity, the interest rate term structure is assumed flat. While not a prerequisite for this strategy, this simplification allows a discount of the coupons from the borrowing at the same rate.

The chapter is structured into eight sections as follows. Section 3.2 discusses the underlying motivations of the work presented in this chapter, and Section 3.3 introduces a basic framework for the hedging and the identification of arbitrage opportunities on Zero-Coupon CoCo bonds (ZCCB). We next extend this model by adding the eventuality of coupon payments and different currencies in Section 3.4. Two numerical examples, including a general application and a ZCCB, are presented in Sections 3.5 and 3.6, respectively. Finally, we introduce a real scenario based on the historical market data for the Icelandic bank Arion Banki (Section 3.7). We discuss the extension risk in Section 3.8 before concluding our findings in Section 3.9.

3.2 Motivations

Contingent Convertibles remain in a maturing phase with evidence of pricing flaws, leading to arbitrage opportunities and the possibility of locking-in risk-free profit for the investors.

With CET1-based designs, different pricing methods exist. However, in illiquid markets where some types of derivatives instruments are missing, such as Credit Default Swaps (CDS), a proxy value must replicate the triggering condition. The so-called implied trigger uses the underlying stock price level as a benchmark. The share price might be considered a forward-looking indicator of the firm's financial health because it encompasses all information known about the company. Additionally, it is objective, transparent, and fixed while being a continuously and publicly disclosed metric [41].

Fourteen years after the financial crisis and the well-known disastrous consequences in Iceland, Arion Banki is the first Icelandic bank to have issued a CoCo bond on the market in February 2020[1]. The existing historical data used in this Chapter has been introduced in Chapter 2, Fig. 2.2.

CoCos are split between equity and debt components. So, at the time of the Q1 2020 financial statement release by Arion Banki, the instrument should have been driven by a debt behaviour as the conversion eventuality is a distant concern. However, this was not the case, as the market price movements were not aligned with changes in the fundamental trigger measure.

3.3 Zero-Coupon CoCo Bond (ZCCB)

Initially, we only consider a ZCCB with a conversion feature that has a maturity value defined by

$$P_T = N\mathbb{1}_{\{\tau>T\}} + \frac{N}{C_p}F(\tau,T)\mathbb{1}_{\{\tau\leq T\}} \tag{3.1}$$

where N is the denomination of the CoCo bond redeemed at T if not converted, C_p the conversion price, τ the time of the conversion, and $F(\tau,T)$ the forward value of the underlying share at conversion time.

By defining S_τ as the share price at conversion time τ, in the case of conversion, the investor receives a recovery value on the CoCo equal to

$$R_{CoCo} = C_r * F(\tau,T) = N * \left(\frac{1}{C_p}\right) * S_\tau \tag{3.2}$$

By purchasing the hybrid instrument on borrowing, the recovery portfolio value Q_R^* at time t can be written as

$$Q_R^*(t) = ND(t,T)\mathbb{1}_{\{\tau>T\}} + \left(\frac{N}{C_p}\right)S_\tau D(t,\tau)\mathbb{1}_{\{\tau\leq T\}} - P_t \tag{3.3}$$

The recovery value on the portfolio Q_R^* is well defined as the sum of (a) the face value N if the CoCo does not convert before T, (b) the recovery value from the new shares if the conversion occurred between two payment times, and (c) the negative cost to purchase the instrument on borrowing.

[1] In September 2021, Íslandsbanki, another Icelandic bank, has also issued an Aditional Tier 1 instrument denominated in SEK. The note is in the form of a floating-rate perpetual bond with Temporary Write-Down feature, triggered if the bank's CET1 falls below 5.125% of the RWA (ISIN: XS2390396427).

Knowing that the payoff of a put option is $Put_T = (K - S_T)\mathbb{1}_{\{S_T<K\}}$, we can build the hedged recovery portfolio by adding a number Δ put option, purchased at time t on borrowing. Then, at this time t, the hedged portfolio is valued as

$$\iff Q_R(t) = Q_R^*(t) + \Delta\left[(K - S_T)\mathbb{1}_{\{S_T<K\}}\exp\left(-r(T-t)\right) - Put(S_0, K, 0, T)\right] \tag{3.4}$$

by setting

$$\begin{cases} \Omega(t,\tau) & = ND(t,T)\mathbb{1}_{\{\tau>T\}} + \left(\frac{N}{C_p}\right)S_\tau D(t,\tau)\mathbb{1}_{\{\tau\leq T\}} + \Delta(K - S_T)\mathbb{1}_{\{K<S_T\}}D(t,T) \\ \Theta(P_t, S_t) & = P_t + \Delta Put(S_0, K, 0, T) \end{cases}$$

By fixing the strike K and the number of put options Δ, we can search for all combinations (P_t, S_t) for which the inequality $\Omega(t,\tau) > \Theta(P_t, S_t)$ is satisfied at any time t (i.e., $\forall t$). When this condition holds, the strategy is profitable as the CoCo payoff upon conversion or redemption is higher than the cost of setting up the strategy.

3.4 Extended framework

Without any loss of generality, the initial model can be extended by (a) incorporating coupon payments (the coupon yield c paid at a frequency f) delivered at fixed times k until conversion or maturity and (b) considering the case of a bond issued in a foreign currency f, with the underlying equity denominated in the domestic currency d. When deploying this investment strategy, the following two unfavourable scenarios exist where the profitability is minimal:

(a) The conversion occurs soon before maturity at a share price equal to the put strike. In such a case, the hedge cost is lost as both the option time and intrinsic values are worth zero.

(b) The conversion occurs before the first interest payment ($k = 1/f$), but the share price at conversion (S_c) is high. In such a case, the hedge cost is not compensated by a coupon payment.

On the one hand, within this extended framework, scenario (a) is circumvented by adding the option maturity T_o as a driver for the model. If $T_o > T$, then the option time value remains positive at the CoCo maturity/first-call date. On the other hand, scenario (b) is ruled out as it would imply a market capitalization that increases drastically over a very short period, followed by a conversion triggered anyway.

To simplify the following equations, the share price at conversion time S_τ is written as S_c, and we define F as the time (in years) between two payment dates, such that

$$F = \frac{1}{f} \tag{3.5}$$

We next rewrite the terminal values of the portfolio Q_R^* given a conversion occurring between two interest payment dates ($k - F$ and k) as

$$\begin{aligned} Q_R^*(T \mid k - F < \tau < k) = N \exp(-rT)\mathbb{1}_{\{\tau > T\}} + \left(\frac{Nc}{f}\right) \sum_{i=1}^{T} \exp(-ri)\mathbb{1}_{\{\tau > i\}} \\ + \frac{N}{C_p} S_c \sum_{i=1}^{T} \exp(-ri)\mathbb{1}_{\{i-1 \le \tau < i\}} - P_0 \end{aligned} \tag{3.6}$$

The conversion price C_p features a floor price FP that limits the dilution for the initial shareholders and is defined as the maximum between FP and the share price at conversion time S_τ, such that

$$C_p = \max(S_\tau, FP) \tag{3.7}$$

Fig. 3.1 presents the behaviour of this portfolio for various conversion eventualities. The conversion time τ drives the number of coupons being paid before the debt turns to equity, which affects the final payoff.

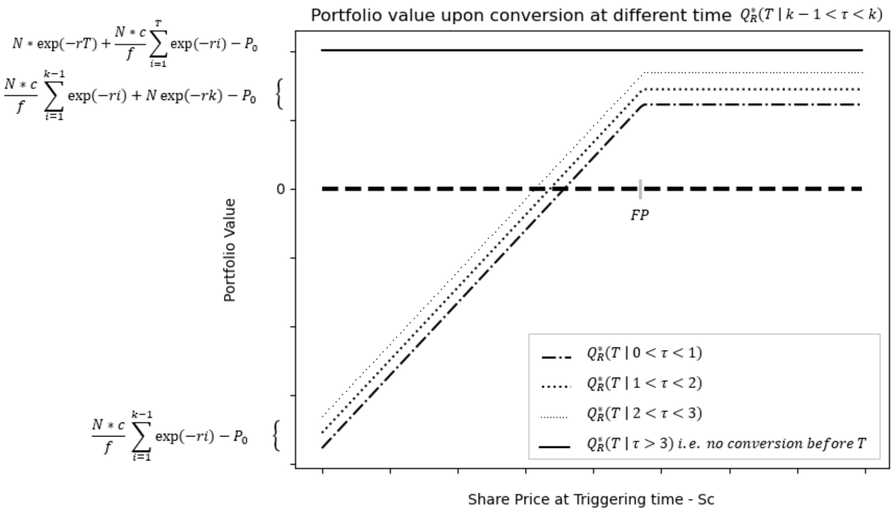


Figure 3.1: Portfolio recovery value $Q_R^*(t)$ as a function of the share price at conversion time S_τ for different scenarios of the triggering time, with $T = 3$ and calculated with Eq. 3.6

.

The Fig. 3.1 highlights the following portfolio characteristics:

- The value of Q_R^* is maximized when the hybrid instrument is redeemed early or matures at T. The portfolio becomes equal to the redemption value plus all the intermediary coupons discounted by the interest rate r less the purchased price, i.e.,

$$Q_{R,max}^*(T \mid \tau > T) = N\exp(-rT) + \left(\frac{Nc}{f}\right)\sum_{i=1}^{T}\exp(-ri) - P_0 \tag{3.8}$$

  In this specific case, Q_R^* is not dependent on the underlying share price.

- The portfolio value is minimized when the hybrid instrument converts into worthless shares, $S_c = 0$. Therefore, Q_R^* is negative due to the purchase cost. This loss is reduced by any potential intermediary coupons paid before conversion, such that

$$Q_{R,min}^* = \left(\frac{Nc}{f}\right)\sum_{i=1}^{k-1}\exp(-ri) - P_0 \tag{3.9}$$

The sum index spans the first coupon payment to the final payment before default, $(k-1)$.

- When the share price at conversion time is above the floor price, FP, the conversion price is $C_p = S_c$ (see Eq. 3.7). Therefore, the investor receives shares valued at the CoCo denomination of $\frac{N}{C_p}S_c = N$. When $S_c < FP$, the slope of the curve is defined by $\frac{N}{C_p}$.
- The more the conversion is delayed, the more the break-even will be shifted to the left, meaning that a lower share price at conversion (S_c) will be sufficient to break even. This is due to intermediary coupons being paid at a frequency f and reinvested at the risk-free yield r.

The intrinsic portfolio risk lies in the conversion eventuality, making Q_R^* sensible to the share price at conversion time S_c. In this scenario, the portfolio value replicates the structure of a short put option.

In the specific case of different currencies being used for the bond and equity, we resort to quanto put options for the hedge. Writing d and f as the domestic and foreign currencies, respectively, the price of a d-denominated equity put option struck in a pre-determined investment currency f is derived in Sec. 6.2.1, Problem 11 and 12c in [18] and Sec. 23.5 in [54]. The proof is demonstrated here in Appendix A.

From the Black-Scholes Equation in the case of a Cross-Currency Option valued as V, we write

$$\begin{aligned}\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2_{stock}S_t^{d^2}\frac{\partial^2 V}{\partial S_t^{d^2}} + \frac{1}{2}\sigma^2_{FX}FX^{f/d^2}\frac{\partial^2 V}{\partial FX^{f/d^2}} + \\ \rho_{stock/FX}\sigma_{stock}\sigma_{FX}S_t^d FX^{f/d}\frac{\partial^2 V}{\partial S_t^d \partial FX^{f/d}} + \\ (r^d - \delta^d - \rho_{stock/FX}\sigma_{stock}\sigma_{FX})S_t^d\frac{\partial V}{\partial S_t^d} + \\ (r^f - r^d)FX^{f/d}\frac{\partial V}{\partial FX^{f/d}} - r^f V(S_t^d, FX^{f/d}, t) = 0\end{aligned} \tag{3.10}$$

Using specifically the payoff of a quanto put option, i.e.,

$$\Psi(S^d(T_o), FX^{f/d}(T_o)) = \overline{FX^{f/d}}\ .\ \max(K^d - S^d(T_o), 0)$$

, it follows that the price of a d-denominated equity put option struck in a predetermined investment currency f is

$$Put^f(K^d, t, T_o) = \overline{FX^{f/d}}\left[K^d \exp\left(-r^f \,.\, (T_o - t)\right)N\left(-d_2\right) - S_t^d \exp\left(-\delta^f \,.\, (T_o - t)\right)N\left(-d_1\right)\right] \tag{3.11}$$

with

$$d_1 = \frac{\log\left(\frac{S_0^d}{K^d}\right) + (r^f - \delta^f + 0.5\sigma_{stock}^2)(T_o - t)}{\sigma_{stock}\sqrt{T_o - t}}$$

and

$$d_2 = d_1 - \sigma_{stock}\sqrt{T_o - t}$$

The dividend δ^d is changed to δ^f to consider a quanto adjustment as

$$\delta^f = r^f - r^d + \delta^d + \rho_{stock/FX}\sigma_{stock}\sigma_{FX} \tag{3.12}$$

The origin of the quanto adjustment is verified in Appendix A, and N is the standard normal cumulative distribution defined as

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}\,dy$$

Now, the equation for our hedged portfolio follows as

$$Q_R(T \mid k - F < \tau < k) = Q_R^*(T \mid k - F < \tau < k) + \Delta\left(Put^f(K, \min(k,T), T_o) \,.\, e^{-r^f \min(k,T)} - Put^f(K, 0, T_o)\right) \tag{3.13}$$

If we show that (a) for any k, the proceed from conversion results in $Q_R(T;k) > 0$, and (b) the exercise of the early redemption right by the CoCo issuer also leads to $Q_R(T;k) > 0$, then we demonstrate the existence of an arbitrage opportunity.

Assuming $k - F < \tau < k$, we calculate the yield y perceived in excess of the risk-free return by setting equal the forward value for our investment and its payoff at conversion, then solve for y. The margin y is an indicator of the discrepancy in terms of conversion risk between the market expectations and its actual impact, such that

$$y(k) = -r^f + \left(\frac{1}{k}\right).$$

$$\log\left(\frac{N\mathbb{1}_{\{\tau>k\}} + \left(\frac{Nc}{f}\right)\sum_{i=1}^{T} e^{i*r^f}\mathbb{1}_{\{\tau>i\}} + \frac{N}{C_p}S_c\mathbb{1}_{\{\tau<k\}} + \Delta Put(K,\min(k,T),T_o-k)}{P_0 + \Delta Put(K,0,T_o)}\right) \tag{3.14}$$

Condition for arbitrage

Decreasing P_0 or increasing S_0 observably results in a decrease of the strategy cost through a lower "upfront cost" or lower hedge price (the put options, respectively. Therefore, we translate the above arbitrage condition at its limits in terms of a maximum initial value for the CoCo price P_0 or the minimum initial share price S_0.

Calibrating the market vector $\mathbf{I} = (P_0, S_0)$ that leads to an arbitrage opportunity becomes possible given a fixed CoCo design by setting $Q_R(T \mid k-F<\tau<k) \geq 0\ \forall k$ from Eq. 3.13 and isolating either P_0 or S_0.

Maximum CoCo price

As we evaluate the recovery portfolio value $Q_R(T \mid k-F<\tau<k)$ for various conversion times $k \in [F, T+F]$, we next define $P_{0,k}$ as the set of initial CoCo prices that satisfy $Q_R(T \mid k-F<\tau<k) \geq 0$.We also set the quantity $\Pi(T;k)$ as

$$\Pi(T;k) = Ne^{(-rT)}\mathbb{1}\{\tau>T\} + \left(\frac{Nc}{f}\right)\sum_{i=1}^{T} e^{(-ri)}\mathbb{1}\{\tau>i\} + \frac{N}{C_p}S_c\sum_{i=1}^{T} e^{(-ri)}$$
$$* \mathbb{1}\{i-1\leq\tau<i\} + \Delta\left(Put^f(K,\min(k,T),T_o)e^{-\min(k,T)r^f} - Put^f(K,0,T_o)\right) \tag{3.15}$$

Based on Eq. 3.13), we are allowed to express a condition on $P_{0,k}$ by isolating it from $Q_R(T \mid k-F<\tau<k) \geq 0$, such that

$$\iff P_{0,k} \leq \Pi(T \mid k-F<\tau<k) \tag{3.16}$$

When this inequality holds, the CoCo price is sufficiently low to ensure a positive profit from the strategy, given a known conversion time $(k-F<\tau<k)$. Then,

$\max_k (P_{0,k})$ gives the arbitrage limit where the total return on the investment is exactly zero for a given conversion period $\tau \in [k-F, k]$.

Because the condition must hold $\forall k$, the absolute profit condition can be written as

$$\min_k Q_R(T \mid k-F<\tau<k) \geq 0$$

Applying the same steps that led to Eq. 3.16, we find the required initial CoCo price P_0 that results in break-even, using the strategy developed within this scope, to be the lower value from the set of prices $\Pi(T;k)$, hence, ensuring $Q_R(T) \geq 0$ for any k.

$$P_0 = \min_k \{\max(P_{0,k})\} = \min_k \{\Pi(T;k)\} \tag{3.17}$$

To summarize, if the dirty price of the CoCo is below the value P_0 defined in Eq. 3.17, then implementing the proposed strategy is less expensive than the minimum payoff from the CoCo plus put investment, regardless of the conversion time τ, which translates to a risk-free profit.

Minimum share price

Similarly, we define $S_{0,k}$ as the set of initial share prices that satisfy $Q_R(T;k) \geq 0$ for a given conversion period. The share price information S_0 is contained in the quanto put valuation formula introduced in Eq. 3.11. When increasing, this variable drives down the initial price of $Put^f(K,0,T_o)$ (cf. option greeks), reducing the overall strategy cost and increasing the profitability.

Due to this sensitivity behaviour, the absolute minimum share price required to set up the strategy is expressed as the maximum of the minimum share price found for each individual conversion period eventualities ($\forall k$), stated as

$$S_0 = \max_k \left\{\min S_{0,k} \;:\; \min_k Q_R(T \mid k-F<\tau<k) \geq 0\right\} \tag{3.18}$$

Because Eq. 3.13 is non-linear, numerical methods are required to calibrate the so-called implied share price that would lead to break-even in the most adverse conversion situation.

Strategy optimization

Given market prices $\mathbf{I} = (P_0, S_0)$, we can compute different conversion scenarios and observe the achievable payouts $Q_R(T;k)$ for all times k. When positive, the strategy delivers a net realizable profit. Extending this idea, we can calibrate the optimal number of puts Δ as well as their respective strike K and maturity T_o, which maximizes the minimum value achieved by Q_R across all periods k. This optimization problem is expressed as

$$(\Delta, K, T_o) \rightarrow \max_{\Delta, K, T_o} \left(\min_{\forall k} Q_R(T \mid k - F < \tau < k) \right) \tag{3.19}$$

The strictness of this optimization problem may be relaxed to better accommodate the practical application of the strategy, provided that conversion does not occur at a price point that exceeds a pre-determined threshold H. Then, Eq. 3.19 adjusts to

$$(\Delta, K, T_o) \rightarrow \max_{\Delta, K, T_o} \left(\min_{\forall k} Q_R(T \mid k - F < \tau < k) \Big|_{S_c \leq H} \right) \tag{3.20}$$

3.5 Numerical example 1: the general case

We provide a numerical example in Table 3.1 of how a portfolio Q behaves depending on the number of Δ puts and their respective strike K. In this first example, the option maturity equals CoCo maturity (i.e., $T_o = T$).

CoCo parameters (numerical example)	
Dirty price CoCo P_0	790
Floor price C_p	$\max(4.10, S_c)$
Maturity T	3 years
Frequency of payment f	2
Coupon (annualized)	5.5%
Macro parameters (numerical example)	
Share price at inception S_0	4.2
Risk-free rate r	1%
Equity volatility σ_{stock}	25%
Dividend rate δ	0.5%

Table 3.1: Inputs for a numerical example on the portfolio optimization and puts design.

Setting the following complementary inputs enables deriving the portfolio Q value at the end of the **first** interest payment period ($k = 1$):

- Put strike $K = 3.9$.

- Initial time $t = 0$.
- The price of each put option follows as $Put(K, 0, T) = 0.5167$.

We observe this numerical example in Fig. 3.2 to see the importance of the appropriate choice in Δ. On the one hand, a low number, such as $\Delta = 200$, reduces the initial cost at the price of not effectively hedging the conversion eventuality. Therefore, the strategy exhibits a negative payoff in the case of early conversion for low S_c values while positive for higher share prices at conversion. On the other hand, a too-high number of Δ adequately hedges the downside risk but might increase the strategy cost by too much, leaving the portfolio with a potential loss on the upside (i.e., conversion at a high underlying share price).

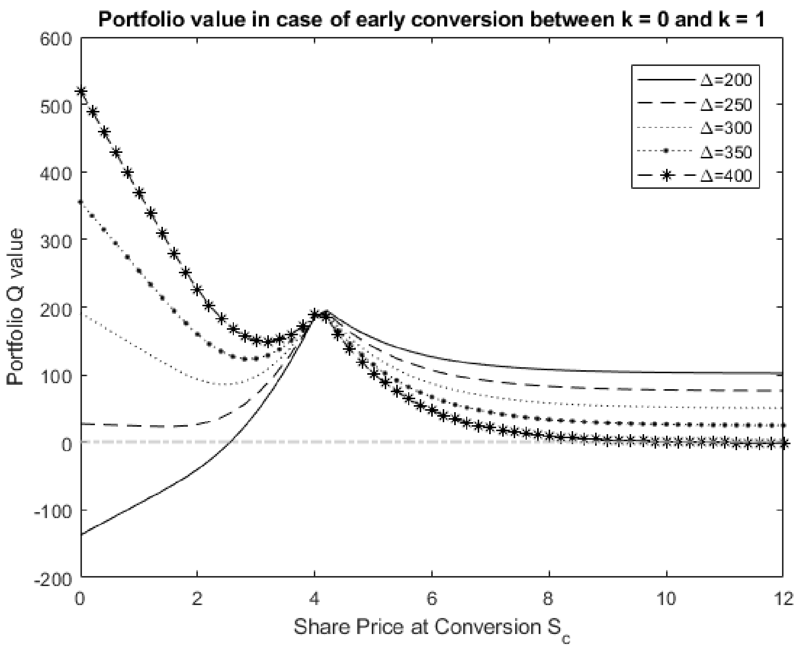


Figure 3.2: Portfolio Q value as a function of the share price at conversion time assuming conversion occurs at $0 < \tau \leq 1$. The figure is plotted for various numbers of puts in portfolio Δ.

Fig. 3.2 demonstrates that under certain conditions, the minimum recovery is always positive, meaning that the achievable return on the strategy would yield a return $y > 0$ over the risk-free rate (see Eq. 3.14). This case is visually observed when $250 \leq \Delta \leq 400$. These results can also be presented in terms of returns over the risk-free rate related to the concept of arbitrage introduced previously, as shown in Fig. 3.3.

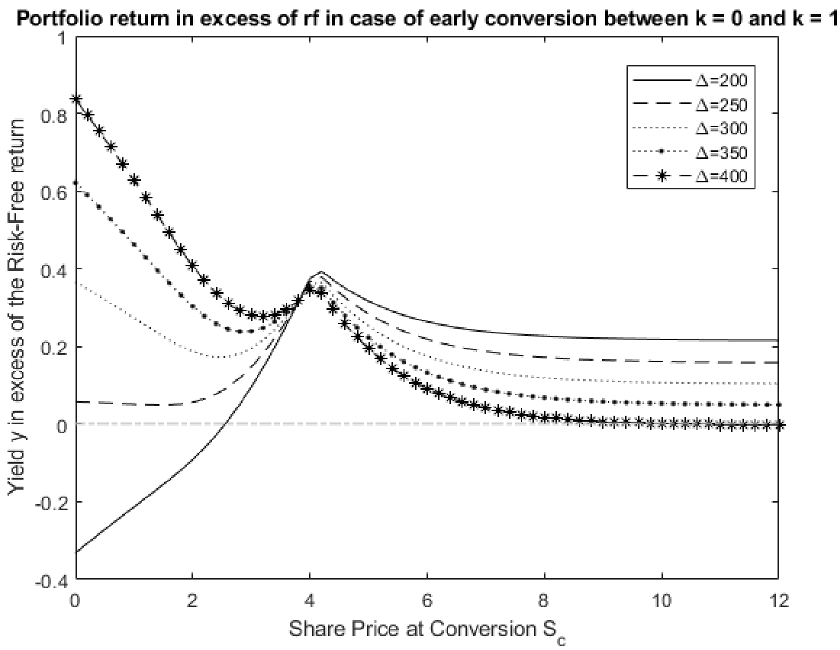


Figure 3.3: Portfolio Q return over the risk-free rate r^f as a function of the share price at conversion time, assuming conversion occurs between $k = 0$ and $k = 1$. The figure is plotted for various numbers of puts in portfolio Δ.

Next, we set a constant $\Delta = 280$ and study the effect of a change in the strike price, mechanically affecting the payoff and initial cost of the option hedge, as well as the strategy's overall profitability. The theoretical costs of each option are presented in Table 3.2, each depending on their strike.

Strike K	3	3.5	4	4.5	5
Put Initial Price $Put(K,0,T)$	0.1745	0.3399	0.5668	0.8492	1.1785

Table 3.2: Theoretical price at inception for $Put(K,0,T)$ with different strike K and the parameters from Table 3.1.

A higher strike K would drastically increase the cost of the strategy, bringing a negative recovery for the portfolio Q in the case of early conversion combined with a high S_c price at conversion time. The put payoff would then not offset its initial cost. Otherwise, a strike configured too low might not sufficiently compensate for the loss on the recovery R_{CoCo}, giving the advantage of lowering the strategy cost setup. Then, the strategy would have a positive PnL for high values of S_c but deeply negative in the case of conversion at a very low market capitalisation. These observations are seen in Fig. 3.4

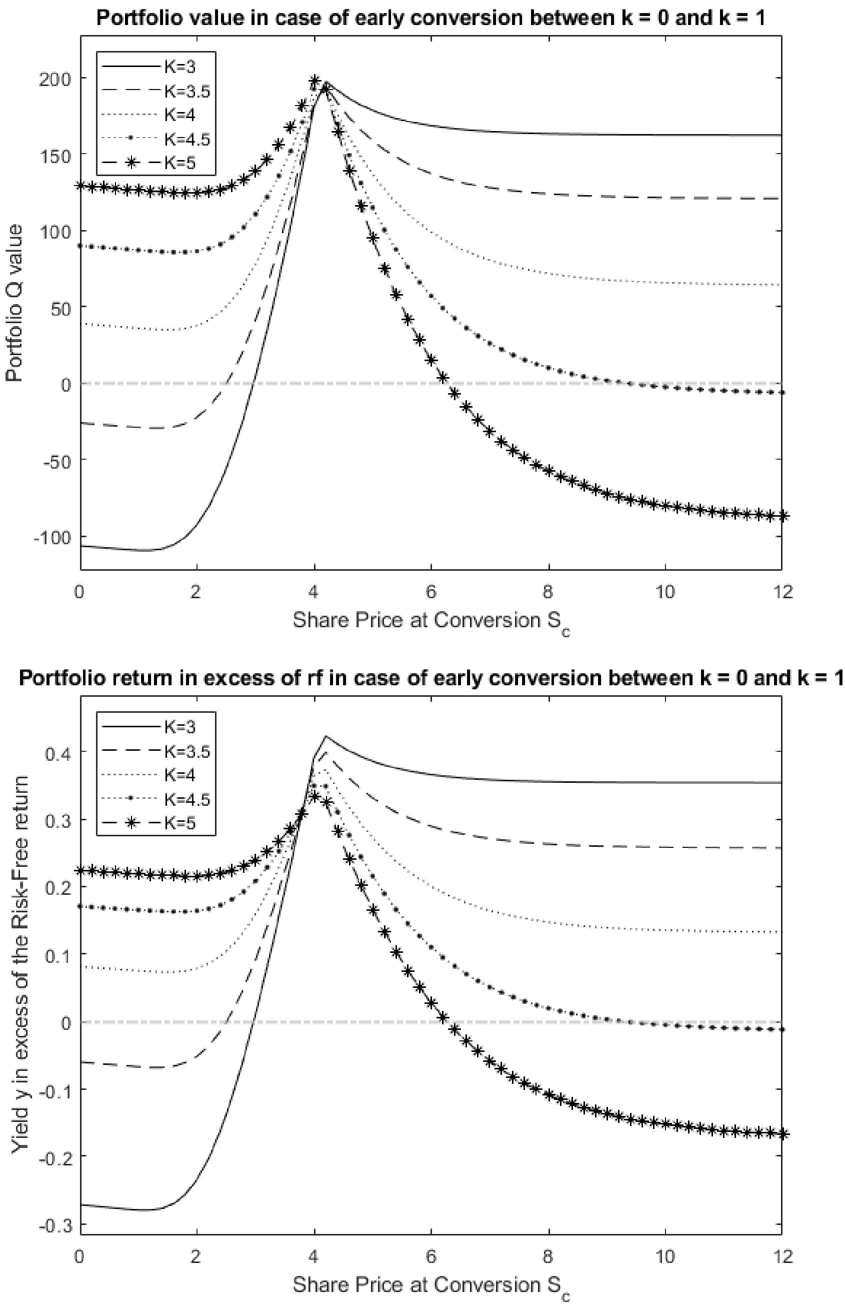


Figure 3.4: Portfolio Q behavior as a function of S_c, assuming conversion occurs between $k = 0$ and $k = 1$. The upper panel shows the value $Q_R(K, 1)$, and the lower panel exhibits the yield y over r^f. The figure is plotted for various put strikes K.

In these two examples derived, we assume the conversion takes place within the inception and first observation period, i.e., between the states $k = 0$ and $k = 1$. Calculation of the remaining option time value is performed in the most unfavourable case, i.e., when $k = 1^-$, ($t = 0.5^-$).

We now derive the portfolio Q behaviour when more payments are released. To do so, we look at the portfolio value when:

- Conversion is assumed to occur between $k = 0$ and $k = 1$, i.e., zero interest payments released.
- Conversion is assumed to occur between $k = 1$ and $k = 2$, i.e., one interest payment released.
- ...
- Conversion is assumed to take place between $k = T * f - 1 = 5$ and $k = T * f = 6$, i.e. 5 interest payments released.

Fig. 3.5 plots the result as a portfolio value surface with $\Delta = 280$ and $K = 3.9$. In this configuration, the price of each put is 0.5167.

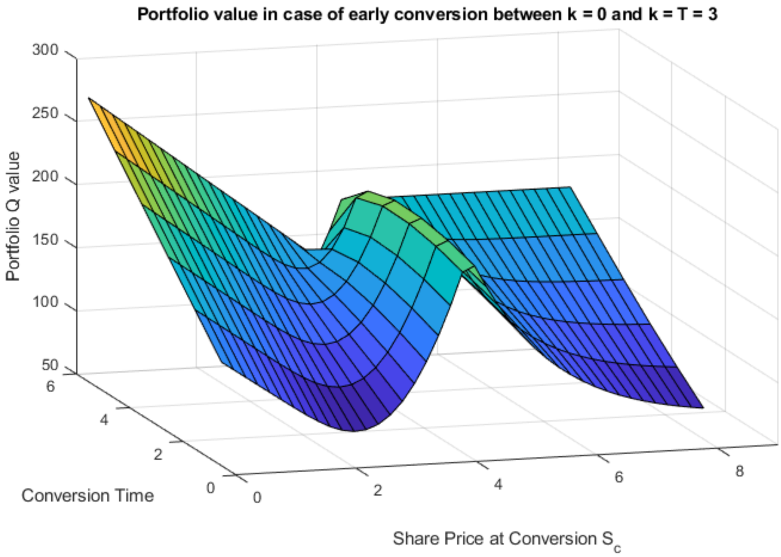


Figure 3.5: Portfolio Q value surface as a function of the time of conversion and the triggering share price S_c in the case of early conversion. The surface exists along all points above 0, and risk-free profitability is achieved by applying this strategy within this theoretical setup.

We observe in Fig. 3.5 that Q has an increasing value over time (in the case of delayed conversion) due to the accrued coupon payments.[2] The number of puts Δ chosen exceeds the "neutral" value $C_p^* = 1000/4.1 = 243.9$ that would perfectly offset the downside risk. Therefore, the hedge incentivizes conversion at a low share price. The figure justifies the decreasing portfolio value when S_c is increasing. Also, when S_c increases too much, the portfolio value is constant over the same time horizon because the put delta is very close to 0 (due to the unlikely payoff) and the recovery rate R_{CoCo} is constant to 100% (due to the definition of C_p).

Another key observation relates to the minimum z-value on the surface, which is strictly positive in all points. This result highlights the existence of a secured and achievable profit, even in the case of early conversion, at any state k.

By shifting the perspective to focus on the achievable annualized excess return y, we provide an objective overview of the strategy benefits in Fig. 3.6.

[2] However, for some S_c values close to the strike, the Q value is slightly decreasing if the conversion occurs very little before $k = T$. This is due to the theta decay of the option.

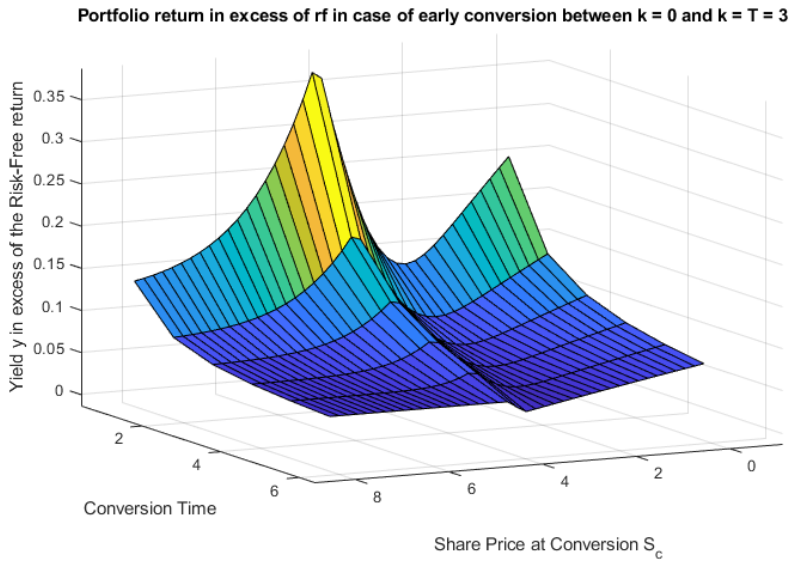


Figure 3.6: Annualized return y surface over the risk-free return for the investment strategy. Here, y is a function of the conversion time and the triggering share price S_c in the case of early conversion. The surface includes all points above 0, so arbitrage is achievable.

Previously, the portfolio Q mostly gained value over the payment periods due to the accrued interest payments (as in Fig. 3.5). Still, Fig. 3.6 accentuates this feature on early conversion as it precedes the annualized yield in excess of the risk-free return achievable through this strategy. This strategy peaks above 35% p.a. if the CoCo converts within the first six months with an underlying share price close to the put option strike price.

When the 5.5% coupon yield is being paid semi-annually and reinvested at $r^f = 1.0\%$, it naturally drives down the annualized return in excess of r^f to a 4.16% p.a. minimum. This occurs when the CoCo turns to equity at time $t = 3^-$ (i.e., $k = 6^-$) with $S_c = K = 3.9$, which demonstrates the importance of th e y calculation.

Arbitrage horizon

The last part of our study in this chapter consists of finding the maximum CoCo price at inception ($t = 0$) for our proposed investment strategy to remain profitable. This problem can be reformulated as the following: *What is the maximum* *price P_0 that still leads to the existence of a value Δ and K that verifies $Q(k) > 0$ for all k between 0 and T?*

Using the same numerical inputs from Table 3.1, we solve this optimization problem to determine that if $P_0 = 86.56\%$ of the CoCo nominal value, we achieve break-even in the worst case with $\Delta = 329.26$ and $K = 3.630$. We represent the portfolio surface in Fig. 3.7 projected in 2D for each of the possible conversion times.

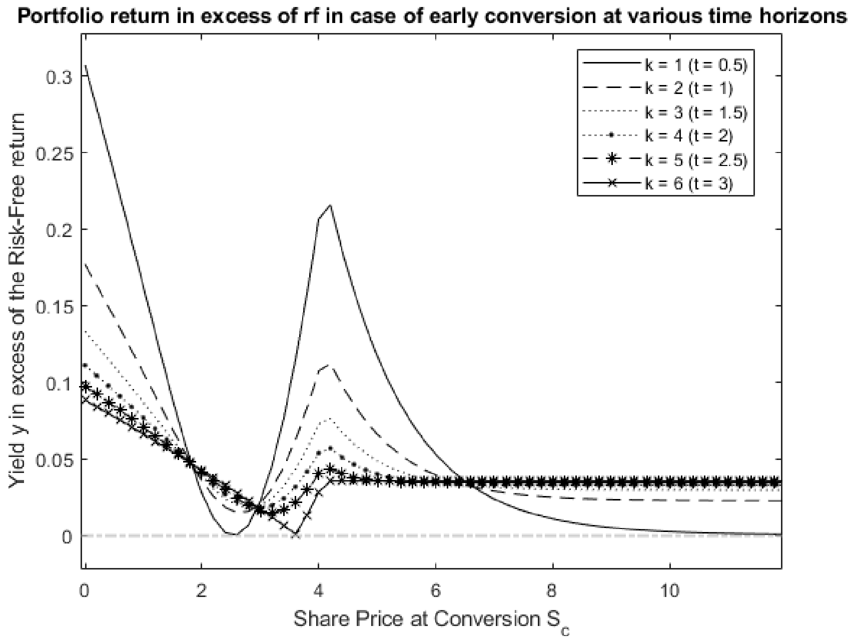


Figure 3.7: 2D projection of the y surface as a function of share price S_c and time horizon in the case of early conversion for an initial CoCo price $P_0 = 865.6$. Break-even is achieved with a number $\Delta = 329.26$ and a strike $K = 3.630$.

Finally, we plot in Fig. 3.8 the so-called arbitrage universe horizon as the surface that allows to at least break even, depending on the three portfolio drivers of the initial CoCo price P_0, the initial share price S_0, and the equity volatility σ_{stock}

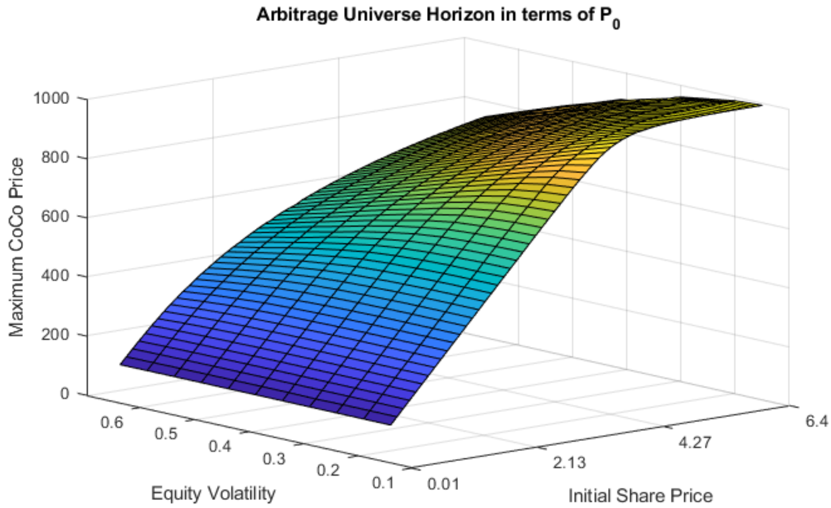


Figure 3.8: Market inputs of the initial CoCo price, share price, and equity volatility allowing to guarantee break-even with the portfolio Q at time $t = 0$.

The entire universe Ω^- below the surface ensures the existence of a risk-free arbitrage, regardless of conversion during the period $[0;T]$. Profitability can not be guaranteed if the market information vector $\mathbf{I} = (P_0, S_0, \sigma_{stock})$ comprised of the initial market prices for CoCo, the underlying shares, and volatility are located in the space Ω^+ above the surface.

3.6 Numerical example 2: Zero-Coupon CoCo Bond

Using the same put parameters and inputs from Table 3.1 and setting the coupon rate to $c = 0\%$, we demonstrate there is no loss of generality associated with studying a ZCCB with a conversion feature. Figures 3.2, 3.3, and 3.4 do not change because the portfolio is evaluated in the case of conversion between $k = 0$ and $k = 1$, i.e., before the first interest payment.

However, we observe that Fig. 3.5 turns into Fig. 3.9 with a logical change in shape because there are no more coupon payments over time. Therefore, the value decrease over time is sharper, as there is a decrease in the options' time value that is no longer compensated by coupon payments. The "extremity" of the surface also flattens for the same reasons.

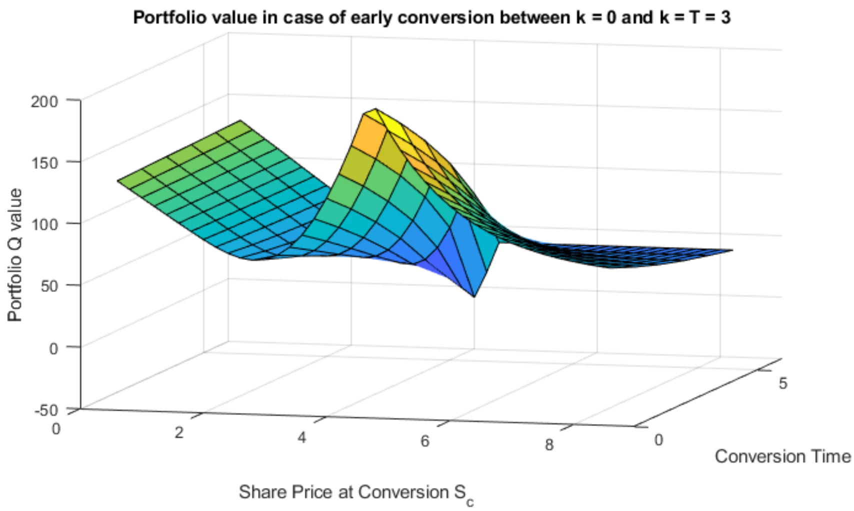


Figure 3.9: Portfolio Q value surface as a function of the conversion time and the triggering share price S_c in the case of early conversion. The setup assumes a Zero-Coupon CoCo Bond.

Fig 3.6 turns into Fig. 3.10 logically, as the yield in excess of the risk-free rate lowers as the maturity time nears, once again, because the coupon payments are eliminated.

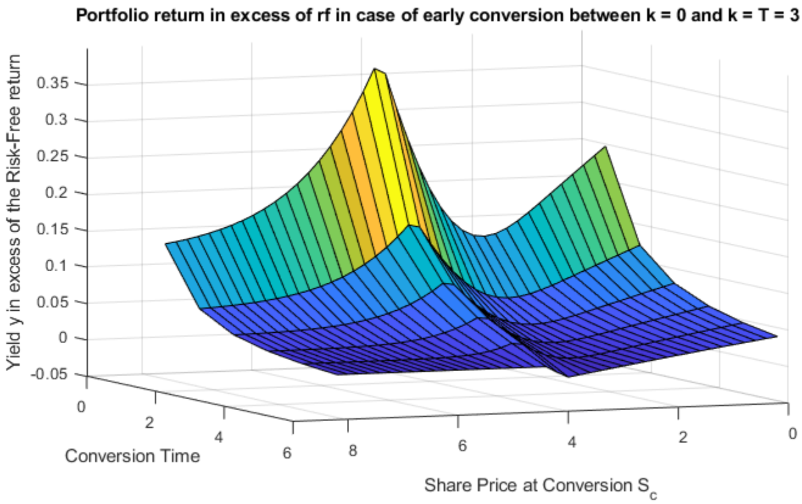


Figure 3.10: Annualized return y surface in excess of the risk-free return for the investment strategy based on a Zero-Coupon CoCo Bond. Here, y is a function of the conversion time and the triggering share price S_c in the case of early conversion.

Recalling, we define in this chapter the occurrence of an arbitrage opportunity by the existence of a pair (Δ, Strike K) that still leads to $y > 0$ across all conversion dates (k), regardless of the share price at conversion S_c. Here, because no coupons are paid, the maximum initial price for the CoCo that ensures an arbitrage opportunity is lowered. Numerically, we find that the maximum dirty price P_0 is 819.20\$ (equiv. 81.92%), which leads to a number $\Delta = 243.90$ put options to buy, with a common strike $K = 4.0976$. For this strike value, each put costs 0.6178\$.

We update Fig 3.7 to Fig. 3.11 for the case of a ZCCB. Now, the worst case for the hedged portfolio is achieved if the ZCCB was initially bought at 81.92 cents on the dollar with a conversion occurring just before maturity. In this specific situation, the return in excess of the risk-free yield would be exactly zero.

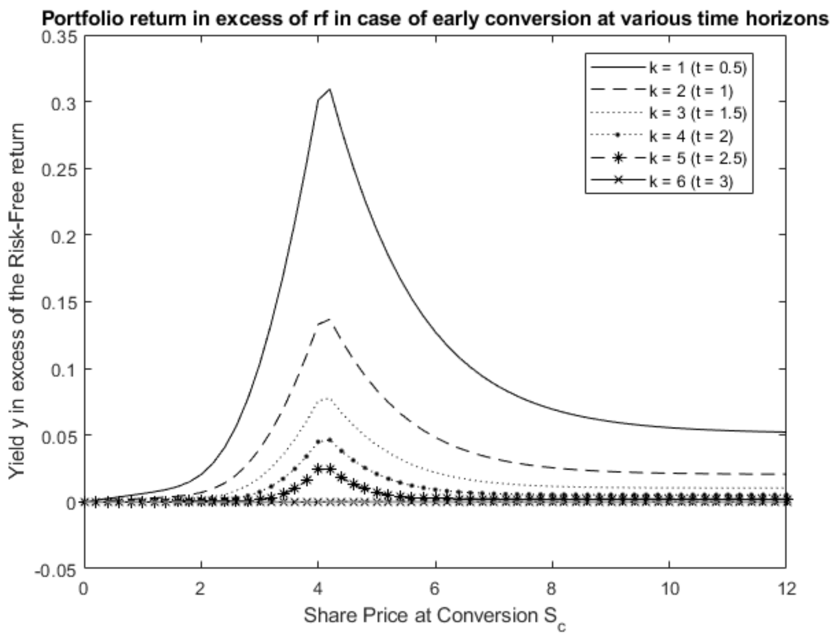


Figure 3.11: 2D projection of the y surface as a function of share price S_c and time horizon in the case of early conversion for an initial ZCCB price $P_0 = 819.2$. Break-even is achieved with a number $\Delta = 243.9$ and a strike $K = 4.0976$.

This result can be derived by noticing that the strike K is extremely close to the floor price C_p, so the cost to set up the portfolio, defined by $\Theta(P_t, S_t)$, is equal to the following:

- The discounted bond denomination if the share price at conversion is above the floor price ($FP = 4.1$), by definition of the conversion price C_p.
- $\Omega(t, \tau)$ with $t = T$ if the share price at conversion is below the floor price, hence $C_p = 4.1$.

In the second case above, we observe that the number Δ put options to buy is exactly $N/FP = 1000/4.1 = 243.9$. So, when S_c decreases, the lower recovery rate on the ZCCB is compensated 1:1 by the increase in put options' intrinsic value.

We plot the so-called arbitrage universe horizon again in Fig 3.12 for the case of a ZCCB, which is the surface that allows achieving break-even by using the proposed strategy, depending on the three portfolio drivers of the initial CoCo price P_0, the initial share price S_0, and the equity volatility σ.

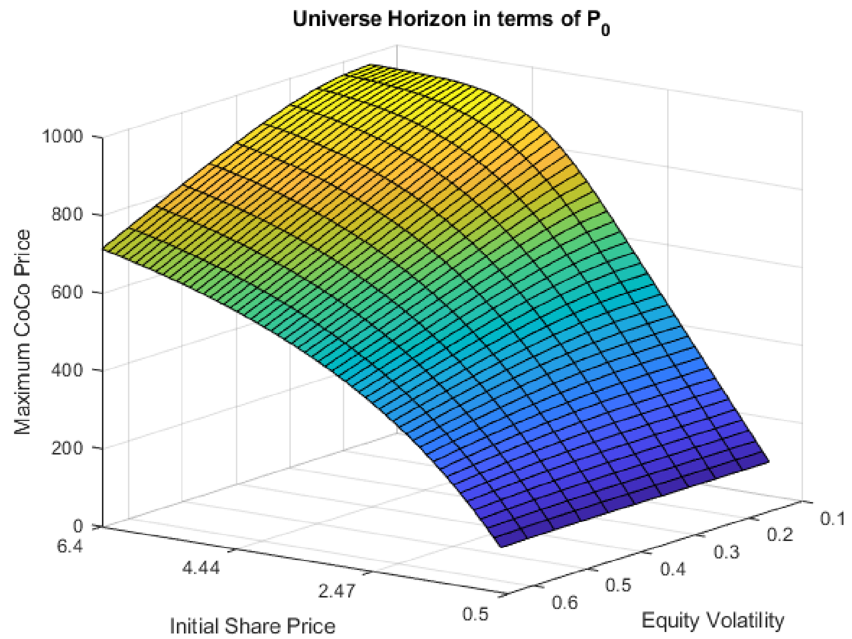


Figure 3.12: Market inputs of the initial ZCCB price, share price, and equity volatility that enables a guarantee of break-even with the portfolio Q at time $t = 0$.

For a given pair (S_0, σ) and because there is no coupon to drive the profit up over time, the maximum initial CoCo price that guarantees $y > 0$ is lower, with a sharper slope than the case with a coupon $c = 5.5\%$ in Fig. 3.8.

Similar to the case with coupons, the entire universe Ω^- below the surface ensures the existence of a pair (Δ, K) guaranteeing $y > 0$ (i.e., arbitrage), regardless of conversion during the period $[0, T]$. Profitability can not be guaranteed if a point of coordinates $\mathbf{I} = (P_0, S_0, \sigma)$ made of the initial market prices for the ZCCB, underlying shares, and volatility are located in the space Ω^+ above the surface.

3.7 Numerical example 3: Arion Banki application

In this use case example, the Arion Banki shares are traded on the market in the domestic currency (ISK), while the portfolio is denominated in the CoCo foreign currency (USD).

Arion Banki micro- and macro-environments	
Date t	March 31, 2020 ($t = 0.0944$)
Arion Banki AT1 Bond Denomination (foreign)	USD
CoCo Bond Coupon (annual)	$c = 6.25\%$
Maturity/First-call date T	5.0 years
Conversion Price	$C_p = \max(S_\tau^f, 0.473 \text{ USD})$
Interest Payment Frequency	Semi-annual ($f = 2$)
CoCo Bond Clean Price	$P_t^{clean} = 73.75\%$
CoCo Bond Dirty Price	$P_t = 74.34\%$
US 5-year Gov Bond Yield, f	$r^{USD} = 0.378\%$
Iceland 5-years Gov Bond Yield, d	$r^{ISK} = 2.325\%$
Arion Banki Share Price	$S_t^{ISK} = 54.9$ // $S_t^{USD} = 0.3879$
Dividend Yield (domestic)	$\delta^{ISK} = 6.6\%$
Dividend Yield (foreign)	$\delta^{USD} = 4.29\%$
USD/ISK FX rate	$FX^{f/d}(t) = 141.53$
Correlation between equity and USD/ISK	$\rho_{\{Arion,FX\}} = -11.85\%$
USD/ISK volatility p.a. (HV)	$\sigma_{FX} = 10.27\%$
Arion Banki Historical Volatility p.a. (HV)	$\sigma_{Arion} = 29.99\%$

Table 3.3: Publicly accessible information for Arion Banki, as of March 31, 2020. Source: Arion Banki [52], Bloomberg/Investing.com - Fusion Media Ltd.

We now calibrate the appropriate strike K, T_o option maturity, and Δ number of options to purchase that maximize the minimum achievable profit, given the current pricing of the CoCo on the market. With the value P_0 given, we solve the following optimization problem for a scope of $0 \leq S_c^f \leq 0.67$ USD equivalent to $S_c^d \leq 95$ ISK at the prevailing FX rate as

$$\max_{K,\Delta,T_o} \left(\min_k \left(Q_R(T \mid k - F < \tau < k)\right)\Big|_{S_c^f \leq 0.67} \right) \tag{3.21}$$

We find $\min_k \{Q_R(T;k)\} = 0$ USD for the following configuration:

$$\begin{cases} \Delta & = 6973 \\ K & = 0.2382 \text{ USD} \\ T_o & = 8.506 \text{ years} \end{cases}$$

The portfolio Q_R value surface is presented in Fig 3.13. With the above option parameters, each put costs 0.0664 USD, and the hedge value is $H = 0.0664 * 6973 = 463$ USD.

As the minimums are extremely local, the probability of achieving a much higher yield through this investment strategy is high.

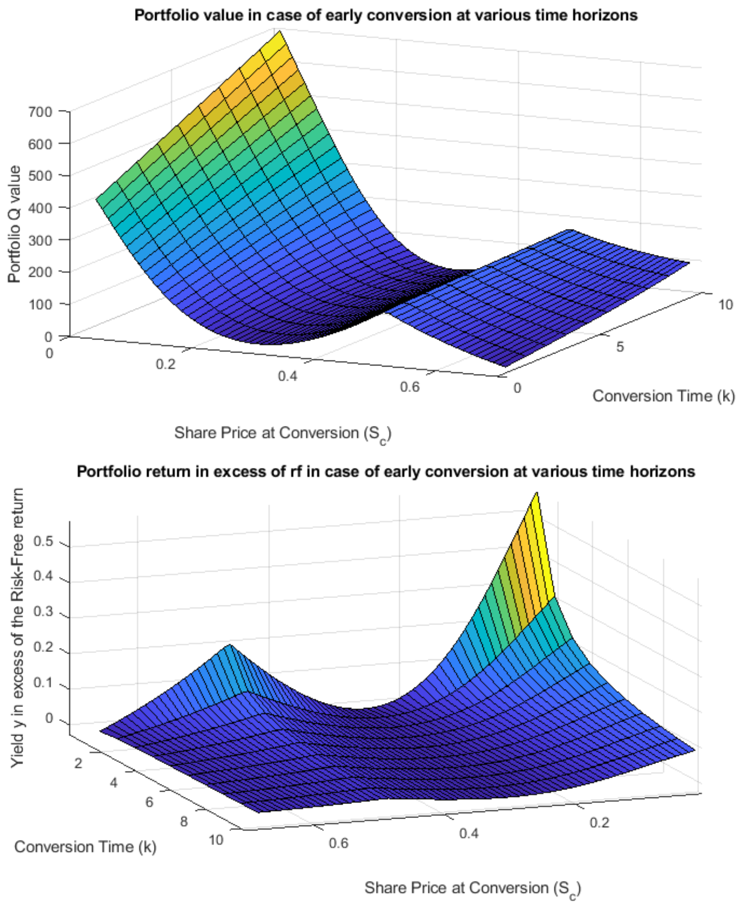


Figure 3.13: Portfolio Q value surface (upper plot) and margin y (lower plot) as a function of the conversion period and share price S_c in the case of early conversion for the Arion Banki CoCo. The portfolio encompasses $\Delta = 6973$ put options with strike $K^f = 0.2382$.

In this simulation, we restrained the scope to satisfying $\min_k Q_R(T;k) \geq 0$, assuming that the share price at conversion time is contained between $0 \leq S_c^f \leq 0.67$ USD. The investment strategy is then valid under this condition.

We next relax this assumption on S_c to solve the optimization problem stated in Eq. 3.19 to find the maximum Arion Banki AT1 price that would still lead to the existence of a triplet (K, Δ, T_o) resulting in a minimum portfolio value $\min_k Q_R(T;k) \geq 0$ regardless of the share price at conversion time. Fig. 3.14) displays the arbitrage universe horizon where any point with coordinates (P_0, σ, S_0) that lies on or below the curve is ensured to achieve $\min_k(Q_R(T;k)) \geq 0$, i.e., portfolio profitability. If a point lies above the curve, arbitrage does not exist, and the profit (or loss) depends on the conversion time.

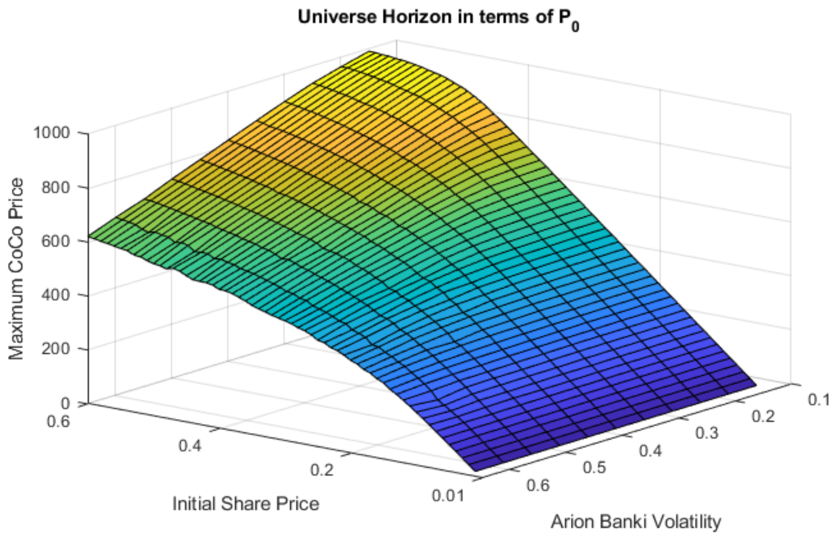


Figure 3.14: Market inputs of the initial Arion CoCo Price, share price, and volatility allowing to guarantee break-even with the portfolio Q_R at time $t = 0$.

Low share prices and higher volatilities increase the cost of the hedge, reducing the maximum CoCo price and guaranteeing break-even. The reciprocal holds, with higher or lower share prices associated with volatility, and the higher CoCo price is still allowed to trade to deliver $Q_R \geq 0$.

To be less restrictive regarding model validity, we simulate the arbitrage horizon surface for various S_c validity thresholds. In Fig. 3.15, we plot the maximum CoCo price P_0 that guarantees break-even (i.e., $Q_R \geq 0$) for three conversion configurations of $S_c \leq 0.67$, $S_c \leq 1.8$ (Eq. 3.20), and $\forall S_c$ (Eq. 3.19).

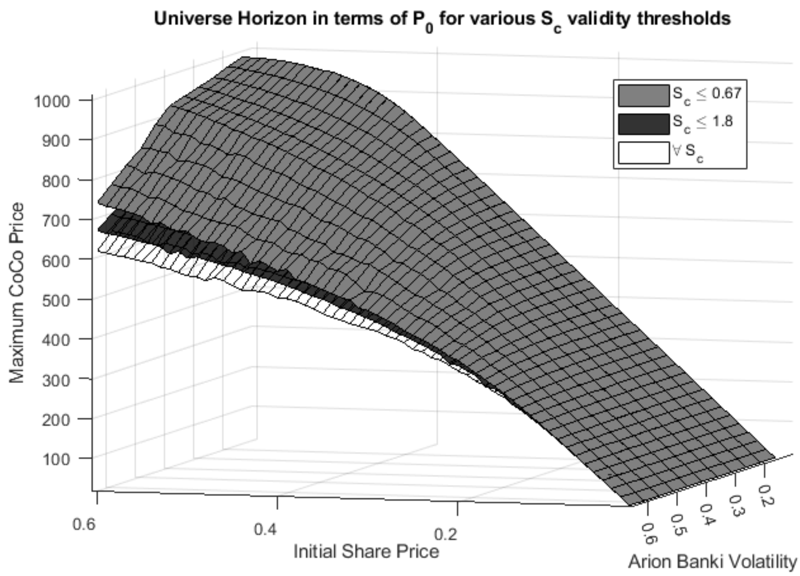


Figure 3.15: Market inputs of the initial Arion CoCo price, share price, and volatility enabling guarantee of break-even with the portfolio Q_R for various S_c validity thresholds. Simulated at time $t = 0$.

With more restrictions, the CoCo must trade lower to ensure positive returns. This observation is interpretable as the hedge cost must be higher to cover a wider scope of conversion configurations. Still, a reasonable assumption is to consider the conversion unrealistic when the share price exceeds a certain threshold.

The model sensitivity to the volatility parameter is significant. As highlighted in Chapter 2, no options on Arion Banki are traded on the market. Consequently, the pricing is only limited to the use of historical annualized volatility instead of the implied volatility that could have encompassed more forward-looking information and, specifically, market expectations in terms of volatility. However, while the implied volatility cannot be derived, the market does not disregard shifts in the market environment, such as the risk presented by the COVID-19 pandemic.

In this numerical example, the model is calibrated to the Arion Banki CoCo with the prevailing data available as of March 31, 2020. We could have selected a different date for our experiment, falling sometime during the second half of the year, especially as it appears in Fig. 2.2 that the Arion Banko CoCo price continued being traded below 80% of the par until October 2020. At that time, the market outlook was more positive (due to hopes for a quick resolution of the global situation), which should have been reflected by lower implied volatility.

This observation affects the practical validity of the model. However, we demonstrated that the framework developed in this chapter is sufficiently flexible to return a solution ranging within a specific validity scope, i.e., a maximum CoCo price assuming the conversion occurs at a share price S_c below a threshold H. In our case, we could increase the volatility level using (σ_{Arion}) and return the upper S_c bound for which a profitable portfolio configuration still exists. For example, we previously found that for $\sigma_{Arion} = 29.99\%$, the maximum CoCo price should be $P_t = 74.37\%$ under the assumption of a conversion occurring at $S_c^f \leq 0.67$ USD (equiv. to 95 ISK). Also, assuming a hypothetical $\sigma_{Arion} = 38\%$ to reflect the increase in the implied volatility, the maximum CoCo price ensuring risk-free profit would remain $P_t = 74.37\%$ under the condition of a conversion occurring at $S_c^f \leq 0.5$ USD (equiv. to 71 ISK).

These validity thresholds are considered acceptable because the Arion Banki share price closed at 54.9 ISK, which was not far from its all-time low, without impacting the AT1 triggering.

3.8 Extension risk

We proposed an investment strategy to potentially benefit from the discount trading price of Contingent Convertible instruments on the market. This strategy does not aim to maximize revenues by leveraging the deviation between the CET1 ratio and the CoCo spread. Instead, it achieves a profit while minimizing the risk to a theoretical null. Borrowing the necessary amount to purchase the CoCo bond and a number Δ (quanto) puts is required to hedge the downside eventuality (i.e., conversion to equity).

More specifically, the maturity for the put options neglects the extension risk, i.e., the risk that the issuing company decides not to call the convertible instrument the first time they can. Even so, empirical evidence shows that only twice did a firm not exercise their early redemption right at the first possible time (see Chap. 5 and 2). We list the following arguments to support this assumption:

- Hybrid instruments usually pay a step-up coupon if not early redeemed at the first callable date, which is in most cases more expensive for the issuer than redeeming the current issuance through the issuance of a new equivalent AT1 (or T2) instrument (as per bond provisions).
- On the one hand, if the company capital and liquidity are sufficient, then the managerial team has no incentive to skip the early redemption possibility. On the other hand, if the cash reserves are too small, then this should be reflected in a low share price, generating an additional payoff through the put options at time T.

- Extending this reasoning, the share price should be negatively impacted by the public decision of not redeeming the existing CoCo issue. Again, a positive payoff from the put options should follow the official announcement (released at a time $t < T$). The same would apply in the case of coupon deferral or cancellation.

3.9 Summary

By reasoning on a term-to-term basis, we established a framework for a proposed investment strategy that takes advantage of the price discrepancy between the CoCo spread and the underlying share price of an issuing entity. This strategy manifests three advantages: (a) it can be used on a systematic basis for a wide class of Contingent Convertibles, (b) it portrays the condition for this arbitrage to exist in terms of the CoCo and underlying share prices, and (c) it does not require assumptions regarding the CoCo conversion probability, known to be an opaque quantity to evaluate.

After introducing the model in a generalized form, we applied it to three numerical scenarios, demonstrating, in one real case, the existence of the required conditions in the Icelandic market for this strategy to be profitable while taking advantage of the discount price on the Arion Banki CoCo bond. We determined the conditions for the strategy developed in this scope to always lead to a cash-positive outcome.

In this chapter, we also introduced a framework for systematic arbitrage identification for CoCo bonds of similar design to the Arion Banki AT1 instrument. The model offers the advantage of not requiring any assumptions on the conversion probability, as its evaluation appears impossible in low liquidity markets, where no CDS information is available.

As the efficient market assumption is an unreachable market paradigm, a continuous and transparent observable control variable as a triggering mechanism for Contingent Convertible would reduce the uncertainty around conversion events for this type of hybrid security and benefit the entire AT1 industry.

When specifically applied to the Icelandic market, implementing such a strategy can be confronted with a lack of liquidity and derivatives instruments. Options such as the *quanto* put required in the portfolio Q would most likely need to be contracted through an OTC deal with a foreign bank.

This study can also highlight the need for more derivative products being issued in the Icelandic market to remove potential arbitrage opportunities from the pricing and increase the efficiency and visibility of low liquidity markets.

Chapter 4

Dynamic Control of Leverage (DCL)

"That is what our money system is. If there were no debts in our money system, there wouldn't be any money."

Marriner Stoddard Eccles, [55].

This chapter is based on a paper co-published with Prof. Sverrir Ólafsson in Finance Research Letters [56].

Abstract

We introduce a version of a CoCo bond to construct a new model for dynamically adjusting a firms leverage. We assume that the firms equity value follows a geometric Brownian motion process. This assumption and the regular and fixed down payments of outstanding debt dynamically change the firms leverage. To maintain the leverage within some pre-fixed boundaries, three controls are applied: pay debt as cash coupons, convert coupon payments into shares, and issue more debt. As leverage inequalities can be converted into share price inequalities, decisions on which control option to apply at any given time can be made directly from observing real-time market data. No time-consuming estimations or decisions by regulatory bodies or risk committees are required. Also, the leverage boundaries can be fixed to keep the probability of default within acceptable values. The resulting leverage dynamics have interesting mean-reverting properties and, to our best knowledge, have not previously appeared in the literature.

4.1 Introduction

An important issue in designing CoCo bonds is the definition of a trigger event i.e. the conditions under which an equity conversion or a write down takes place. Here, transparency and clarity are very important. Some accounting ratios do not satisfy this transparency criterion as different accounting standards would produce different trigger events. In addition, the decision as to whether an event has occurred should not depend on external intervention. Some accounting numbers, such as Core Equity Tier 1 (CET1), are not continuously disclosed information but may only be revealed quarterly. Making conversion depend on any such accounting numbers will delay the trigger, perhaps even until the issuer has failed, making it unlikely that the CoCo will achieve its primary goal [14, 58]. There is some evidence that CoCo bonds would not have prevented the bankruptcy of Lehman Brothers due to poorly designed triggers [40]. In the wake of the Credit Suisse tumult and its forced CoCo write-off, these observations are still relevant today. In essence, every business relies on different optimal capital ratios depending on the nature of their operations, calling for additional flexibility, transparency and automaticity in the CoCo designs and triggering process.

Various versions of CoCo bonds have been proposed, where the main differences relate to the conditions under which conversion of debt to equity takes place and then at what conversion rate. Flannery [47] introduced Reverse Convertible Debentures (RCD), which automatically convert to equity if the issuers market-based capital ratio falls below some pre-fixed values. Another notable contribution is the Call Option Enhanced Reverse Convertible by Pennacchi, Vermaelen and Wolff [10] where the conversion price is set lower than the trigger price, and equity holders are given the option to buy the shares back from the debt holders at the conversion price. Duffie [59] and Flannery [60] have discussed the trigger mechanism and emphasised the importance of market-based triggers.

Several other authors have emphasized on the need for market-based triggers, see for example [61, 62, 63]. Bulow and Klemperer [38] have proposed Equity Recourse Notes where payments currently due are converted into equity in case the issuers share price has suffered substantial decline. The ERN have some similarities with the Dynamic Leverage Control (DLC) suggested in this work as will be discussed in more detail below. Dual-trigger products depending on both micro and macro parameters i.e. at the firm level and up to the whole banking sector have been recommended by McDonald [61], Calomiris and Herring [62].

A 2014 review of contingent convertibles for large financial institutions by Flannery [63] pointed at three challenges raised by firms with too-high leverage and faced by regulating entities:

- Resulting losses could be saddled by taxpayers, as history has shown.

- Firms with high leverage have an incentive to take more risks because of an asymmetric risk-return trade-off.

- Stakeholders in these firms do not favour issuing more shares, as it would shift the benefits to shareholders.

Flannery concluded that the monitoring process by the financial authorities was not suitable to ensure that major financial groups were meeting adequate capitalization requirements. An important role of CoCo bonds is to supplant discretionary decisions based on pre-established rules stated in the bond covenants.

Without considering local initiatives from applicable jurisdictions (that tend to be more constraining), banks must operate in accordance with Basel III key principles that urge the CET1 Capital Requirements (the ratio of Core Equity Tier 1 and Risk-Weighted Assets) to be above 4.5% and T1 Leverage Ratio, calculated as the ratio between Tier 1 Capital and Total Assets to be above 3%. Because banks were historically the first CoCo issuers, they likely shaped the current market practices in this asset category by linking the conversion requirements to the CET1 or T1 of the issuing firm. When implemented on CoCos as a control trigger, it leads to flaws already evoked, as these opaque accounting quantities neither give forward-looking indications on the financial health of the firm nor a clear picture of the indebtedness that could be related to a default probability measure. Nonetheless, other measurements already disclosed in the quarterly financial reports exist and seem more appropriate to serve the purpose of conversion thresholds, such as:

- The Debt-to-Equity Ratio is calculated as the ratio between total debt and shareholders equity.

- The Debt-to-Capital Ratio is calculated as the ratio between total debt and total capital (i.e., Tier 1 plus Tier 2 capital).

As described previously, traditional CoCos can be converted into equity, written down, or off. When the conversion is total rather than partial, it involves large amounts creating a contagion risk. In 2016, following a fear of skipped coupons, the price of major quoted instruments from BBVA, Santander, Banco Popular, UniCredit, and the influential Deutsche Bank sharply decreased [14, 64]. More recently, the Swiss Financial Market Supervisory Authority (FINMA) created controversy when deciding to completely write off the $17 billion Credit Suisse AT1 instruments, while shareholders could recover partially from their investments: it is a clear violation of the absolute priority rule. It has sparked a contagion effect on other CoCo instruments as investors realized that under a Point of Non-Viability (PONV) clause, AT1 might be junior to Equity in the hierarchy of restitution.

Operationally speaking, the control instrument introduced in this chapter is an annuity bond, i.e., one that pays down its residual value in equal payments instead of having a nominal payment at maturity. Then, by only converting the current coupon payment, the nominal value is not affected and can be considered as a partially convertible instrument (with limited conversion). Converting the current coupon payment offers the advantage of not being concerned by small payments or conversions arising with partial conversion proposals in the literature, as in [17].

In this chapter, we introduce leverage dynamics that maintain the issuers leverage ratio within predetermined boundaries where the likelihood of default on outstanding liabilities is small. The requirement that the leverage does not exceed a certain critical level can be formulated in terms of a minimum acceptable share price for the given debt level. If the market value of the share price goes below this minimum price, when a coupon payment is due, the coupon is converted into the firms shares at a predefined conversion price.

This aspect of our proposed model is similar to the ERNs proposed by Bulow and Klemperer [38]. However, a key difference is that we relate the trigger share price to the leverage level of the issuing firm, which results in a mean reversion to acceptable leverage levels. In contrast, [38] convert the payment due to equity if the equity price is below a fixed fraction of the share price on the issue date. Such a feature turns the conversion decision into an automatic process, decreasing the regulatory surveillance and approval needed. The authors of [38] refer to this as a credible conversion event. In our model, the conversion criterion is also automatic and dynamically adjusts to the share value associated with an acceptable leverage ratio. Our model enables the issuer to determine, in advance, what leverage levels are acceptable and to fix the share value-based trigger to achieve that goal.

For the trigger, Flannery designed its RCD in a similar way to deliver a programmed unlevering, using the banks market capital ratio. In other words, RCD transfers the companys ownership from the shareholders to the bondholders in the presence of a credit event [47]. In our model, there is also scope for increasing the leverage level. If the leverage goes below a pre-fixed minimum value, then more debt can be issued to take the leverage back up to the minimum level. The conditions for this additional debt issue are expressed in terms of the share price, information that is available in real-time. This action improves the firms leverage ratio, and the decision to convert, pay in cash, or issue more debt is based on real-time observable market events.

The self-adaptative model we introduce appears to be adequate for any business through its framework that enables additional degrees of freedom, such as endogenous parameters, a relevant control measure for any firm (i.e., the debt-to-asset ratio), and a conversion limited to the current interest payment, thereby reducing the systemic risk and making the previously mentioned chain-reaction less likely. By acting at the same time on limiting the leverage from reaching too high and too low a level, our instrument is more suitable for the real behaviour of firms and performs as a leverage 'watchdog' by mechanically self-adjusting the debt-to-asset ratio without external interference. The resulting leverage dynamics have interesting mean-reverting properties, and to our best knowledge, our proposed model has not previously appeared in the literature.

In the simulations presented in this chapter, all debt is issued for the same maturity, which we take to be ten years, the firms debt situation consists of several simultaneous loan agreements, and each coupon payment or equity conversion pays down a fraction of each outstanding loan. The total outstanding loan RQ_k^T at time T_k consists of the residual values of M outstanding loans, such that $RQ_k^T = \sum_{j=1}^{M} RQ_{k,j}$, where $RQ_{k,j}$ is the residual value of loan j at time T_k. If at time T_k the maximum accepted leverage L_c is exceeded, then all M loan payments due are converted into equity at predetermined rates. Repetition of this procedure strives to keep the outstanding leverage value below the maximum critical value L_c. Similarly, if at time T_k the leverage value is below some predetermined minimum value L_{min}, then new debt is issued to take the leverage up to this minimum value. When leverage values are between L_{min} and L_c all coupon payments are paid in cash, as prescribed by individual loan agreements.

Despite the similarities of the work in this chapter with that of Bulow and Klemperer [38], the differences are fundamental. In our case, conversion occurs at a share price associated with some critical maximum acceptable leverage level. In [38], on the other hand, conversion occurs when the share price reaches or goes below some fraction of the share price at the notes issue date and does not adjust to changes in the firms leverage ratio or capital structure. This is critical as the leverage ratio is an important factor determining the probability of default in some of the major models for corporate capital structure [24, 42, 34], whereas the share price, as a stand-alone value, is not a significant determinant for the probability of default.

Traditional CoCo bonds existing in the market failed to meet expectations and in an attempt to address the current flaws related to the various alternative CoCo proposals [65], we contribute to the research by proposing a new self-adaptive model labelled Dynamic Control of Leverage (DCL). This proposed framework rectifies the issues observed in traditional CoCo bonds by limiting the conversion to the coupon value, which significantly mitigates the potential negative impact of a conversion event on existing shareholders. The DCL model also incorporates a simpler, more transparent control mechanism by using the debt-to-asset ratio as the control variable. This approach enhances the understanding of the conversion mechanism for investors and provides a clear, measurable trigger point, thus increasing market stability. By simplifying the mechanism and improving transparency, the DCL model increases the predictability of conversion events, which can prevent panic selling and market instability observed in previous instances of CoCo conversion [66].

The assumptions regarding capital structure are intentionally simplified to focus on the framework of the instrument and its interplay with the existing capital structure of the issuing firm. As a result, DCL is appropriate for all types of businesses, not just banks or insurers, representing an economic advancement towards broader acceptance of CoCo bonds.

The structure of this study presented in this chapter is as follows. In Section 4.2, we provide a brief introduction to the model, including the repayment of outstanding debt and the stochastic nature of equity prices. We define leverage value in terms of the remaining book value of total outstanding debt and the market value of equity [17]. In Section 4.3, we illustrate how the necessary inequality for the leverage ratio can be expressed in equivalent real-time inequalities for a firm's share price. We also estimate the probability that acceptable share price values are violated and provide a simple example. In the following Section (4.4), we introduce and implement the leverage dynamics and demonstrate through simulation its impact on the realized leverage levels, number of shares issued, and frequency and size of the top-up loans. We then present in Section 4.5 a case study that implements the model introduced in the previous section. In the subsequent Section 4.6, we analyze the efficiency of a CoCo when conventional debt also exists in a firm's capital structure. We discuss in Section 4.7 the value of the CoCo bond to investors and compare it to the value of a fixed payment bond where no equity conversion occurs. We extend our proposal to offer a Write-Down alternative (section 4.8). Lastly, we summarize the key takeaways from the analysis in section 4.10. Our results and conclusions are in the final section 4.11.

4.2 A firm's capital structure

Our scenario assumes that a company finances its operations with debt and equity. On the first issue date, the nominal or book value of outstanding debt is Q, and the annual interest cost of debt is R. The debt is due to be paid down over a period of N years, with n payments per year. The $N_n = n * N$ equal payments are given by [67]

$$P_{N_n}(T_i) = \frac{rQ}{1-(1+r)^{-N_n}} \tag{4.1}$$

where $i = 1, ..., N_n$ and $r = R/n$. The debt Q is issued at time T_0 and paid down at the times $\mathbf{T} = \left(T_1, T_2, ..., T_{N_n}\right)$ with $T_k = T_{k-1} + \Delta T$ where ΔT is the payment tenor and $k = 1, 2, ..., N_n$. At time T_k, when k payments have been made, the residual value of the loan is given by

$$RQ_k = Q\left((1+r)^k + \frac{1-(1+r)^k}{1-(1+r)^{-N_n}}\right) \tag{4.2}$$

Equations 4.1 and 4.2 are demonstrated in the Appendix B.

At time T_0, the firm also issues NS_0 shares and the market value of each share is S_0. The share price is assumed to follow a geometric Brownian motion (gBm) process defined by the stochastic differential equation (SDE) [18]

$$dS_t = (\mu - \delta)S_t dt + \sigma S_t dW_t \tag{4.3}$$

where μ is the expected annual return on equity, δ is the annual dividend yield paid by the firm to equity holders, σ is the annual volatility of returns, and dW_t is an infinitesimal change in the Wiener process W_t in the objective world. The share value at time T_k is given by the solution to the gBm process [18]

$$S_k = S_0 \exp\left(\left(\mu - \delta - \frac{1}{2}\sigma^2\right)(T_k - T_0) + \sigma\left(W(T_k) - W(T_0)\right)\right) \tag{4.4}$$

where $W(T_k) - W(T_0) \sim N\left(0, \sigma^2 (T_k - T_0)\right)$, i.e., is normally distributed with mean 0 and variance $\sigma^2 (T_k - T_0)$. At time T_k, the debt-to-asset, or leverage ratio, is given by

$$L_k = \frac{RQ_k}{RQ_k + NS_{k-1} * S_k} \tag{4.5}$$

We use the same time values for the leverage ratio L_k as for RQ_k and S_k, but for the number of shares, we use the values NS_{k-1}, which is the number of shares fixed at the previous loan payment date T_{k-1}.

L_k presents a stochastic process driven by three variables, RQ_k, which is reduced in a deterministic manner but increases stochastically due to the payment schedule, and the stochastic variables NS_{k-1} and S_k. However, at any time T_k, the leverage ratio L_k is known.

4.3 The CoCo bond structure of the loan

The firms loan arrangements are in the form of a hybrid instrument of the type of a CoCo bond [29]. Under normal financial circumstances, i.e., when the firms debt-to-asset ratio does not exceed a previously fixed critical value L_c, the bondholders receive regular coupon payments $P_{N_n}(T_i); i = 1, ..., N_n$ as cash payments. If this condition is not satisfied, then the cash payment due to the bondholders is converted into the firms shares at some fixed price S_p per share. The CoCo structure of the loan shares a key design specification as with the ERN that the conversion is limited to the due payment. Converting future cash flows does not help the bank solve its capital problem and provides no rational economic benefits. If these payments need to be converted, then the process is performed in a timely manner at the presupposed payment time. Even from an accounting perspective, the gain generated by future cash flow through early conversion is marginal [38].

Between the payment times T_k and T_{k+1}, the equity price evolves according to a geometric Brownian motion process. At payment time T_k, the share price S_k and the firm's leverage value L_k are known. If at time T_k, when a debt payment is due, the debt-to-asset ratio satisfies the inequality $L_k \leq L_c$, then the debt payment is made in cash. Otherwise, it is converted into $P_N(T_k)/S_p$ shares, which have the market value $(P_N(T_k)/S_p)S_k$ at time T_k.

The inequality in the debt-to-asset ratio translates into the inequality of the share price at time T_k as

$$S_k \geq \left(\frac{1 - L_c}{L_c}\right) \frac{RQ_k}{NS_{k-1}} = S_{c,k} \tag{4.6}$$

Provided this inequality is satisfied, the debt payment is made in cash. Otherwise, the amount $P_{N_n(T_k)}$ is converted into $P_{N_n}(T_k)/S_p$ shares. For the stochastic evolution of the number of shares, we write

$$NS_k = NS_{k-1} + \frac{P_N(T_k)}{S_p} * \mathbb{1}_{\{S_k \leq S_{c,k}\}} \tag{4.7}$$

where $\mathbb{1}_{S_k \leq S_{c,k}}$ is the indicator function defined as

$$\mathbb{1}_{\{S_k \leq S_{c,k}\}} = \begin{cases} 1 \text{ if } S_k \leq S_{c,k} \\ 0 \text{ otherwise} \end{cases}$$

The benefit of this scheme is that at time T_k, from the market value of the firms shares, the payment is clearly known to be in cash or shares. No human intervention, decision-making, or involvement of a risk management or regulatory committee is required. This criterion, in terms of the share price, is non-ambiguous, and the decision on how to settle the payment due can be made in real-time.

From the above equation (4.7), the conversion price S_P controls the equity dilution. Therefore, it is in the interest of the initial shareholder to keep it high and, conversely, for the CoCo bondholders to convert at low values. However, a conversion to equity should be viewed as the last resort to keep the issuer as a going concern, resulting in shareholders having little choice. The literature identifies this problem but asserts that the shareholders prefer dilution over bankruptcy. Here, there are two decisions that must be made that impact equity dilution: the leverage level at which the trigger activates and the conversion price. However, market considerations might not be aligned with this claim, as we expect CoCos to be issued during times of economic prosperity and not while distress is ongoing. Rational managers (or shareholders) could disagree with the need for CoCo issuance by the firm. As our CoCo variation is similar to ERN, the argument that the issuing entity can still buyback shares after conversion if the financial situation improves still holds [38]

With increased transparency, the market could continuously price the change in conversion probability, which is in fact what we could call the "price of dilution." The probability of conversion is strongly correlated with the issuers probability of default. Some high leverage situations may require acceptance of equity dilution as the price for keeping the issuer as a going concern.

The DCL model emerges as a viable alternative to traditional CoCo bonds, retaining their core objectives while addressing their shortcomings. Specifically, the DCL can be designed to include a controlled negative wealth transfer to equity holders, spread over time to mitigate abrupt changes in firm control. Additionally, its new trigger mechanism minimizes the risk of accounting manipulation aimed at delaying trigger breaches. Overall, the DCL model serves to reduce the debt overhang problem inherent in traditional CoCo bonds [68].

The probability that no conversion to equity occurs at time T_k is given by (with $T_0 = 0$) [18]

$$P(S_k \geq S_{c,k}) = \Psi\left(\frac{\log\left(\frac{S_0}{S_{c,k}}\right) + \left(\mu - \delta - \frac{\sigma^2}{2}\right)T_k}{\sigma\sqrt{T_k}}\right) \tag{4.8}$$

where Ψ is the standard normal cumulative distribution and S_0 is the share price at time T_0 when the debt Q is issued, i.e., time ΔT before the first payment is due. Inserting the expression for $S_{c,k}$, we find that this equation can be rewritten as

$$P(S_k \geq S_{c,k}) = \Psi\left(\frac{\log\left(\frac{S_0 L_c N S_{k-1}}{RQ_k(1-L_c)}\right) + \left(\mu - \delta - \frac{\sigma^2}{2}\right)T_k}{\sigma\sqrt{T_k}}\right) \tag{4.9}$$

Example

In this simulation, we assume a firm's capital structure is made of equity (100 shares valued at 20$ each) and a liability in the form of a DCL, amounting to 5000$. The share price is simulated over a ten-year period based on the numerical inputs given in the top row in Table 4.1. The equal payment amount $P_N(T_k)$, the residual loan value RQ_k, and the leverage L_k at time T_k are calculated with Eqs. 4.1, 4.2, and 4.5, respectively. This approach allows deriving the probability of conversion and the expected outstanding number of shares for a given time horizon T.

	Q	**y**	**N**	S_0	μ	δ	σ	L_c	NS_0	**dt**	S_p
Inputs:	**5000**	**0.05**	**10**	**20**	**0.1**	**0.025**	**0.35**	**0.8**	**100**	**1**	**18**
Time [k]	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
P_N	647.5229										
RQ_k	5000	4602.477	4185.078	3746.809	3286.627	2803.435	2296.084	1763.365	1204.011	616.6885	0
S_k	20	8.541503	6.940614	9.80454	10.32128	11.15332	16.97717	8.927845	6.152837	4.384188	5.879976
$S_{c,k}$		11.50619	10.26789	8.938954	7.628205	6.352364	5.102632	3.86249	2.612513	1.331833	
$P(S_k > S_{c,k})$		0.94726	0.91962	0.918713	0.92724	0.939825	0.954442	0.969929	0.98489	0.996512	
$E[NS_k]$	100	101.8972	104.7888	107.713	110.3304	112.4951	114.134	115.2158	115.7593	115.8848	
L_k		0.843466	0.855441	0.78734	0.747239	0.694954	0.545916	0.633772	0.629412	0.54558	

Table 4.1: For the case specified in the first numerical row, P_N states the fixed regular debt payments, RQ_k is the residual values of the loan, and S_k presents the possible share price at future times. $S_{c,k}$ is the critical share values at different times. $P(S_k > S_{c,k})$ gives the probability that the share price exceeds the critical value $S_{c,k}$. $\mathbf{E}[NS_k]$ is the expected number of issued shares and L_k the leverage value at time T_k.

4.4 Dynamically controlling debt levels

Debt Q issued at time T_0 is paid down at time T_k; $k = 1, ..., N_n$ in deterministic quantities P_{N_n} in cash or by conversion to equity as described in the previous section. Equity conversion occurs when the debt-to-asset ratio exceeds some predefined fixed level L_c or equivalently when the equity value goes below the critical value $S_{c,k}$ at time T_k. However, constantly striving towards an efficient leverage level and desirable cost of capital arrangements, a firm also seeks to keep the leverage above some minimum level indicated by L_{min}.

A firm or bank with too low a level of leverage (compared to its peers) is not as capital efficient as a company with higher leverage, as it does not fully enable the potential for achieving (higher) returns. To create value, the firm's management should seek higher leverage levels to ensure that the company generates more profit from its operations than the cost of its financing capital.

Also, banks typically run on higher leverage than other firms because of the lending nature of their business. A low debt-to-asset ratio depicts a less significant ability to lend money to customers, thereby affecting profitability. The re-issuance process featured here prevents this occurrence but can be withdrawn by setting $L_{min} = 0\%$.

Banks and other financial institutions typically have higher leverage ratios than other firms, a flexibility that is accounted for in the model. By keeping L_{min} and L_{max} to reasonable (conservative) values, the leverage only marginally impacts the equity dynamics.

We consider how to construct an adaptive leverage dynamics that strives, at all times T_k, to keep the issuer's leverage level within the boundaries $L_{min} \leq L_k < L_c$. Assume that at time T_k, the firms debt-to-asset level reaches a value that satisfies the following inequality

$$L_k = \frac{RQ_k}{RQ_k + NS_{k-1} * S_k} < L_{min} \tag{4.10}$$

To maintain the minimum desirable leverage levels, the firm issues new debt Q_k to achieve a total debt level of $TQ_k = RQ_k + Q_k$, which satisfies at time T_k

$$L_k = \frac{TQ_k}{TQ_k + NS_{k-1} * S_k} = L_{min} \tag{4.11}$$

In other words, the outstanding debt is increased to bring the leverage up to a minimum critical leverage level L_{min}. To achieve this, the firm must increase the debt by the value

$$Q_k = -RQ_k + \frac{L_{min} NS_{k-1} S_k}{(1 - L_{min})} \tag{4.12}$$

As with previously issued debts, the new debt is also paid down in N_n equal payments n times per year over N years. The total debt payment due at time T_k is given by

$$P_{N_n}(k) = \frac{r}{1 - (1 + r)^{-N_n}} \sum_{i=\max(k-N_n,0)}^{k-1} Q_i \tag{4.13}$$

In the following, we summarize the three possible situations at times $\mathbf{T} = (T_1, T_2, ..., T_{N_n})$ when the debt payment is due:

- **Case 1:** $L_k > L_c$ *i.e.* $S_k < \frac{RQ_k(1-L_c)}{NS_{k-1}L_c}$. The leverage level is too high at time T_k and the payment due $P_{N_n}(T_k)$ is converted into $P_{N_n}(T_k)/S_p$ shares, which have the value $(P_{N_n}(T_k)/S_p)S_k$ at time T_k.
- **Case 2:** $L_{min} \leq L_k \leq L_c$. The firm is within its leverage target zone, so the payment due $P_{N_n}(T_k)$ is made in cash.
- **Case 3:** $L_k < L_{min}$. The firm is below its minimum leverage target L_{min}, so the payment due $P_{N_n}(T_k)$ in made in cash and issues more debt Q_k given by $Q_k = -RQ_k + \left(\frac{L_{min}}{1-L_{min}}\right) NS_{k-1}S_k$. If the payment results in the inequality $L_k < L_{min}$, then more debt will be issued such that $L_k = L_{min}$.

Under the Basel III guidelines and for practical implementation, a fourth case is possible where the regulatory body can force the conversion if a so-called Point of Non-Viability (PONV) is reached ($L_k > L_c > L_{PONV}$).

A company looking to enhance returns will need to increase their risk exposure, and a balance must be identified to obtain the optimal leverage. The framework introduced in this scope can be related to the probability of default or bankruptcy costs via other models, such as Merton [24], Leland [42], or Leland and Toft [43]. Therefore, the choice of the critical leverage levels L_c and L_{min} is left to the firms management.

On the one hand, in our model, conversion to equity is dynamically controlled by the issuers leverage at the times the coupon or principal payments are due. All payments due from different CoCo bonds, issued at different times, are subjected to the same conversion criteria. This approach is different from ERN, where the conversion of each bond depends on the issuer's share price at the time the note was issued. This leads to inconsistencies that are not present in the model introduced in this chapter.

On the other hand, traditional Contingent Convertibles do turn into equity when a violation of the CET1 (or alternatively T1) requirement is observed. Practically, the illusion of an automatic conversion is limited by the quarterly disclosure of these ratios by the bank and subject to regulatory approval in the case of when a conversion is required [14]. DCL follows the same logic, with conversion only occurring at interest payment time. However, the following three modifications provide improvements:

- The payment frequency f can be set to reduce the time interval between two leverage observations.

- The European-like structure (observation of the leverage on a payment date) can be replaced by an American-like mechanism with continuous observation of the leverage to restore the leverage within acceptable limits immediately.

- A PONV can be installed, allowing the regulator to trigger the conversion if they evaluate the situation as a requirement. This third option already exists in the scope of traditional CoCo bonds but interferes with the self-adaptative model proposed here.

By dynamically controlling the leverage, implicitly considering any potential change in the firms capital structure, the DCL offers an enhanced version of ERN. The attractive and new aspect of DCL is its tendency to make the issuing firm operate in a pre-specified leverage interval between the values L_{min} and L_c.

These two key leverage levels at which the debt instrument turns the current interest payment into equity (L_c) or triggers re-issuance of additional debt through a top-up loan (L_{min}) are decided by the issuing entity according to their risk targets and goals in terms of efficient use of resources. The higher L_c is set, the later the loss-recognition and stabilizing manoeuvres are triggered. Alternatively, L_{min} operates as a powerful tool to bind the leverage within an efficient zone, depending on the optimal debt-to-leverage ratios applicable to the firm and in line with business risks. Although regulators might restrict some parameters (including L_c), the issuer designs the DCL to align its core mechanism to the goal pursued. This is achieved through changes in the frequency of payments f, the initial loan maturity T, the initial loan nominal Q, the cost of debt R, the leverages triggering conversion, and the re-issuance mechanisms.

4.5 Case study

We simulate the dynamic debt control for a firm with the relevant specifications listed in Table 4.2.

Inputs	**Values**
Nominal debt value, Q	5,000
Maturity of loan in years, N	10
Loan payment per year, n	2
Annual cost of debt, R	5.0%
Expected return on equity, μ	10.0%
Dividend yield, δ	2.5%
Volatility, σ	35.0%
Initial number of shares, NS_0	100
Initial share value, S_0	20
Conversion price, S_p	18
Triggering leverage, L_c	0.8
Lower leverage level, L_{min}	0.5

Table 4.2: All relevant parameters for the simulation discussed in Section 4.5.

Fig. 4.1 shows two simulation scenarios for the same parameter set. The top two graphs show the share price and leverage evolution. The bottom three graphs show how loans are topped up, the evolution of the residual loan value, and the total number of issued shares. Share numbers above $NS_0 = 100$ result from debt conversion to equity.

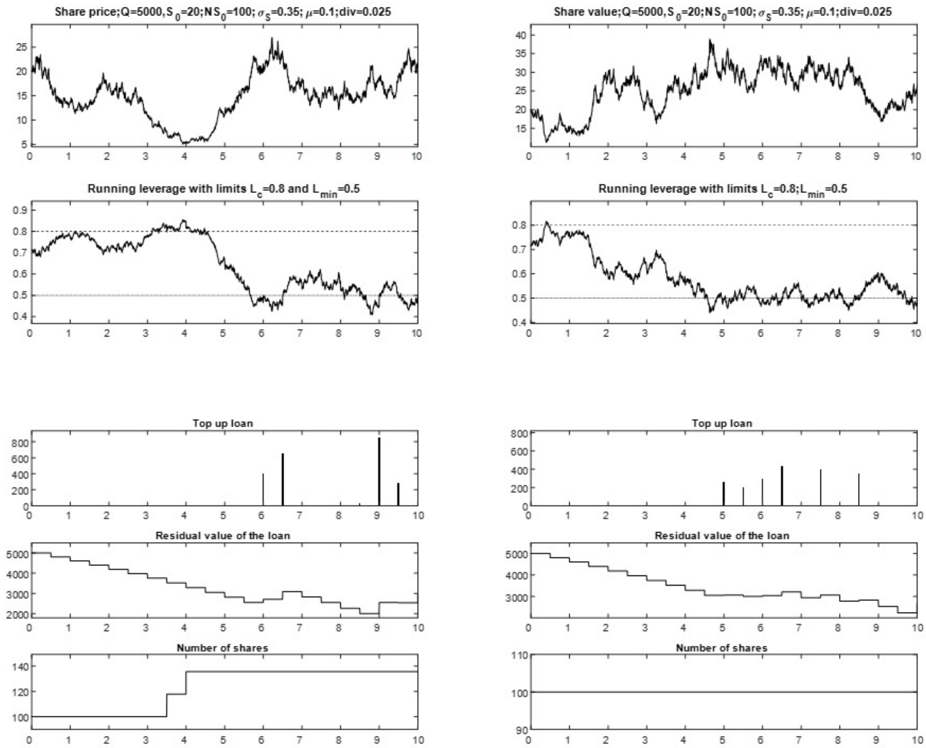


Figure 4.1: The top two graphs present the evolving share price and the associated leverage levels for the same set of parameters. The three lower graphs present the actions taken in response to the events unfolding in the top graphs. Each lower graph, from top to bottom, represent the issued top-up loans, the residual value of all outstanding loans, and the evolving number of shares. Both sets of plots are the results from a simulation of the same parameter set given in Table 4.2. Only a subset of these parameters are presented in the figure titles.

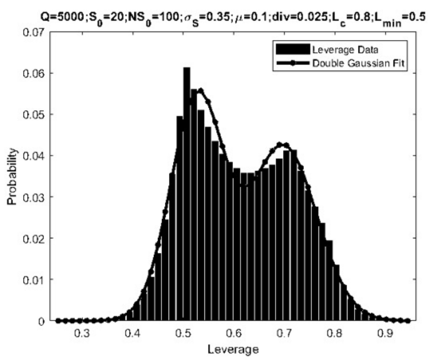
As seen in Fig. 4.1, the top graphs suggest how anti-correlated the leverage levels are to the evolving share price. The leverage level goes down with the reduced residual loan values and increasing share values. From the lower graphs, we observe how top-up loans are issued and how that stops the downward trend in the residual value of all outstanding loans and increases leverage. The bottom panels of the lower graphs also show the total number of issued shares.

On any payment date k, when the re-issuance process is triggered by the inequality $L_k < L_{min}$, a new loan is issued with identical seniority as all previous loans. The new bond acts as a top-up loan, with the same frequency of payment f, identical control levels L_c and L_{min} to the initial loan but different nominal value, which creates a different cash flow over the next N years. When the initial DCL is paid back, the sub-loans created over time remain active. This mechanism ensures the infinite maturity requirement to qualify as an AT1 instrument. The seniority is maintained if the same re-issuance or conversion process is applicable to all active DCLs (if L_c and L_{min} remain unchanged).

With our proposed DCL design, leverage levels are computed on fixed dates, making it possible to re-balance the equity-to-debt ratio and change the choice of debt instruments (AT1 or T2) during the company's lifetime. Consequently, the management remains free to adapt its choice of funding vehicles.

This feature is a significant improvement compared to ERNs that convert their next coupon based on a share price fixed at a fraction of the initial share price at the ERN issuance date. By setting the triggering condition in the leverage space (defined between 0% and 100%), the complexity due to multiple triggering thresholds or scaling properties is eliminated.

Figure 4.2 presents the distribution of leverage for fixed values of L_{min} and L_c and how this can be fitted with double and triple Gaussians. Table 4.3 lists the empirical and fitted probabilities that the leverage values are below L_{min}, above L_c, or between the values L_{min} and L_c.



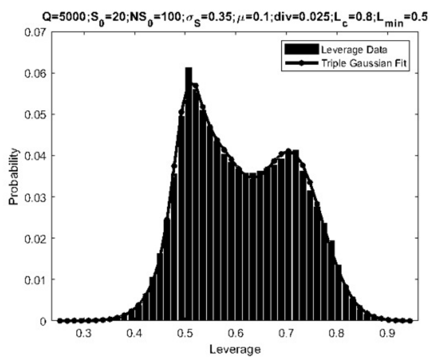


Figure 4.2: The left and right plots show the distribution of leverage for fixed values of L_{min} and L_c. with N_{ex} = 5000 experiments. The left plot is fitted with a double Gaussian and the right with a triple Gaussian.

	Empirical	**Double Gaussian**	**Triple Gaussian**
$P(L < L_{min})$	15.30%	15.27%	15.42%
$P(L > L_c)$	2.85%	2.98%	2.88%
$P(L_{min} \leq L \leq L_c)$	81.85%	81.75%	81.70%

Table 4.3: The empirical probabilities and those fitted with double and triple Gaussians for the leverage values satisfying three inequality scenarios.

The leverage distribution is bimodal, with a considerable mass around the turnaround levels L_{min} and L_c. We fit the leverage distribution with the following probability density function

$$f_T(t, \mathbf{a}, \mathbf{b}, \mathbf{c}) = \sum_{i=1}^{N} \frac{a_i}{\sqrt{2\pi c_i^2}} \exp\left(-\frac{1}{2}\left(\frac{t - b_i}{c_i}\right)^2\right) \tag{4.14}$$

where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are N component real-valued vectors. For $N = 2$, we have a double Gaussian and for $N = 3$, a triple Gaussian. For the cumulative distribution, we find

$$F_T(t, \mathbf{a}, \mathbf{b}, \mathbf{c}) = \sum_{j=1}^{N} a_j \Psi(t, b_j, c_j) \tag{4.15}$$

where $\Psi(t, b, c)$ represents the cumulative normal distribution with mean b, standard deviation c, and evaluated at t.

Parameter Values	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3
Double Gaussian	0.5048	0.4952	x	0.5272	0.6981	x	0.0523	0.0656	x
Triple Gaussian	0.1102	0.4816	0.4082	0.5032	0.551	0.713	0.0246	0.0653	0.0591

Table 4.4: The parameter values fitted to the leverage ratio by the double Gaussian and triple Gaussian distributions.

In Fig. 4.3, we compare the leverage dynamics for two identical firms, one that has its debt in the form of a DCL, and the other as straight "re-issuance debt," by setting $L_c = 80\%$ and $L_c = 100\%$, respectively, to rule out the conversion feature. Extreme leverage values are attained with a higher probability in the absence of the DCL instrument. This result suggests the efficiency of DCL in limiting the default risk for the issuing firm, given the known relationship between leverage and the probability of default.

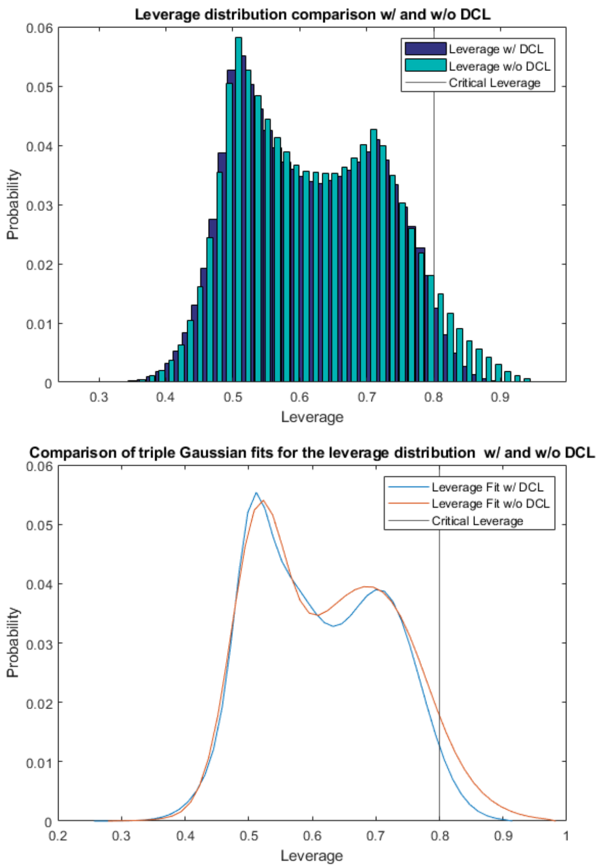


Figure 4.3: Comparison of the leverage distribution for firms with and without DCL in their capital structure (upper plot) and a triple Gaussian fit representation (lower plot). $N_{ex} = 5000$ experiments.

Table 4.5 lists the empirical probabilities that the leverage values are below L_{min}, above L_c, or between the values L_{min} and L_c for the two cases considered here (firms with and without DCL). The DCL instrument in the firm's capital structure acts as a protective barrier and fulfils its role by providing a source of ready cash for the firm to draw in times of financial need, ensuring liquidity and solvency in the long term. The calibrated parameters from Eq. 4.15 are presented in Table 4.6 for the two capital structures.

Table 4.5: Empirical probabilities for the leverage values satisfying different inequalities for the two scenarios of a firm with and without DCL in its capital structure.

Empirical Probabilities	**With DCL**	**Without DCL**
$P(L < L_{min})$	15.30%	14.34%
$P(L > L_c)$	2.85%	6.31%
$P(L_{min} \leq L \leq L_c)$	81.85%	79.35%

Table 4.6: Calibrated parameters of the triple Gaussian for the firm with and without DCL in its capital structure.

Triple Gaussian Parameters	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3
without DCL	0.3078	0.6359	0.0563	0.5134	0.6652	0.7409	0.0417	0.0989	0.0487
with DCL	0.1102	0.4816	0.4082	0.5032	0.551	0.713	0.0246	0.0653	0.0591

Considering a firm with an existing mix of senior and subordinated debt, the overall efficiency of DCL depends on the fraction it represents of the capital structure. This could pose a potential drawback of the proposed instrument, as it could only marginally impact the firms leverage position.

Additional research considerations of DCL presented in this thesis include (i) a deeper discussion on the instrument efficiency (Section 4.6 and Chapter 6) and (ii) a hedging experiment to help issuers design the instrument appropriately to balance the transfer of wealth at conversion time (Appendix C).

4.6 Fraction of CoCo bonds within the capital structure

The efficiency of CoCo bonds in the standard or DCL form depends on their weighting within a firm's capital structure. Using our definition of the debt-to-asset ratio (Eq. 4.5), we create a simple model where a hypothetical firm is financed with a mixture of conventional debt, CoCo bonds, and equity E. The split of debt D is governed by the control variable α_{CoCo}, and the pre-conversion leverage follows as

$$L_0 = \frac{D}{E+D} = \frac{\alpha_{CoCo}D + (1-\alpha_{CoCo})D}{E + \alpha_{CoCo}D + (1-\alpha_{CoCo})D} \tag{4.16}$$

When $\alpha_{CoCo} = 0$, the debt component in the firm's capital structure is entirely made of conventional debt. For $\alpha_{CoCo} = 1$, the debt is comprised entirely of CoCo bonds, potentially lowering the leverage to 0% upon conversion.

At conversion time τ, the bail-in security becomes equity with a coefficient η. This ratio controls the conversion and is implicitly related to the conversion price S_p. Specifically, $\eta = 1$ corresponds to a conversion *at par* (where $S_p = S_\tau$) and

$\eta = 0$ is a full write-down feature. The leverage post-conversion is then defined as

$$L_\tau = \frac{(1-\alpha_{CoCo})D}{E + \eta.\alpha_{CoCo}.D + (1-\alpha_{CoCo})D} \tag{4.17}$$

When designing an appropriate AT1 buffer, a firm's management has a few options to enable effective liquidity cushions. Fig. 4.4 displays the post-conversion leverage for a firm as a function of α_{CoCo} and η under a complete conversion. Assuming a target leverage $L = 60\%$ (the horizontal line in the figure), the firm can adequately design the CoCo depending on its issuance nominal value with respect to the conventional debt book value. If the CoCo nominal is 85.75 (i.e., $\alpha_{CoCo} = 28.6\%$), then the CoCo requires $\eta = 0.5$ to fulfil its role. A coefficient below 1 punishes the CoCo-holders in the case of conversion but limits the dilution for the original shareholders. Enhancing the firm's ability to absorb losses to increase its financial stability and lower the risk of insolvency comes at the price of increasing the firm's overall cost of capital, given its higher coupon rates compared to traditional bonds, potentially impacting profitability.

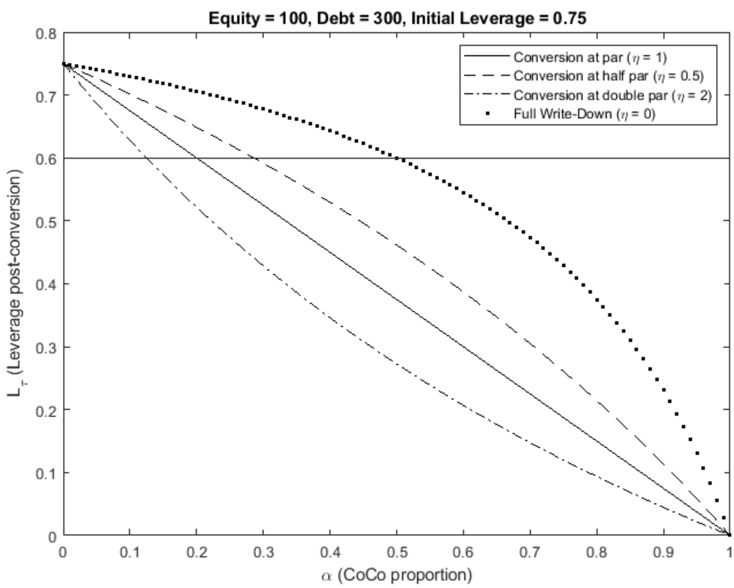


Figure 4.4: Leverage behaviour under various CoCo designs and capital structures. The post-conversion leverage is displayed as a function of the CoCo fraction in the capital structure (α_{CoCo}) and the dilution factor (η).

4.7 Value to bondholders

In this section, we consider the value of the CoCo bond to the bondholders. Under certain financial conditions, the bond pays the bondholder in cash. If these conditions are not satisfied, then the coupons nominal values are converted into shares. Here, the realized value to the bondholder depends, amongst other things, on the relationship between the equity conversion price, S_P, and the market price of the shares, S_k, at the time of conversion. As the CoCo bondholders become shareholders upon equity conversion, the value of their holding depends on the market price evolution of the equity between the conversion and the maturity of the CoCo bond.

We compare the price of the CoCo bond, discussed in this chapter, with an equivalent position in a bond that pays a stream of coupons $P_N(T_k)$ over the life of the bond. The present value of this bonds cash flow is represented as

$$P_{RF}(t, \mathbf{T}) = \sum_{k=1}^{N_n} D(t, T_k) P_{N_n}(T_k) \tag{4.18}$$

As discussed above, the CoCo bond pays, at time T_k, the bondholder the fixed cash coupon $P_{N_n}(T_k)$ as long as the leverage ratio satisfies the inequality $L_k < L_c$. If the leverage ratio satisfies the inequality $L_k > L_c$, then the bondholder receives $(P_{N_n}(T_k)/S_p)$ shares in the company, which at time T_k have the market value

$$(P_{N_n}(T_k)/S_p)S_k$$

.

One possible price of this bond, which can switch between fixed coupon cash payments and conversion into shares, is expressed in a semi-analytical form, where knowledge of $S_{c,k}$ at time T_{k-1} is required to compute the expected payoff at time T_k, such that

$$\begin{aligned} P^{ex}_{CoCo}(t, \mathbf{T}) = & \sum_{k=1}^{N_n} P_N(T_k) D(t, T_k) \mathbf{E}_t[\mathbb{1}_{\{S_k > Sc,k\}}] \\ & + \sum_{k=1}^{N_n} \left(\frac{P_N(T_k)}{S_p} D(t, T_k) \mathbf{E}_t[S_k \mathbb{1}_{\{S_k \leq Sc,k\}}] \right) \end{aligned} \tag{4.19}$$

where the superindex denoted as ^{ex} indicates the value of the CoCo bond after one experiment.

Using the relationship [18] with $t = 0$, we have

$$\mathbf{E}_t[S_k \mathbb{1}_{\{S_k \leq Sc,k\}}] = S_t D(t, T_k)^{-1} \exp\left(-\delta(T - t)\right) N(-d_1(S_0, S_{c,k}, T_k)) \tag{4.20}$$

with which we can write the price of the CoCo bond as

$$
\begin{aligned}
P^{ex}_{CoCo}(t,\mathbf{T}) &= \sum_{k=1}^{N_n} P_{N_n}(T_k)D(t,T_k) - \sum_{k=1}^{N_n} P_{N_n}(T_k)D(t,T_k)\Psi(-d_2(S_t,S_{c,k},T_k)) \\
&+ \sum_{k=1}^{N_n}\left(\frac{P_{N_n}(T_k)}{S_p}\right)\exp\left(-\delta(T-t)\right)S_t\Psi(-d_1(S_t,S_{c,k},T_k)) \\
&= \sum_{k=1}^{N_n} P_{N_n}(T_k)D(t,T_k) - \sum_{k=1}^{N_n} P_{N_n}(T_k)p_d(S_t,S_{c,k},T_k) \\
&+ \frac{1}{S_p}\sum_{k=1}^{N_n} P_{N_n}(T_k)p_a(S_t,S_{c,k},T_k)
\end{aligned}
\tag{4.21}
$$

where

$$
p_d(S_t,S_{c,k},T_k) = D(t,T_k)\Psi(-d_2(S_t,S_{c,k},T_k)) \tag{4.22}
$$

is a digital put and

$$
p_a(S_t,S_{c,k},T_k) = S_t\exp\left(-\delta(T-t)\right)\Psi(-d_1(S_t,S_{c,k},T_k)) \tag{4.23}
$$

is an asset or nothing put [18].

Then, the expected price for the DCL follows as

$$
P_{CoCo}(t,\mathbf{T}) = \frac{1}{N_{ex}}\sum_{i=1}^{N_{ex}} P^i_{CoCo} \tag{4.24}
$$

where N_{ex} is the number of experiments.

The expected loss of holding the CoCo bond, as compared to an equivalent risk-free government bond, can be written in terms of a series of digital puts and asset or nothing puts as

$$
\mathbf{E}_t[L(T)] = \frac{1}{N_{ex}}\sum_{i=1}^{N_{ex}}\left\{\sum_{k=1}^{N_n} P_{N_n}(T_k)p_d(S_t,S_{c,k},T_k) - \frac{1}{S_p}\sum_{k=1}^{N_n} P_{N_n}(T_k)p_a(S_t,S_{c,k},T_k)\right\} \tag{4.25}
$$

In Fig. 4.5, the price of a risk-free coupon bond versus the price of a CoCo coupon bond is plotted as a function of the conversion price S_P for five share prices of the issuer firm. For each share price S_0, the expected loss $\mathbf{E}_t[L(T)]$ can be positive or negative depending on the conversion price S_P. The lower the conversion price, the higher the expected value of the CoCo bond.

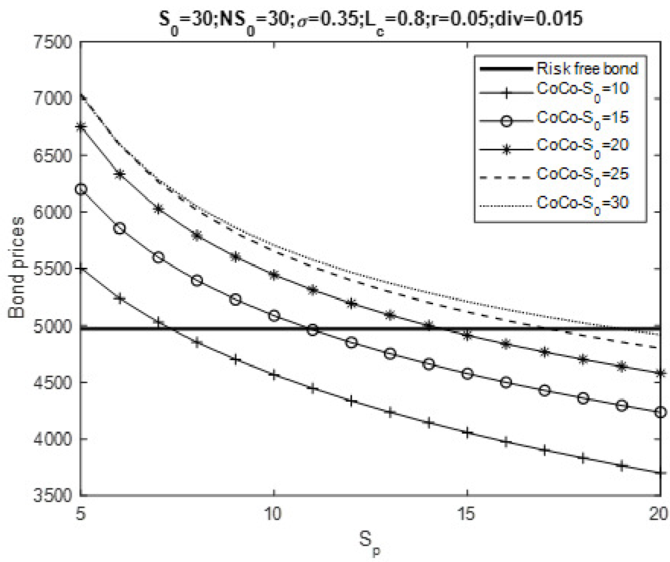


Figure 4.5: Price of a risk-free bond plotted together with prices of five CoCo bonds as a function of S_p for multiple initial share price values.

In Fig. 4.6, the price of a risk-free bond versus the price distribution of a CoCo bond is plotted. This distribution is the result of 20,000 simulations, where each consists of daily iterations over a ten-year period.

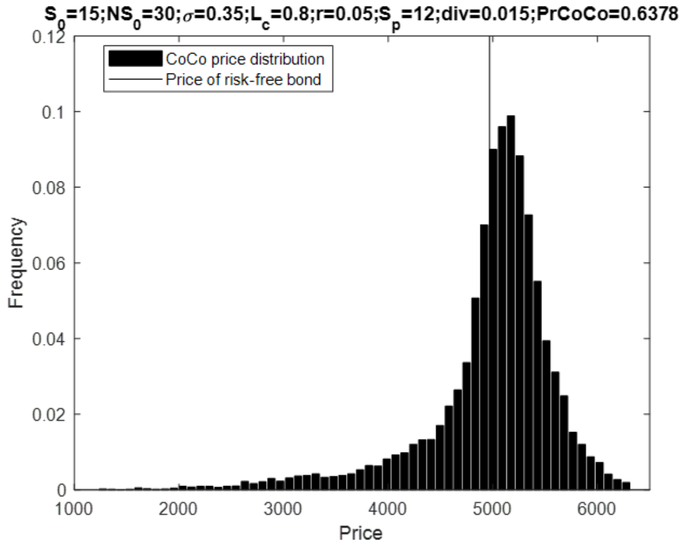


Figure 4.6: Possible price distribution for a CoCo bond with the indicated specification in the title. The price of the risk-free bond is $P_{RF}(t, \mathbf{T}) = 4969.3$. The probability the value of the CoCo exceeds the value of the fixed-paying bond is 63.78%.

According to the above price equation (4.21), the value of the CoCo bond depends strongly on how the value of S_p is set. In Fig. 4.7, we display the probability that a CoCo bond performs better in terms of its present cash flow value than an equivalent fixed payment cash bond as a function of the conversion price S_p.

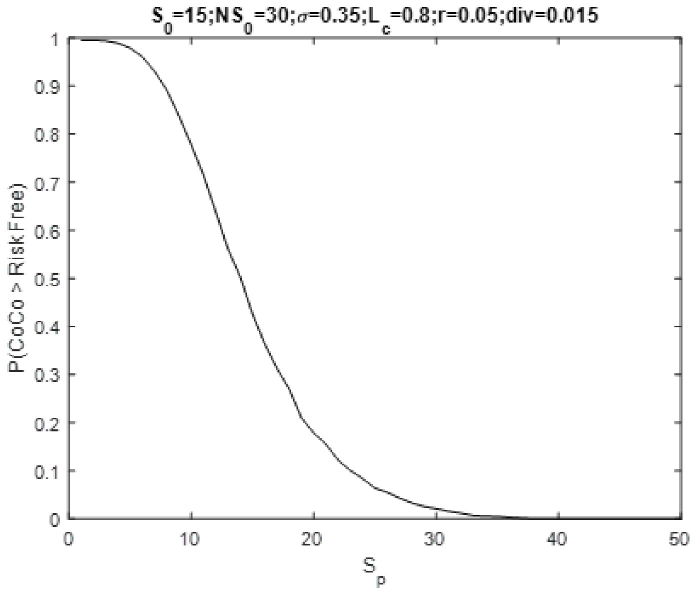


Figure 4.7: The probability that the CoCo bond performs better than an equivalent fixed payment cash bond as a function of the conversion price S_p.

Fig. 4.5 suggests there is considerable variance in the value of the CoCo bonds considered in this work. So, the probability that the value of the CoCo bond is higher than that of the corresponding risk-free bond is considerable. A good-performing CoCo bond is based on the temporarily poor equity performance of the issuer, leading to bond coupons being converted to equity, which subsequently recovers and regains values in excess of the conversion ratio S_p. The value of the CoCo is sensitive to the conversion ratio, as demonstrated in Fig. 4.7, and to equity prices post-conversion.

4.8 Write-down feature

The model developed in this chapter is flexible enough to feature a write-down (WD) and conversion to equity. Instead of converting the next coupon into shares, the interest payment could simply be cancelled when the leverage exceeds its critical value on a payment date. In such a case, the second term, intrinsic to the equity component in $P^{ex}_{CoCo}(t,\mathbf{T})$ is removed, and the pricing of DCL reduces to

$$P^{ex}_{CoCo}(t,\mathbf{T}) = \sum_{k=1}^{N_n} P_{N_n}(T_k)D(t,T_k)\mathbf{E}_t\left[\mathbb{1}_{\{S_k>S_{c,k}\}}\right] \tag{4.26}$$

Such modification to the design diminishes the price of the security due to increased risk associated with the investment, consequently leading to a higher required yield. Considering the recent observations in the CoCo market behaviour, such an option could enhance investor interest.

4.9 Sensitivity incurred by the model choice

In this section, we review the sensitivity of the results to the choice of model, which is an essential aspect of financial modelling. By examining the implications of our modelling decisions on the DCL output response, we aim to provide a qualitative approach to understanding how variations in model assumptions and parameters can impact the results and interpretations of our study.

A firm's capital structure can be refined in many ways. A key consideration was provided by Modigliani and Miller's Proposition II [27] that postulates a shift in a firm's leverage from its initial level correspondingly influences the cost of equity. If we assume, for example, an increase in the firm's debt-to-equity ratio, either through a rise in debt or a fall in the market value of equity, would typically lead to a higher expected return on equity, all else being set equal. This outcome is because equity holders require a higher return for the increased financial risk they bear.

Here, we reintroduce the leverage L as the ratio of debt D over the assets A as

$$L = \frac{D}{A} = \frac{D}{D+E} \tag{4.27}$$

where the equity E is composed of shares valued at S_t. Following a geometric Brownian motion process defined by the SDE (Eq. 4.3), when then have

$$dS_t = (\mu - \delta)S_t dt + \sigma S_t dW_t \tag{4.28}$$

We observe that the return on equity μ drives the dynamics of the equity value while also being influenced by the changes in equity value via changes in the leverage ratio. So, we obtain a feedback loop that leads to a complex dynamical process.

We provide an overview of the cascading effect through individual causal relationships:

(1) We begin with a given leverage L_0 and an associated expected return on equity μ_0, determined by Modigliani & Miller's Proposition II.

(2) We assume some factors cause the leverage to increase at time T_1, making it expected that $\mu_1 > \mu_0$ so as to compensate equity holders for the increased risk.

(3) This increased μ parameter could result in a faster-than-expected increase in the share price. (However, the actual increase in E could be lower or higher due to the random component).

(4) An increase in E would, all else being set equal, slightly reduce the leverage ratio L, counteracting the initial increase in leverage.

(5) Finally, the conversion probability might be slightly lower than the one estimated in the context of Section 4.5.

The opposite effect may also be true, where a decrease in leverage might tend to be altered by a decrease in the cost of equity, thereby lowering $P(L_t < L_{min})$, i.e., the re-issuance probability.

While the simplified approach adopted in this discussion offers a useful perspective and intuition to address the research question, the model may not exactly capture all "real-world" dynamics occurring in these variables. The DCL is solely a mechanism that "responds" to the change in leverage by triggering an appropriate reaction, such as re-issuance, conversion, or cash payment. Changes in the modelling of the equity component will affect the pricing of the instrument, but its purpose of preserving the debt-to-asset ratio between the two boundaries L_{min} and L_c remains unaffected.

4.10 Key Takeaways from the Analysis

In the ever-changing financial sector, CoCo bonds have consistently sparked debate. While they offer unique features, research has highlighted their shortcomings, particularly in their design and triggering mechanisms. This chapter's primary contribution is the introduction of the DCL model, addressing some of the identified limitations:

1. Self-Adaptive Mechanism: The DCL model is distinguished by its self-adaptive nature. Unlike traditional CoCo bonds, which rely heavily on external judgment, the DCL model offers a dynamic approach to managing a firm's leverage. Whether it's about increasing the leverage when it drops below a threshold or converting debt to equity when it exceeds a limit, the model operates with minimal external interference.

2. Limited Conversion Impact: By confining the conversion to the interest payment value, the DCL model significantly reduces the potential dilution effect on existing shareholders. This design choice is crucial in preventing market panic and ensuring stability during conversion events.

3. Real-time Monitoring: The DCL model's emphasis on real-time monitoring, particularly through the use of the debt-to-assets ratio as a control variable. This not only simplifies the conversion mechanism but also offers transparent, real-time triggering risk information. This is a marked improvement over traditional CoCo bonds, which often suffer from delayed triggers due to reliance on quarterly disclosed accounting numbers.

4. Flexibility: The DCL model offers inherent flexibility, allowing for the integration of various in-built features. This includes the Write-Down feature, as well as the refinement of the control variable through continuous observation of the leverage [69] (Chapter 8).

5. Applicability Across Businesses: Our analysis suggests that the DCL model's design makes it suitable for a wide range of businesses. Its intrinsic adaptability and automated processes are well-suited to diverse capital structures and financial requirements.

6. Mitigating Default Risk: By ensuring that the leverage remains within acceptable bounds, the DCL model acts as a 'watchdog' against the risk of default. Through our analysis, we demonstrated that a firm issuing a DCL exhibits a reduced probability of default compared to a firm with regular debt.

4.11 Summary

In this chapter, we extended a recently developed model by reintroducing a specific type of a CoCo bond with fixed cash payments of regular and equal down payments of one or several simultaneously outstanding loans. The leverage level for the firm, at any given time T_k, is determined by the residual value of all outstanding debt RQ_k^T and the firm's total equity value $TS_k = NS_{k-1}S_k$, where NS_{k-1} is the total number of shares issued at time T_{k-1} and S_k is the share value at time T_k. Coupon payments are converted to equity (shares) whenever the leverage exceeds some critical value L_c. These are made in cash whenever the leverage L_k as of the payment date T_k satisfies the inequalities $L_{min} \leq L_k \leq L_c$. If the leverage L_k on the payment date T_k falls below L_{min}, then more debt is issued to increase the resulting leverage to L_{min}. The implemented leverage dynamics is stable and, with high probability, maintains the leverage level within the boundaries defined by the two values L_{min} and L_c. We calculated the probability $P_k(L_{min}, L_c) = P(L_k \in [L_{min}, L_c])$, which is high to very high for all reasonable scenarios, implying good leverage stability and a low probability of default.

The benefits of the proposed approach are that the decision for action at each payment date T_k is clear as it only requires the knowledge of the residual loan amount RQ_k^T, the number of shares issued at previous time T_{k-1}, and the market value of the firm's shares at time T_k. Bureaucratic or judgement-based involvement of external bodies or internal risk committees is not required. The model leads to a stable leverage position, where the probability of default or bankruptcy is low to moderate. The fixing of the two key leverage levels L_{min} and L_c should be done based on the probability distribution of the leverage L_k, which can be inferred from the book value of the outstanding debt, the payment schedule, and the share dynamics.

We demonstrated how the value of the CoCo bond depends on the conversion price S_p and that for a considerable range of conversion price values S_p, the holders of the CoCo bond are in high probability better of than the holders of corresponding risk-free bonds. The key is that at high leverage levels, coupon payments are converted into equity, improving the leverage position. This occurs as long as necessary or until the leverage value has been brought within acceptable boundaries.

The strength of this approach is that the decision on how coupons are paid, in cash or equity, is automated in real-time. No delay in the settlement of payments is required, as is the case in some previous versions of CoCo bonds, where the form of payment depends on accountancy-related quantities that are not immediately available.

Chapter 5

Legal Framework and Practical Considerations

"There is certainly a role for regulation, but regulation should always take into account the impact that it has on markets – a balance that must be constantly weighed [...] More regulation is not the best answer to every problem."

Jerome Powell, [70].

5.1 Current context

The discussion around legal frameworks must begin with an overview of the historical background of how existing contingent convertibles fall within regulatory incentives, depending on the product setup and the location jurisdiction where the CoCo instrument is sold. We will see to what extent modifying the triggering design affects the current principles and how the bond covenant or legislation should be adapted to retain the bank interest and regulator advancement.

Glasserman & Perotti showed that the equity component in existing CoCos is very low [14], and we explain this below with an assumption related to the low conversion probability.

First, we consider product design. In gone-concern trigger products (i.e., low-triggers), the debt converts to equity only when a breach in the CET1 ratio requirement occurs, or worse, at the PONV for the firm, leaving no room for the equity conversion to be efficient. Nevertheless, even with going-concern triggers (i.e., high-triggers), where the "debt-induced collapse"[1] is supposed to be avoided, conversion remains conditional upon regulatory approval. Market triggers such as share price or CDS spread are not encouraged because of this need to monitor the situation discretionarily. With accounting-based triggers, if a bank had to breach a conversion trigger, then it would implicitly require a discretionary endorsement from its dependent financial authority.

Second, we consider the market expectation. The 2016 events in the CoCo market provided insights into the gap between the intrinsic uncertainty held by CoCo and investor beliefs. Following fear of skipped coupons, the price of the primary quoted instruments from BBVA, Santander, Banco Popular, UniCredit, and the influential Deutsche Bank sharply decreased. With the coupon being higher for this type of risky debt, a deferral would be a significant disruption. The correction was so virulent that it is now transparent to the market contributors (investors and regulators) that they were underestimating the associated risks with this instrument and were unprepared for the retained coupon's eventuality. Once again, the equity role contained in CoCo was reduced as soon as the financial authorities decided on legal changes to encourage banks to pay coupons even under challenging circumstances, such as the 2016 adjustment by the European Central Bank (ECB) following the yearly Supervisory Review and Evaluation Process (SREP) that turned a legal constraint into a recommendation [71].

The following sections provide a closer look at the capital requirement pillars. The so-called "Pillar 2 add-ons" that set rules on additional capital requirements are split into a must (P2R) and an only for guidance part (P2G). If the necessary condition is not satisfied, then the bank is limited in its profit distribution by a Maximum Distributable Amount (MDA[2]). Otherwise, if the voluntary (but encouraged) part is not fulfilled, then the bank is allowed to distribute profits through dividend payments or, in our use case, coupon remittance on Additional Tier 1 instruments. The side effect is the decrease in absorption capability of the bankruptcy risk, originally a target of contingent convertibles. To date, Bremer Landesbank is the only issuer to have skipped a coupon on AT1 in June 2017.

Glasserman & Perotti also found it understandable that regulators try to circumvent additional market stress by not assuming losses. This willingness is in opposition to post-crisis resolutions, including acting ahead of excessive threats to the economy and ensuring that financial actors, instead of taxpayers, bear the risk.

Even if focusing more on the weak equity component, these observations represent criticisms about the current CoCo design. On the one hand, the idea proposed with the DCL instrument allows for increasing the equity portion, bringing back the prominent role of a contingent convertible in that it is convertible to ease

[1] This is the language used by Glasserman & Perotti to describe a default border above the conversion trigger.

[2] As defined in Article 141 of the European Directive No 2013/36 (Capital Requirements Directive, CRD IV).

a hazardous situation for a firm approaching a financially distressed state. This mechanism is permitted by combining a higher equivalent convertible trigger and a less dominant position from regulating entities resulting from the mix of market values (market capitalisation) and outstanding debt, which are both continuously and publicly observable data.

On the other hand, this new triggering condition will not further foster the spread of CoCo issuance in countries that treat the contingent convertible as not a sufficient debt product, to allow the issuer tax deductions on the coupons, including the United States, Ireland, and more recently Sweden (as of 2017) [72] and the Netherlands (since January 2019). The Netherlands government also expects the European Commission to follow this trend, bringing to an end the tax-deductibility of coupon payments for AT1 instruments, and then fulfilling their initial desire to unburden taxpayers (who are indirectly assuming the cost of such tax deductions on AT1) [73].

In the US (and other countries where the regulation is undergoing change), the AT1 related to CET1 demand is satisfied by the issuance of preferred shares instead of contingent instruments. In Asia, the inverse phenomenon occurs, with a switch from preferred shares to CoCo instruments. Section 4.2.2 from [28] examines this aspect.

The initial purpose of redefining the core design of contingent convertibles was not to make it more attractive for the issuer and the investor. Instead, it was to reconcile with its main interest of creating a true safety buffer that makes the bank more resilient, decreasing the systematic risk, and by extension improving the financial stability and the weight of the burden in the case of a financial crisis for the society. In the long run, regulators and financial authorities are believed to overtake the sole coupon deductibility advantage through other incentives.

Additionally, Avdjiev highlighted other considerations justifying a lack of interest in CoCos [74]. Considering that about 11% of the European bank debt is owned by life insurers [75], the large capital charge load surrounding insurance companies, according to Solvency II, can justify such a reluctance. Also, the Solvency II regulation introduced in 2016 constrains the proportion of owning funds through tier-layering constitution. While Tier 2 can be up to 50% of internal funding and Tier 3[3] is limited to a 15% boundary, a minimum of half the Solvency Capital Requirement (SCR[4]) is required to be Tier 1, with an expectancy of 80% in unrestricted T1, and at most 20% of Restricted Tier 1 (RT1), such as contingent convertibles. We assume that with our enhanced instrument, the conversion risk is more significant, leading to higher capital requirements while lowering the

[3]This does not apply to Basel III, as Tier 3 was eliminated from the framework.

[4]The minimum required capital to fit a 99.5% probability that the firm can meet its obligation in the following twelve months, as defined in Article 101, Section 4 (Solvency Capital Requirement) of the European Directive [76].

probability of default for the CoCo issuing bank. Thus, we assume the undesired impact induced by the regulation remains unchanged.

The challenge for credit rating agencies in assessing the risk represented by a CoCo is a recurring issue that has been considered before in the research. As clarified by Kiewiet, Lelyveld, and Wijnbergen, the obstacles are either due to jurisdiction discrepancies (e.g., around the MDA trigger definition), unforeseen triggering possibilities by regulators (e.g., when the firm is close to the PONV), or the likely violation of the absolute priority rule, intended to refund creditors (debt holders) ahead of equity holders in the case of default [30]. These scenarios are detailed in the following:

- The infringement of the liquidation preference is more likely to arise for a high trigger CoCo ($> 7.125\%$) and induce losses to the contingent convertible holder before the stakeholder. With low-triggers, the shareholder should suffer a loss before the CoCo-holder, given the existing stress on the firm's financial health at the trigger level.
- The freedom given to monetary supervision authorities through the PONV is criticised because of its the potential to occur before the conversion threshold.
- In some cases, the absence of ratings prevents some market makers from investing in this asset class, such as with pension funds.
- When a rating exists, Hybrid Financial Instruments (HFI) are typically not allowed to be part of the bond indexes because they do not reach "investment grade"[5]. Applying the notching methodology allows differentiation of issues from a given financial entity that considers its creditworthiness and dependence on the issuance position in the claims hierarchy. In a 2011 Standard & Poor's publication, the rating agency advocated for an adjustment of two notches below the Issuer Credit Rating (ICR) base if the ICR is BBB- or higher and three notches downgrade if the ICR is BB+ or lower. Also, the highest grade accessible is at most BBB+ for a hybrid security. (See Appendix D).

We further discuss the role of rating agencies in Section 5.5.

Even though the large number of features that can be built into CoCos makes their potential funding impact for the issuing bank very flexible and their pricing methods challenging, comparing two bonds remains difficult. This lack of normalisation led the European Banking Authority (EBA) to release in October 2016

[5]Nonetheless, some indexes are specially designed for CoCos, such as the iBoxx Contingent Convertible Liquid Developed Europe AT1 Index (AT1 index, ISIN: GB00BF9Q3T33, BBG: IBXXCCL1).

standardised templates for banks interested in AT1 issuance [77]. The document provides details on the prudential terms and conditions relative to AT1 instruments. More recently, in June 2021, as part of the EBA regular reports on AT1 issuance through Europe, the use by EU institutions of the provisions suggested in these standardised templates was still rising. The banking authority revels from this continuous trend in standardisation [78]. The market expectations are considered in the next section.

5.2 Introduction to capital requirements and Basel III implementation through CRD IV

Although policies on contingent convertibles widely differ depending on the region, Basel III should be considered a standard in the industry and result in a convergence of treatment. Basel III refers to the third update of the regulatory framework (following Basel I and Basel II) to increase the quantity and quality of capital held by banks. Historically, the directive set minimum levels of capital-to-asset ratios. However, experience showed that a more accurate definition of capital and assets was required. For example, viewing individual asset classes in terms of their risk contribution to a bank's overall asset portfolio is vital. For this purpose, Basel III established new financial ratios and minimum thresholds with which banks must comply.

The vagueness around the numerical requirements is founded in calculating these metrics. The triggers for instruments issued so far are expressed as the issuing company's CET1 ratio to its RWA. The lack of transparency that exists for this calculation is a concern that could potentially lead to distortion when qualitatively gauging the benefits of triggering ratios [31]. Ideally, the metric is objective, transparent, fixed, risks no jurisdiction disruption, and is publicly disclosed with at least a quarterly frequency.

The three layers of the capital stack are typically defined as the following:

- CET1 (Common Equity Tier 1) is supposed to be the safest holding, composed primarily of the bank's common share and retained earnings, as well as intangible assets, certain qualifying issues, and adjustments. Basel III urges this ratio to be at least 4.5% of the RWA.
- AT1 (Additional Tier 1) performs as a security buffer for the financial stability of the bank, where the sum of AT1 and CET1 must be at least 1.5% of the RWA to guarantee quality in the held assets. AT1 also includes preferred shares[6] and high-trigger CoCos. The mechanical trigger must be at least 5.125% of RWA to qualify as a contingent convertible.

[6] Preferred shares do not meet the maximum subordination condition to qualify as CET1.

- T2 (Tier 2) applies to other non-CoCo subordinated debt and low-trigger CoCos (< 5.125% of RWA). This layer is summed with the prior layer (AT1 + CET1) and is required to account for 8% of the overall RWA.

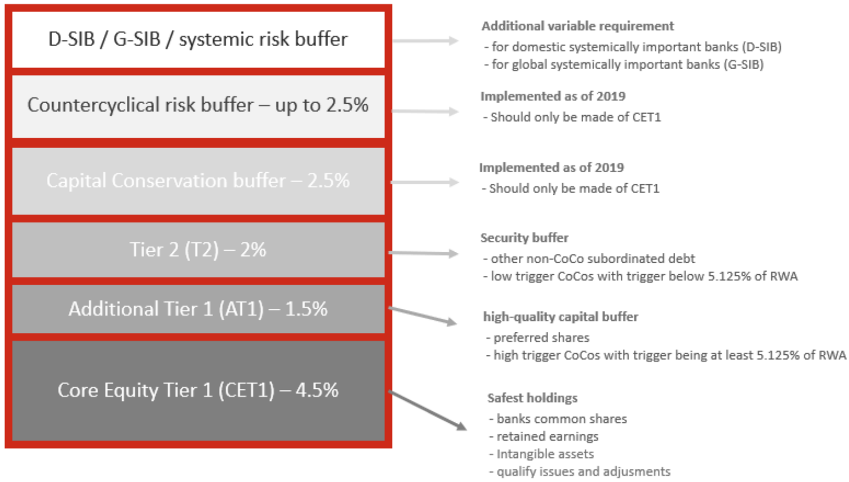


Figure 5.1: The stacked structure of capital, the qualifying underlying characteristics, and the minimum capital levels, according to the Basel III regulation.

The package composed of CET1 and AT1 is called Tier 1. Adding the Tier 2 layer provides the Minimum Own Fund Requirement, also called Pillar 1. On top of this interlocks the P2R mentioned before (standing for the Pillar 2 Requirement), which is specific to banks[7]. This first stack creates a first minimum requirement boundary. The regulation also provides a provision portion identified as the Combined Buffer Requirement (CBR) that contains three capital buffers. If a company fails to fund this provision, then it will be affected by an MDA restriction. Also, as mentioned before, a Pillar 2 Guidance (P2G) that is specific to banks exists. All capital up to the MDA is called the Overall Capital Requirement (OCR). The split between P2G and P2R in 2016 allowed for flexibility in the ratio. The fourth execution of SREP (in 2018) led to a new risk assessment (applicable in 2019) and included a 10-bps increase of P2R (up to 2.1%) and a 10-bps decrease of P2G (down to 1.5%) [79]. As a result, the P2G buffer could no longer be used as an AT1/T2 shortfall. Thus, the requirement enables the holding of OCR, P2G, and any shortfall in AT1/T2.

[7] Under the Capital Requirements Directive (CRD IV), which is the implementation of the Basel III guidelines in the European Union.

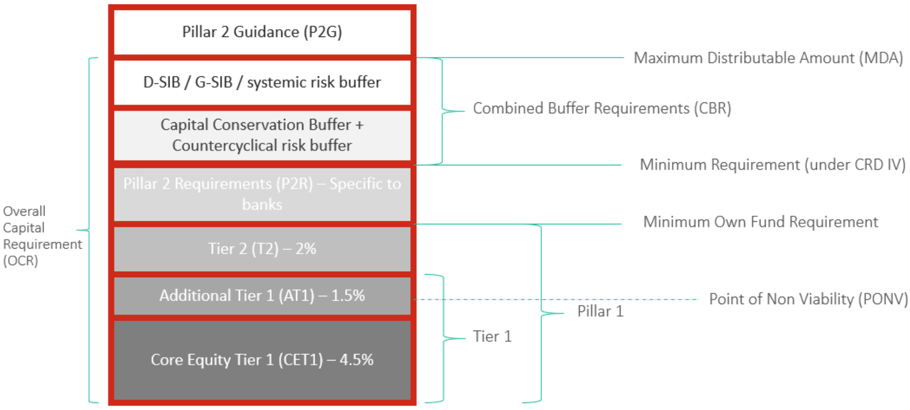


Figure 5.2: Graphical representation of the restricting regulatory levels as defined in the regulation.

The CET1 is highly sensitive to the RWA. So, in the future, the introduction of Basel IV accompanied by an increase in risk weights (including the introduction of floors) is expected to severely impact banks with risky or sizeable asset exposures in their portfolios. A transition phase is planned to soften this effect.

In addition to the numerical constraints, an infinite maturity is required to qualify as AT1 under the Basel regulation (while only five years are sufficient for T2 qualification[8] under Basel III and ten years under Solvency II). Finally, a discretionary trigger in the form of a regulatory control if the firm reaches a supposed PONV is needed to qualify as AT1 or T2. However, the definition of this PONV within the existing bond covenant is unclear and ambiguous.

With unlimited and regular cash-flow transfers, perpetual instruments are often treated as equity-paying dividends. This practice is aligned with IAS 32 and IAS 39[9] and allows the issuing company to receive accounting benefits [25]. As seen before, this asset specification enables banks to fulfil their capital requirement. Today, most include a callability option for the issuers, allowing them to redeem the bond after some years[10]. The standard in the industry (and, by extension, what became the market expectation) is in favour of constantly calling the

[8] According to the article 63-64 of the Capital Requirement Regulation (CRR), attention is drawn on the T2 eligible part being only a ratio of the nominal value.

[9] International Accounting Standards.

[10] Under Basel III, usually ten years, but not less than five, which is defined as a call protection period. However, even during this period, a Regulatory Event Clause allows the issuer to call back the bond if a regulation adjustment made it ineligible for the capital category from which it was primarily purposed. A note with a 10-y maturity and 5-y non-call period is dubbed 10NC5. Under Solvency II, the 30NC10-format is more usual.

issue on the first possible date. In other words, if not employed, then creating market stress around the solvability of the issuer. This type of asset is relatively new for some top-name issuers, as the first callability date was in 2019. Santander surprised and negatively affected the market as soon as the Spanish bank announced skipping the first call date on their 1.5 billion euros issue in February 2019. A similar event occurred with a Deutsche Bank CoCo bond in 2020. Some analysts argue that the decision is proof of a maturing marketplace, driven by economic decisions and not "reputational" ones [80]. This analysis could be in accordance with the decision taken by Santander's management if some of the non-exhaustive Basel III criteria listed in the following were not satisfied at the callability date:

- Inability to demonstrate that its capital would remain well above the minimum capital requirement once the instrument is called.
- No disclosure or behaviour that would create a call event expectation.
- No call exercise without substitution of the early-redeem issuance for any capital of the same or better quality[11].

In addition to the above requirements, the supervisory authority's approval is required to call an AT1 instrument.

The cost of issuing a new AT1 is an essential factor to consider, as most of the CoCo issues reset the coupon level to a floating *plus* fixed rate after the first call date. Looking at the yield-to-call offers a hint of the market expectation. If the new coupon level is lower than the yield-to-call, then the issuer has no interest in calling the instrument to emit a new one (and vice versa).

No specification or requirement is issued regarding the loss-absorption structure, meaning that either a Conversion to equity (C) or a Principal Write-Down (WD) can be used. In fact, Glasserman & Perotti asserted that no research unanimity managed to favour one over the other. These are not the two only options, as a Write-Up feature (WU) can be embedded on top of WD, allowing only a temporary write-down that is cancelled if the bank's situation later improves. Such as the coupon payment on the AT1 instrument[12], the WU is considered a payment, so the amount to be written-up must not exceed the MDA limit for the given bank. Yet, regulators tend to not favour the WU mechanism as it would go against their only equity increasing principle.

In Fig. 5.3, we observe the split between the different existing loss-absorption mechanisms for the CoCo market. The upper plot considers the CoCo issuances since 2010 and the lower plot considers the CoCo issuances since June 2019. As

[11]This statement applies less to the Santander case as the bank issued a 1.2 bn euros CoCo a few weeks before the first call date of the 1.5 bn issue.

[12]RT1 instruments issued by insurers are different from the AT1 in the sense that any MDA does not limit payments.

the data are cumulative, there is a transitional stage over the first observations before reaching a permanent regime. The lower plot also illustrates the change in issuance behaviour over recent years. Whereas 'Conversion to Equity' represents 44% of the global issuance, this number is reduced to 28% when considering only the previous three years, suggesting an uptrend in the use of Write-Down features. The cumulative ratio of WD mechanisms amounts to 56% when considering the entire data set, and 72% when considering data since June 2019.

This observation can be construed as a sign that the market is doing what is best for raising investor interest. The WD mechanisms (temporary or permanent) avoid dilution and then receives stronger support from the initial shareholders than conversion to equity. Furthermore, it prevents most operators that only have a mandate (or simply expertise) in the Fixed Income market from being forced to sell the newly issued shares after conversion, often in an anxious market climate.

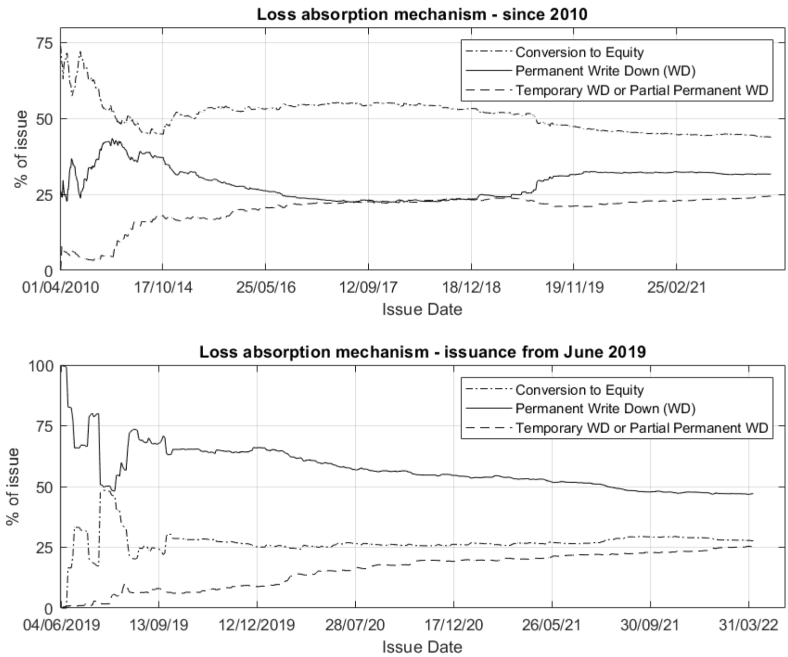


Figure 5.3: The two plots present the choice of the loss absorption mechanism associated with the global CoCo market. The upper plot considers the CoCo issuances since 2010 and the lower considers the CoCo issuances since June 2019. As the data are cumulative, having the second plot allows for the observation of the change in issuance behaviour over recent years.

In some countries, financial supervision may appear to be more stringent for banks. In the Swiss example, the FINMA[13] calls for additional capital margins (in addition to Basel III) that amount to a minimum of 9% of RWA in loss-absorbing instruments.

With capital ratio-based triggers, the distance to the trigger is a metric computed from bank-disclosed information, which can easily link to the conversion probability. However, as CoCo deals with loss absorption in tense situations, the results from the stress tests run by the EBA in 2016 and 2018 are important factors in the quantitative assessment of the conversion eventuality. The key facts from this study are outlined in [28] that aggregated a 428 bps average decrease in

[13]The acronym for Eidgenössische Finanzmarktaufsicht, which is the Swiss Financial Market Supervisory Authority.

the CET1 ratio between the end of December 2015 data and the simulated setup (three years adverse forward outlook). One Italian bank out of 51 tested across Europe resulted in a breach of the 5.125% trigger. Unable to raise funds while still not insolvent, the Italian state decided to purchase (at a discount) shares for more than 5 billion euros in 2017. Raiffeisen, the penultimate in the ranking outcome from the 2015 stress test, motivated the issue with its first two CoCos on the market in 2017 and 2018 totalling 1.15 billion euros. Following the stress test conducted in 2018, the global situation improved, with an average of 419 bps drop on the virtual CET1 of 48 banks under an unfavourable setup. Of these banks, none broke a 7% level.

5.3 The rollout of newly designed CoCos in the regulatory and economic environment

For many criteria, leverage-based CoCos already fall within existing regulations. Besides the stronger bank resilience to absorb economic shock, investor worries are concentrated on the skipped-coupon risk, PONV,[14] and the acceptance of the incoming Basel IV guidelines. Redesigned contingent convertibles should find support in this economic landscape to address some of these concerns. However, such an update will involve a higher trigger, which increases the possibility of conversion while still protecting the bank more efficiently. Some precisions could be needed, especially if the optimum conversion time (defined as a percentage of the debt-to-asset ratio) would always lead to qualifying the admission as AT1. Depending on the outcome, a second trigger that formally complies with Basel III/CRD IV could be an extra option that neither changes the triggering probability nor the pricing of such an instrument. Multi-variate triggers are possible and have appeared in research papers and are already issued in cross-assets products. Marcin Liberadzki and Kamil Liberadzki provided examples of CoCos from various issuers that incorporate a double trigger for banking firms with a parent-holding company or groups with a central entity.

In this case, the first trigger relates to the CET1 ratio at the bank scale and the second at the group scale. An alternative using a combination of macro- and micro-triggers exists that consists of a globally economic-related trigger, indicating a threat to the financial system (macro) and a bank-related indicator (micro) [29]. Also, if an institution issued different CoCos with a trigger set at different levels, and assuming these are hit simultaneously, then the European Banking Authority specifies that the loss-absorption should be made by all the instruments concerned

[14] The PONV concept being not applicable to qualify as T2 under Solvency II, as only a default event could bail-in the instrument.

by the conversion to restore the minimum requirement, on a pro-rata basis (point #124-125 [78]).

In June 2019, the EU released the final version of CRR II (Capital Requirements Regulation II) in its official journal that fine-tuned the existing regulation with specific attention to leverage and net stable funding ratios, own funds requirements, eligible liabilities, and market and credit risks [81]. The provisions reported in CRR II are aligned with international standards and drive banks to identify legacy instruments that do not meet the new requirements for being considered as part of their regulatory capital under the latest definition [82]. These instruments will then be downgraded to a lower tier. The transitional period for these "grandfathering provisions" ended on December 31, 2021[15], whereas 20% of these bequest debt instruments were still qualified as own funds in 2020, this figure was expected to be capped by 10% in 2021 [83]. The AT1 quarterly note from the Scope Ratings also points at the infection risk due to distribution and subordination restrictions as specified in the legacy instrument covenants, potentially limiting the flexibility around AT1 payments when higher capital tier instalments are cancelled. To avoid such complications, the EBA advocates remedies including but not limited to either calling, redeeming, or buying back the security[16].

If our proposed DCL instrument proves to be a better capital cushion than traditional ones, then the soon callable AT1 instruments could be rolled over with this improved quality hybrid asset. The regulation and low-interest rate environments could be perceived as suitable and supportive of this switch.

The position of a CoCo, whether capital-ratio or leverage-based, within the liquidation hierarchy that remains between the debt and equity, the holders of the upper secured/senior tranches[17] would not favour a mechanism over the other in the case of established bankruptcy or forced restructuring because given an equal outstanding, both triggers provide an equivalent shield. Yet, while the institution continues operations, the upper secured/senior tranches holders would perceive triggers built on debt-to-asset ratios as less dangerous. The conversion time is more distant to the level incurring a possible induced loss for them, which creates a divergence with equity holders, as they would be less supportive of the alternative trigger that increases their dilution risk. A compromise could be with DCL that includes a write-down feature instead of conversion (WD DCL). As observed in Fig. 5.3, the use of WD is already predominant across the industry.

Additionally, the rollout of the newly designed CoCo could be accomplished according to the green finance trend. In recent years, most of the European Finan-

[15] Further requirements extended the transitional period until June 2025.

[16] Another option amends the provisions for existing non-qualifying securities

[17] The claims are ordered depending on the debt seniority. First, the secured debt (senior and junior), then the unsecured (senior, subordinated, and then junior, like AT1 and T2).

cial Institutions Group (FIG) have proceeded toward green bond issuance. The purpose here is to provide investors with an instrument that answers ESG standards (Environment, Social, Governance) to finance eligible assets in the issuer portfolio. This type of security is sought by investors worldwide and supported by regulators to support the social and environmental efforts of financial institutions [84]. Because of this high demand, the market observes a green premium on qualifying issuance, also called "greenium." In a December 2020 working paper from Amundi Asset Management, green bonds were identified to come with additional costs from the issuer's perspective, whereas, for the investors, there remain significantly equivalent to the bond without the green characteristics [86].

To date, only BBVA has issued a green CoCo bond qualifying as AT1[18], in 2020. However, the rally into green-labelled T2 instruments is strengthened by the European FIG that issued for nearly 6 billion euros during the first half of 2021 [85]. The EBA reminds us that on the pretext of the ESG label, the loss-absorption effect should not be deflated from qualifying CoCos. Another apprehension from the regulator arises from the maturity of green assets that might not be aligned with the time horizon of the CoCo (supposedly perpetual with an initial non-callable period and no incentive to redeem the hybrid security). Being more flexible due to the re-issuance process, DCL could help meet this requirement.

Finally, the performance of the note should be independent of the green assets, and missing an ESG goal does not constitute a default event. Conversely, to the best market practice in place for sustainability-linked bonds, AT1 should not be allowed to encompass a clause of coupon step-up or fee based on the success or failure fulfilment of any ESG scheme [78, 85].

5.4 Practical considerations

A well-known paper written in 2002 and published in 2005 pioneered the research on CoCo [47]. At that time, the disastrous consequences of the crisis that followed were already anticipated with exactitude. Flannery was struck by the lack of advance arrangements to restructure firms if significant losses occurred and so provided a 'minimum viable product' (MVP) to prevent government intervention and facilitate bank recapitalization. Such a product proposition received indirect support from the Squam Lake Group following the 2008 crisis[87], which referred to Flannery in their Recommendation 1 Chapter 7 for a "long-term debt instrument converting to equity under specific conditions." Here, we qualify this RCD by the term MVP, as it includes some draw-downs. Still, this product provided the foundation of traditional CoCo and paved the way for the article [17] by Glasserman & Nouri five years later.

[18] Tier 2 Green CoCos have also been issued, notably by Bank Hapoalim and de Volksbank.

A natural idea that could be explored with our proposed DCL instrument is to increase the leverage observation frequency (mechanically reducing the period Δt between two payments or observations). Theoretically, the immediate effect should be improved efficiency, although it would gloss over operability issues that are incurred by small payments or conversions.

Similar to our suggestion, Flannery suggested the same feature in the case of RCD. We offer a key improvement in Chapter 8 with American-like monitoring of the leverage, allowing it to split into the payment and observation dates. A DCL featuring an American observation would differentiate itself with its continuous monitoring of the leverage and taking action as needed while continuing to pay interest at predefined times.

In our specific case with DCL, a difference exists to decrease the Δt to zero in our European-like monitoring of the leverage. Using an American-like observation, we observe that

1. In the EU version with Δt -> 0, coupons are fractions of dollars, paid almost continuously (cash or in shares).

2. In the US version, coupons remain important, paid at specific times, but the conversion can still occur, only once, at any time between two payments. This is equivalent to separating the observation time from the payment time.

Flannery underestimates the market imperfection risks by asserting that it only affects the initial shareholders. Punishing the bank's shareholders for the excessive risk they take by setting the conversion price low enough to make the hybrid security highly dilutive in the case of conversion might be aligned with the purpose of contingent convertibles that lower the risk-shift of the firms and prevent excessive debt overhang recourse [20]. The practical considerations behind this are that shareholders will not approve any CoCo issuance that could affect them negatively.

Believing they need incentives to support the issuance of CoCo, Wolff, Pennacchi, and Vermaelen recently[19] presented a top candidate under the name of COERC, or Call Option Enhanced Reverse Convertible. This instrument, similar to traditional CoCos, incorporates an option for the initial shareholders to buy the newly issued shares at their conversion price that is set voluntarily at a low level. In a way, the absolute priority rule is restored, as COERC-holders would be repaid completely, whereas the initial shareholders, responsible for the risky decisions up to the conversion, pay the price of this risk (that led to the conversion) [10]. The report on the monitoring of AT1 instruments of EU institution points to existing instruments that include the preemption right for shareholders to allow them to buy shares issued at conversion and redistribute the associated fiat to the former

[19] Post the 2008 crisis.

AT1 holder. This feature faced initial disinclination from the European regulator before being considered acceptable (as noted in the bullet point #79-82 of [78]).

Sometimes the concept of dilution is not accepted. A substitute DCL may be possible with a design that, in the case of conversion, would simultaneously issue and sell new shares on the market to release a cash payment equivalent to the value at the conversion time. Alternatively, the payment could be skipped, which is considered a higher risk, leading to a lower price for the instrument or equivalently a higher yield. We expect to not see any 'jump' or 'death spiral' in the share value at conversion time. This product is completely transparent, relying on publicly and continuously disclosed data, which should allow all market participants to price the associated risk. Market operators are considered equal when they access the same data simultaneously and do not rely on opaque or manipulable metrics, such as the CET1 Ratio and regulatory approvals.

The sketch of CoCo is similar to a structured product note, so it is suitable to consider additional in-built options. One of these consists of adding a conversion right to the bondholder if the underlying stock value reaches a given price. This approach is designed to balance the higher intrinsic risk of these instruments. As highlighted by the EBA [78], such an issuance, labelled 'Contingent Conversion Convertibles' already occurred at least once and should increase the interest from a broader category of investors.

Such mechanisms, based on CoCo with upside conversion, could be argued to be perfectly suitable for the Private Equity (PE) world with only a few adjustments. We may observe a tendency to partially replace existing SAFE (Simple Agreement for Future Equity) agreements. The concept of SAFE previously aimed to provide an easier vehicle than convertible notes to startups looking for funds. The first SAFE was issued in 2013 [88]. Under the terms of the agreement, the investor, usually a venture capital (VC) firm proceeds toward investment in a company at a price per share defined later. An enhanced DCL featuring an upside conversion would allow a VC to make an indirect investment in a company as a loan that could be turned to equity under profitable terms (by following an upside development in the valuation) or because the leverage becomes critical, with a dilution saddled mostly by the founders (equivalent to the initial shareholders). This type of tailored-made DCL would offer private equity firms a flexible hybrid security that bridges debt and equity financing. This flexibility allows firms to optimize their capital structures, potentially leading to improved valuations and financial performance.

Considering the trigger design and mechanism, RCD must not incentivise short sellers to drive the share value down after investing in this debt instrument (and then benefit from the conversion). Therefore, the conversion occurs at the current share price. Also, Flannery supported the use of an Asian-like observation instead of the value on the day before closing for its "triggerability" (i.e., an average of the market value over a certain period). Later, other models for CoCo suggested averaging metrics for triggers [62] and conversion into common shares upon a breach in the "quasi-market value of equity ratio" (QMVER), defined as a 90-day average of the market capitalization divided by the sum of the book value of liabilities and market capitalization. Any averaging, regardless of the time window, would result in a delay in loss recognition. Paradoxically, CoCo was created and designed to prevent this observed lag.

RCD-holders have no right to force conversion, so also have no arbitrage opportunity to exit. In Flannery's 2014 review, he criticized his product, arguing that short-sellers could still benefit from forcing a dilution, characterized by an increased number of outstanding shares [63]. Therefore, RCD can simply be seen as a way to issue equity.

Before concluding on the opportunities enabled by his product, Flannery raised unresolved issues remaining at that time. Table 5.1 lists these concerns along with the existing hindsight and current best market practices, comparing the alternatives between traditional CoCos and, to some extent RCD (Flannery [47]), DCL (Segal & Olafsson [56]), and ERN (Bulow & Klemperer [38]).

Unresolved issues (as quoted by Flannery)	Our observations (with hindsight from best market practice)
Mandated Ratios. What level of equity capital should be required?	Basel III addresses this point and requires an AT1 tranche that could be filled with some CoCos.
Replenishment. How quickly should a bank be required to replace converted RCD?	DCL addresses this issue in an automated way, based on the re-issuance process. To our knowledge, no other instrument has been suggested with such a feature (apart from the Write-Up option).
Maturity. Should supervisors care about the maturity?	Basel III addresses this issue. DCLs are compliant.
Market. Is there likely to be a deep market?	While unknown at that time, a market now exists. The catalyst was, unfortunately, the 2008 crisis. However, not all the products would have deep liquidity. Only the "practical" project will be implemented, and neither RCD nor Glasserman & Nouris instrument [17] succeeded this way. DCL and ERN could be expected to be liquid instruments (as for current CoCos)
Scope. Is it possible to implement a scheme for a bank without traded equity?	Principal Write-Down with payment in cash is conceivable. Rabobank was the first company to issue such a CoCo in 2011.
Ownership restrictions. At least in the United States, supervisors must approve the identity of anyone who controls a banking firm. The SEC requires investors to report when they control 5% of a traded firms shares. Is there a sufficient grace period within which an RCD owner can dispose of his shares in order to avoid such regulations?	This requirement is fueling the existing jurisdiction discrepancies surrounding the CoCo market. However, the disclosure is not expected to occur at the exact time the ownership threshold is crossed. In the US, the SEC requires investors to fill the Schedule 13D within ten days after crossing the 5% ownership threshold. In some cases, this period could be extended to 45 days following the end of the calendar year of a breach.

Table 5.1: Unresolved issues identified by Flannery in 2005, with proposed answers based on best market practices, existing CoCo specifications, and theoretical proposals, including RCD, ERN, and DCL.

To effectively target a wider audience of investors and issuers, we can make our proposed DCL instruments more 'practical and 'issuable' by considering best market practices, from the increasing issuance proportion of Principal Write-Down (WD) to the limited mandate held by fixed income desks to handle equity (in the case of conversion). Similarly, Flannery has a forward-looking view of the CoCo market by claiming that some investors will not have the knowledge to evaluate the value of the firms equity, so, upon conversion, the shares from his RCD would be immediately sold, potentially at a discount, because it would be executed at the worse time, supporting the risk of initiating a death spiral in the share value.

To circumvent this issue, footnote #25 of [47] suggests including a few days between the observation date and the effective conversion date, allowing the bondholder to sell the 'pre-converted' bond with decreased costs associated with the disposal of the new shares. As highlighted here in the later chapters associated to DCL with the US monitoring of the leverage[20], two approaches exist to design the product. One releases the new shares immediately following the conversion (at time τ), and the other pre-converts the shares, but effectively distributes them at the next pre-determined payment time. This second case (i.e., separating the observation date from the payment date) is aligned with Flannery's previous suggestion.

As AT1 needs to encompass cancellable coupons, we can envision a DCL that would, in the first instance, cancel the coupon payment if the leverage becomes too high as of an observation date, which is the ratio $\frac{1}{f \, . \, T}$ of the overall nominal (usually sufficiently large when compared to a traditional CoCo's coupon). If this conversion is not enough, then additional interest payments could be cancelled. However, if the situation improves, then they will be restored. Such a product could remedy concerns from the regulators and the investment community. Taking this further, if the situation significantly improves for the bank and while the lifetime of the product is not expired[21], we can consider paying interest that was previously skipped. In the structured products area, such a mechanism is referred to as a 'conditional coupon with memory.

5.5 Rating agencies

The three top rating agencies[22] developed distinct methodologies regarding the rating assessment of contingent convertibles. The different provisions built into the CoCo design, such as coupon deferral, PONV, and loss absorption mechanism through write-down or conversion, affect their intrinsic equity and debt balance to a non-transparent extent. In these rating processes, agencies also face the same jurisdiction discrepancies as investors, making the CoCo treatment complex. In this sense, CoCo bonds differ from traditional fixed income and equity markets where rules are well known, and risk is appropriately evaluated.

Adverse effects pointed out by [29] include a deep divergence between the rating judgements from the three entities concerning the same bond and sometimes the lack of equivalence between rating scales. Such complexities prompted rating agencies to score hybrids with a so-called "equity credit," scaled from 0 to 100%

[20] Chapters 8 and 9.

[21] In opposition to traditional CoCos, DCL notes have a fixed maturity. But, when considered as a whole (including the top-up loans due to re-issuance), DCL appears to be an open-ended maturity.

[22] Standard & Poors, Moodys, and Fitch.

(corresponding to straight debt to straight shares, respectively) and proportional to the instrument's loss-absorption capacity on its issuer. Depending on the outcome, the agencies place the hybrid instrument to be rated in a different class. Standard & Poor's, for example, assesses the security as having high, intermediate, or no equity content [89]. Moody's created five categories expressed from A to E depending on the debt-equity weight intrinsic to the CoCo, (100-0 being A, 75-25 being B, 50-50 being C, 25-75 being D, and 0-100 being E) [90].

The CoCo callability is a source of divergence between market expectations, regulators, and rating agencies. For instance, S&P deteriorates the equity credit percentage if the bond is not called after the first callability date [91]. In opposition to the S&P initiative, the European Banking Authority recalls in their AT1 Report from June 2021 that the non-automatic call exercise on the first possible date should be favoured, especially for AT1 instruments [78]. This recommendation is contrasted by the absence of pre-set frequency preferences regarding the callability schedule by the regulatory body but keeps track of potential pressures to exercise the call on subsequent dates.

One in-built feature that affects the rating assessment is infinite maturity, seen as perpetual security that increases its equity character in the view of the agencies. Nonetheless, a call provision potentially triggered at the issuer's discretion would mitigate this aspect. Still, new uncertainties are created, including the "extension risk," where a company skips the exercise of their call option on the call date permitted by some obligation covenants. Such a case occurred previously to the Spanish bank Santander in February 2019 when, after ignoring the standard uses and practices in the industry, the management decided to skip the first callable opportunity on their 1.5 billion CoCo issue[23]. To compel the exercise of the callability option associated with hybrid security, issuance with a step-up coupon appears to start after the non-callability period could attract "extension risk"-adverse investors alongside reducing the equity-behaviour considerations from the rating agencies.

Due to the larger risk held by this type of investment, rating agencies advocate for lowering by a few notches the rating of these instruments compared with the issuer rating (cf. Section 5.1). But, in light of the virtuous spiral brought by contingent convertible to strengthen the capital structure of the firm and graphically represented in Fig. 5.4, CoCo can offer more confidence to the most senior debt tranches of a firm, resulting in increases of the issuer rating.

[23] ISIN: XS10435350

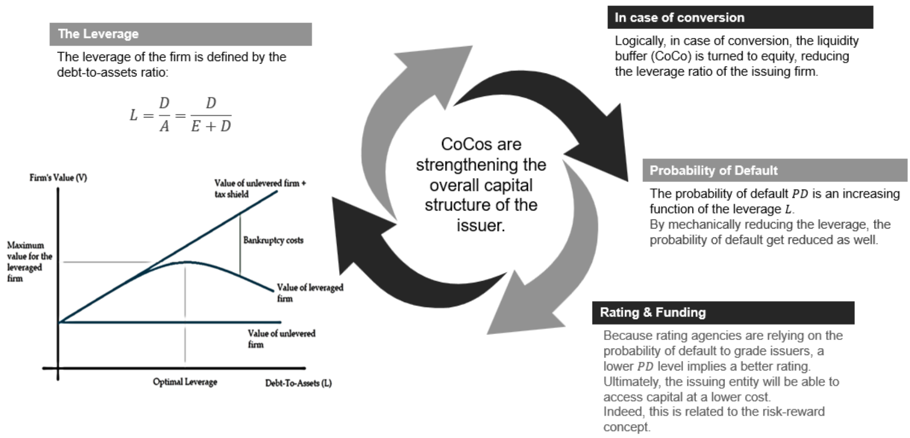


Figure 5.4: Illustration of the virtuous cycle resulting from the issuance (and potential conversion) of CoCo bonds. In this context, contingent convertibles are seen as load-bearing instruments.

To support this standpoint, rating agencies recommend to issuers a Replacement Capital Covenant to be mentioned in the security prospectus. Under such legally-biding terms, as included in [29], the issuer must use the proceeds of a new hybrid capital or new share issuance to call the original contingent convertible. Such a feature guarantees the perpetual existence of an acutely subordinated buffer in the firm's capital structure.

A less traditional approach in the CoCo valuation process for pricing the risk or the pre-issuance design optimisation consists in performing a **rating calibration**. To our best knowledge, no prior research directly considers this task, and we outline a framework in the following.

The proposed idea involves calibrating an unknown CoCo-related variable given a known conversion probability. Such a variable could be the Recovery R, the spread s, or the market value of equity S_c at conversion time. Rating agencies annually release empirical data in terms of default probability as a function of the issuer (or issuance) rating. We utilize the Global Corporate Average Cumulative Default Rates from 1981 to 2020 (Table 5.2) issued by Standard & Poors as a proxy for the conversion probability [92].

Rating	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AA-	0.03	0.08	0.16	0.23	0.31	0.41	0.47	0.52	0.57	0.62	0.68	0.73	0.75	0.8	0.84
A+	0.05	0.09	0.19	0.31	0.41	0.50	0.60	0.71	0.83	0.96	1.08	1.21	1.36	1.54	1.68
A	0.05	0.14	0.21	0.32	0.44	0.61	0.78	0.94	1.11	1.32	1.48	1.60	1.72	1.79	1.95
A-	0.06	0.16	0.25	0.36	0.51	0.66	0.87	1.03	1.15	1.27	1.37	1.50	1.62	1.74	1.84
BBB+	0.09	0.26	0.47	0.67	0.90	1.15	1.35	1.56	1.82	2.07	2.30	2.46	2.64	2.87	3.12
BBB	0.15	0.37	0.59	0.93	1.27	1.62	1.94	2.24	2.56	2.88	3.22	3.49	3.72	3.82	4.03
BBB-	0.24	0.69	1.27	1.93	2.63	3.24	3.78	4.28	4.69	5.04	5.43	5.75	6.05	6.51	6.86
BB+	0.32	0.97	1.76	2.55	3.35	4.14	4.82	5.32	5.92	6.52	6.93	7.42	7.92	8.27	8.82
BB	0.48	1.52	2.96	4.34	5.76	6.88	7.92	8.81	9.67	10.43	11.25	11.86	12.34	12.68	13.08
BB-	0.96	2.92	5.01	7.15	9.03	10.83	12.34	13.78	14.92	15.92	16.68	17.46	18.21	18.94	19.62
B+	1.98	5.42	8.82	11.73	14.02	15.80	17.43	18.86	20.17	21.37	22.41	23.14	23.92	24.65	25.35
B	3.13	7.35	11.11	14.19	16.69	18.97	20.62	21.87	23.07	24.26	25.02	25.78	26.37	26.89	27.44
B-	6.52	13.69	19.28	23.16	25.97	28.07	29.63	30.86	31.72	32.45	33.61	34.32	34.89	35.46	35.88
CCC/C	28.30	38.33	43.42	46.36	48.58	49.61	50.75	51.49	52.16	52.76	53.21	53.68	54.23	54.69	54.76
Investment Grade	0.09	0.24	0.41	0.63	0.86	1.09	1.30	1.50	1.69	1.88	2.05	2.20	2.35	2.49	2.65
Speculative Grade	3.71	7.19	10.18	12.63	14.64	16.30	17.68	18.83	19.86	20.81	21.61	22.29	22.93	23.49	24.04
All rated	1.53	3.00	4.27	5.35	6.25	7.01	7.64	8.18	8.67	9.12	9.50	9.83	10.13	10.41	10.69

Table 5.2: Global Corporate Average Cumulative Default Rates by Rating Modifier from 1981 to 2020, reproduced from Table 26 in the Default, Transition, and Recovery: 2020 Annual Global Corporate Default And Rating Transition Study. Sources: S&P Global Ratings Research and S&P Global Market Intelligences CreditPro [92].

- If we assume the conversion is an exponentially distributed event, then the above conversion probability could be used directly to find the recovery R or spread s. This method relies on the famous credit triangle formula [31] stated as

$$s = \lambda(1 - R) \tag{5.1}$$

  On the one hand, s is the spread that the CoCo should pay in excess of the risk-free rate r_f and λ is the conversion intensity that reflects the probability of conversion between two phases t and $t + dt$. On the other hand, R is the recovery rate on a converted CoCo received by an investor.

  In this case, we must bridge the conversion intensity λ to the effective conversion probability $P(\tau \leq T)$, suggesting that :

$$P(\tau \leq T) = 1 - \exp(-\lambda T) \tag{5.2}$$

  The spread s or the Recovery R is accessible by manipulating the equation. We can further detail the Recovery R by defining it as a ratio between the market value of the shares issued through the conversion (and received at time $t = \tau$) and the nominal investment in the CoCo N with

$$R = \frac{S_T^*}{N}$$

  The total market value of the newly issued shares is obtained by multiplying the value of one share by the conversion ratio C_R, representing the number of shares resulting from the conversion. By introducing C_P as the conversion price, we then write

$$S_T^* = C_R S_\tau \text{ and because of } C_R = \frac{N}{C_P}$$

The Recovery rate R for a CoCo-investor in case of conversion is then

$$R = \frac{S_\tau}{C_P} \tag{5.3}$$

From what has been introduced here, we know that CoCo credit spreads can be linked to the triggering probability in an intensity-based approach by inserting Eq. 5.3 into Eq. 5.1. Then, after isolating λ in Eq. 5.2, we have

$$\begin{aligned} s &= -\frac{\log(1 - P(\tau \leq T))}{T}(1 - R) \\ \iff s &= -\frac{\log(1 - P(\tau \leq T))}{T}\left(1 - \frac{S_\tau}{C_P}\right) \end{aligned} \tag{5.4}$$

- An alternative approach consists in using the same empirical probability as a proxy for the conversion, but in an equity-based model. This is achieved by replacing the right-hand side from Eq. 2.6, as derived in Section 2.3.2. The bond rating, being a piece of fully-fledged market information, with such a technique allows finding the rating agency's belief in terms of the underlying stock price at conversion time.

  We assume the share price at conversion time is at its minimum over the interval $[0; \tau]$ with $\tau < T$ conversion time. This assumption is equivalent to saying that the share price S_τ cannot be reached before the conversion is effective. In this case, even if we do not have a 1:1 mapping between the underlying share price and the CET1, for example, a calibration would provide the conversion threshold.

  Assuming the underlying share price follows a dynamic process of geometric Brownian motion, [51] defines the Cumulative Distribution Function (CDF) for the first-hitting time as

$$P(m_t \leq S_c) = \phi\left(\frac{\log\left(\frac{S_c}{S_0}\right) - \alpha T}{\sigma\sqrt{T}}\right) + \left(\frac{S_c}{S_0}\right)^{\frac{2\alpha}{\sigma^2}} \phi\left(\frac{\log\left(\frac{S_c}{S_0}\right) + \alpha T}{\sigma\sqrt{T}}\right) \tag{5.5}$$

> This method includes the downside of only working for issuers that are public companies. As discussed in Chapter 1, privately-owned companies, such as Rabobank, already issued CoCo bonds, so this equity-based approach cannot be applicable.

To evaluate how the results from this rating calibration differ from the standard approach, we consider the case study from [49] as a benchmark. In their 2012 paper, De Spiegeleer and Schoutens calibrated an implied trigger based on market values available at that time for the largest ECN issued by Lloyds Banking Group CoCo and the first Credit Suisse CoCo bond (Buffer Capital Notes - BCN). We use the data listed in Table 5.3, from [49], for our calibration.

Issuer Data	**Lloyds Banking Group ECN**	**Credit Suisse BCN**
Issue Date	Dec 1, 2009	Feb 17, 2011
Pricing Date	Mar 21, 2011	Mar 21, 2011
Maturity	Dec 21, 2019	5.5 Years (First Call Date)
Interest rate r	3.42%	2.42%
Dividend rate q	0%	3%
Volatility σ	39%	49.5%
Market price (share or hybrid note)	0.6075č (S_0)	488 bps (credit spread cs) 42.84 CHF (S_0)
Conversion Price	0.5900č	max(USD 20, CHF 20, S^*)
Rating	Expected: BB- [93] At issuance: BB On March 9, 2011: BB+ [94]	BBB+

Table 5.3: Micro- and macro-environment parameters for two CoCo bonds issued by Lloyds Banking Group and Credit Suisse. The data is reproduced from [49], unless otherwise specified. The ratings are made by S&P and Fitch, respectively, for Lloyds and Credit Suisse. Both rating scales are equivalent.

Approximating at best the conversion probability at the time of pricing from [49] requires the use of the default probability data existing at this time. We present in Table 5.4 a sample from the 2011 Annual Global Corporate Default Study and Rating Transitions issued by Standard & Poor's [95].

Default Probability	Tenor 5Y	Tenor 6Y	**Interpolation T=5.5Y**	Tenor 8Y	Tenor 9Y	**Interpolation T=8.5Y**
BBB+	1.43%	1.84%	**1.635%**	2.47%	2.84%	**2.655%**
BB+	5.01%	6.19%	**5.60%**	7.95%	8.93%	**8.44%**
BB	8.32%	9.99%	**9.155%**	12.68%	13.76%	**13.22%**
BB-	10.96%	13.08%	**12.02%**	16.74%	18.33%	**17.535%**

Table 5.4: Corporate average cumulative default rates by rating modifier. A data sample reproduced from Table 26 of the 2011 Annual Global Corporate Default Study and Rating Transitions. Data is interpolated for the tenor T=5.5 Years and T=8.5Y needed in our scope [95].

The Lloyds CoCo bond

We plot in Fig. 5.5 the conversion probability at the time horizon of 8.5 years from Table 5.4, alongside Eq. 5.5 as a function of the share price at triggering time (data from Table 5.3).

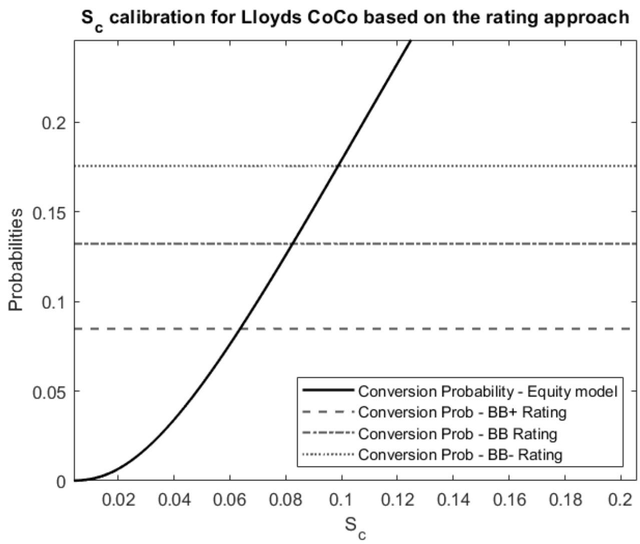


Figure 5.5: Calibration of the implied-trigger S_c for the Lloyds Banking Group ECN bond obtained through the rating-based approach. The solid line is based on the equity model (Eq. 5.5), and the dashed lines are taken from the S&P 2011 report (Table 5.4).

The rating-based approach appears to offer an implied triggering price S_c far below the 22.5 pence initially obtained by [49] in Table 5.5.

	Rating		
Lloyds ECN	**BB-**	**BB**	**BB+**
Implied Trigger	0.0987 £	0.0825 £	0.0635 £

Table 5.5: Results of the rating-based approach for the Lloyds ECN. For each rating, the implied trigger is obtained by the intersection between the probability given by the equity-based model (Eq. 5.5) and the conversion probability (Table 5.4).

With such an important difference between the standard model prediction (in [49]) and our rating-based approach (Table 5.5), we formulate two hypotheses regarding the role of the rating agencies and how to consider their position on CoCo notes:

(a) The agency adapts its grading outlook with a delay compared to when the market data becomes publicly available. As this decision is made on outdated information, any change of position (such as a downgrade) should have been made earlier, which reflects an implied share value S_c that is potentially higher than the one suggested by the rating-based approach.

(b) The agency provides some reliable information regarding the CoCo risk. The issuance grade related to the associated probability of default should be used as a proxy for the conversion risk. This grade highlights a lower implied share value at conversion time compared to models solely based on what market parameters suggest, which is due to a bad risk assessment from the market makers, bearing an exacerbated adversity to conversion risk.

Although conflicting, these two conjectures are not mutually exclusive.

The Credit Suisse BCN

The treatment of the Credit Suisse note here is more delicate due to the very high rating of BBB+ for the issuance received from Fitch[24] that leads to a highly unlikely conversion over the 5.5 years before the first callability option can be exercised by the issuer. Inputting a 2.655% conversion probability (from Table 5.4) into Eq. 5.4 results in a negative candidate implied-trigger S_c, which must be excluded from the scope of possible share values at conversion time[25].

Alternatively, deploying the same equity-based method as the Lloyds ECN, the case of the Credit Suisse note gives a very low intersection point $S_c = 1.396$. If such a level happened to be true, then we could fear the occurrence of a debt-induced collapse upon conversion, where the company could not recover from

[24] Equivalent to the same rating in the Standard & Poor's scale

[25] $S_c = C_P\left(1 + \frac{cs\,.\,T}{\ln(1-P(\tau \leq T))}\right) < 0$ with the data considered.

suffering such a significant loss of a 96.7% drop in equity value (from the initial price $S_0 = 42.84$ to the candidate value $S_c = 1.396$). Then, the conversion alone would be unable to entirely bear the loss.

This case study puts into question the significant discrepancy between the CoCo credit spread (488 bps) and the high issuance rating, implying a low conversion risk, according to the rating agency. However, the rating agencies are not to blame, given the early times for the CoCo market that these instruments were issued. Later, rating agencies refined their model and provided more accuracy in their methodologies for granting the "equity credit" score to the emerging contingent convertible bonds (from [96] in 2011 to [89] in 2019 for S&P). In fact, the most recent methodology booklet for hybrid capital released by Standard & Poor's supersedes 12 other guidance articles effective from 2007 (See the Related Publications section in [89]).

5.6 Equilibrium & Arbitrage

In particle physics, an equilibrium point arises if the sum of all vector forces acting on the particle is zero. In the context of contingent convertibles, the equilibrium price must be defined by the value of the instrument (either the underlying or the CoCo bond) for which no arbitrage exists. Considering the parallel with the physics definition, we consider that the vector forces are market participants taking positions (long or short) on the instrument, resulting in price movements (up or down) until reaching the equilibrium, fair, non-arbitrage price, given a set of publicly accessible parameters.

From the PT article, a unique equilibrium stock price is ensured by the candidate pre-conversion stock value being an increasing function of the bank's asset, for all assets' value being larger or equal to the triggering asset value. A closed-form formula is derived for this candidate stock price (assuming the existence of the equilibrium) that relies on the continuity at the conversion level between the pre-conversion equity value (assuming contingent convertibles exist in the capital structure as well as n shares) and the post-conversion equity value (assuming only senior bonds exist in the capital structure, and $n + m$ shares are due to the conversion of the CoCo).

Having an unverified unique equilibrium condition is equivalent to having a pre-conversion price lower than the triggering share price for some asset value above the threshold triggering the conversion. The consistency would then be lost, resulting in no equilibrium stock price due to a contradiction in Proposition 1 that states conversion takes place the first time the share price falls below the triggering price. Again, this is not the case with ERN and DCL because these are permitted to have the pre-conversion price lower than the triggering share price between two observation dates.

From the equilibrium definition above, we observe that the equilibrium and arbitrage concepts can be bridged in the case of CoCos. On the one hand, the most straightforward case is if a unique equilibrium exists based on the pricing model. In this specific scenario, having the ability to buy the underlying cheaper than the equilibrium stock price (from the model) can be arbitrage by taking a long position in the underlying shares and Δ short positions in the CoCo. Here, Δ represents the number of bonds that annihilate the price risk.

Alternatively, if the underlying price is more expensive than the equilibrium (pre-conversion) price S_t (A_t), the opposite trade should be entered, short-selling the underlying and buying Δ times the CoCo bond nominal. On the other hand, when no equilibrium can be mathematically found, but a tradeable price still exists on the market, arbitrage logic loses its neutral benchmark or reference. Even if PT rules out this eventuality in most realistic bank asset volatility cases in perpetual CoCos [97], the nonexistence of equilibrium remains an issue for contingent convertibles with finite maturity as soon as the conversion terms benefit the initial shareholders by converting the debt instrument into less equity than the nominal of the CoCo.

As later observed in Chapter 7, the absence of equilibrium is ruled out for CoCos with discrete behaviours, such as ERN and DCL when modelled continuously. Similar to barrier options, infinite values of Δ are reasonable when the next observation date is close and the spot price flirts with the triggering price.

Chapter 6

Efficiency Examination of DCL Instruments

"The idea that growth will remedy our debts is so addictive for politicians, but the citizens end up paying the price."

Michael Burry, [98].

6.1 Introduction and hypotheses

This chapter studies the impact and effectiveness of DCL on a firm's capital structure. DCL are a new class of contingent convertibles that are leverage-based and facilitate improved capital management via dynamic payment conversion to equity. In this context, "efficiency" pertains to the ability of a DCL to serve its intended purpose of capital control. This study adapts the well-known Vasicek model and compares different scenarios to a benchmark company without DCL in the firm's capital structure.

We begin by re-examining the primary characteristics of DCL instruments, then progress into the adapted Vasicek model, commonly used for modelling interest rate behaviour [99]. Finally, we use a numerical example that illustrates how DCL instruments, by their leverage-control properties, effectively serve their intended purpose. Such efficiency is demonstrated within the context of maintaining stability in a firm's capital structure by lowering the probability of extreme values for the leverage and turning the leverage into a mean-reverting process, as evidenced by the provided confidence intervals.

The DCL in the form of a leverage-based contingent convertible is a regular loan with a nominal Q_0. The interest rate the firm needs to pay on this debt is defined by r, and the bond is paid down with N equal payments P_N. Additionally, DCL enhances some unique characteristics, such as:

- Physical delivery of shares instead of cash payment if the firm's leverage is beyond the conversion threshold L_c at the *observation date* $t-1$ of the *payment date*. The number of shares newly issued is equal to the firm's payment obligation divided by a predefined conversion price C_p.
- A re-issuance process keeps the debt-to-assets above a given threshold to ensure the firm's market competitiveness. If, at the *observation date* $t-1$ of the *payment date*, the firm's indebtedness ratio is below L_{min}, then the firm issues a new DCL with the same parameters and a nominal Q_k, such as $L(Q_k) = L_{min}$ at the following payment date k. Then, L_{min} becomes the minimum leverage percentage allowed for re-issuance.

Because the factors L_{min} and L_c are now influencing the firm's leverage, we observe a mean reversion mechanism that drives us to consider the Vasicek model as an appropriate candidate for the debt-to-assets modelling and the financial instrument efficiency examination. We derive the Vasicek stochastic differential equation followed by the firm's leverage L_t as

$$dL_t = a(b - L_t)dt + \sigma_L dW_t \tag{6.1}$$

with the initial condition on the leverage $L(0) = L_0 = \frac{RQ_0}{RQ_0+NS_0S_0}$. Here, NS_t is the number of shares in circulation, S_t the share price, and RQ_t the total residual value of the loan[1]. Then, identifying a as the speed of reversion parameter, b as the long-term mean leverage, σ_L as the instantaneous volatility of the process, and W_t the Wiener process in a risk-neutral world become possible. Solving the stochastic equation leads to

$$L_t = L_0 \exp(-at) + b[1 - \exp(-at)] + \sigma_L \exp(-at) \int_0^t \exp(as) dW_s \tag{6.2}$$

By parallelism with an Ornstein-Uhlenbeck process, the expected value of L_t appears to be [100]

$$E[L_t] = L_0 \exp(-at) + b[1 - \exp(-at)] \tag{6.3}$$

whereas the volatility of such a process is defined by

$$Std[L_t] = \sqrt{\frac{\sigma_L^2}{2a}[1 - \exp(-2at)]} \tag{6.4}$$

[1] The residual value of the loan at $t = 0$ equals the initial loan nominal $RQ_0 = Q_0$.

The long-term standard deviation tends toward $\lim_{t\to\infty} Std\left[L_t\right] = \sqrt{\frac{\sigma_L^2}{2a}}$, which will be a key indicator of how the DCL acts to prevent extreme leverage values. No stabilising effect exists in the long run due to the perpetual re-issuance process. The 95% confidence interval (around the mean) can be written in the following form because of the normally distributed process, such that

$$IC_{95\%} = L_0 \exp\left(-at\right) + b\left[1 - \exp\left(-at\right)\right] \pm 2 \,.\, \sqrt{\frac{\sigma_L^2}{2a}\left[1 - \exp\left(-2at\right)\right]} \tag{6.5}$$

To begin, we must calibrate the Vasicek parameters using the sets of a firm's leverage values across n distinct paths. The Vasicek model can be calibrated by considering the entire leverage values throughout the life of the instrument or until $t = T^*$. In this context, we define $N = T^*/\delta t$ as the total number of leverage observations to be considered. We then employ the maximum likelihood estimator (MLE) to maximize the log-likelihood function [101, 102] of

$$\begin{aligned}
\ell(L_i, a, b, \sigma_L) &= \log \prod_{i=0}^{N-1} f(L_{i+1} \mid L_i \,;\, a \,;\, b \,;\, \sigma_L) = \sum_{i=0}^{N-1} \log\left(f\left(L_{i+1} \mid L_i \,;\, a \,;\, b \,;\, \sigma_L\right)\right) \\
&= \sum_{i=0}^{N-1} \log\left(\frac{1}{\sigma_L\sqrt{2\pi\delta t}} \exp\left(-\frac{1}{2\sigma_L^2\delta t}\left(\Delta L_i\right)^2\right)\right) \\
&= \sum_{i=0}^{N-1} \log\left(\frac{1}{\sigma_L\sqrt{2\pi\delta t}}\right) - \frac{\Delta L_i^2}{2\sigma_L^2\delta t} = \left[\frac{1}{\log(\sigma_L\sqrt{2\pi\delta t})}\right]^N - \sum_{i=0}^{N-1}\frac{\Delta L_i^2}{2\sigma_L^2\delta t} \\
&= -\frac{N}{2}\log\left(\sigma_L^2 2\pi\delta t\right) - \sum_{i=0}^{N-1}\frac{\Delta L_i^2}{2\sigma_L^2\delta t}
\end{aligned} \tag{6.6}$$

In this equation, $\Delta L_i = L_{i+1} - L_i - a(b - L_i)\delta t$ denotes the adjusted change in leverage at time i, which accounts for the mean-reversion process described by the Vasicek model. The function $f(L_{i+1} \mid L_i \,;\, a \,;\, b \,;\, \sigma_L)$ is the conditional probability density function for a normal distribution that models the change in leverage from L_i to L_{i+1}, given the parameters a, b, and σ_L and the current leverage L_i. The log-likelihood function $\ell(L_i, a, b, \sigma_L)$ represents the likelihood of observing a specific leverage path $L_t = \{L_t/t \in [0; T^*]\}$, given the parameters a, b, and σ_L. The goal is to find the parameter values that maximize this log-likelihood function, provided the observed leverage path.

For a given payment frequency f, the triple estimators

$$\begin{pmatrix} a \\ \hat{b} \\ \sigma_L \end{pmatrix}$$

are derived from the calculation of

$$\begin{pmatrix} \frac{\partial \ell(L_i,a,b,\sigma_L)}{\partial a} \\ \frac{\partial \ell(L_i,a,b,\sigma_L)}{\partial b} \\ \frac{\partial \ell(L_i,a,b,\sigma_L)}{\partial \sigma_L} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The following solution exists and represents the values that maximise the likelihood of obtaining the dataset simulated. We demonstrate this result in Appendix E.

$$\begin{cases} a & = \frac{\sum_{i=0}^{N-1}(L_{i+1}-L_i)(b-L_i)}{\delta t \sum_{i=0}^{N-1}(b-L_i)^2} \\ \hat{b} & = \frac{1}{N}\sum_{i=0}^{N-1} \frac{L_{i+1}-L_i(1-a\delta t)}{a\delta t} \\ \sigma_L & = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1} \frac{(L_{i+1}-L_i-a(b-L_i)\delta t)^2}{\delta t}} \end{cases} \tag{6.7}$$

with b as the average leverage observed from the dataset $L_i = \{L_t/t \in [0;T^*]\}$.

6.2 Numerical example and simulations

We provide in this section numerical examples to assess the leverage ratio's behaviour when constrained with a DCL instrument in the balance sheet of a firm. Our base case scenario assumes the same parameters input as in Chapter 4, and are listed in Table 6.1. Here, we set the time interval constant between $t_0, t_1, ..., t_N$, such as $\delta t = t_{i+1} - t_i = 0.05$.

Input	**Value**
Nominal Value	5000$
Number of Payments	10
Interest Payment	5%
Expected Annual Return on Equity	10%
Dividend Rate	2.5%
Volatility	35%
Initial Number of Shares	100
Stock Value	20$
Conversion Price	18.5$
Triggering Leverage	80%
Min leverage to not reissue debt	70%

Table 6.1: Numerical inputs used for the simulations. The data are the same as used in Chapter 4.

Fig. 6.1 presents the results from the Vasicek calibration for a firm incorporating the aforementioned hybrid security in its balance sheet. Visually, this appears to impact the leverage evolution range depending on the payment frequency set (annually or bi-annually). The result might be qualitatively justified by the shorter period between two payment dates, leaving less room for the leverage to evolve before potentially leading to either a payment conversion or debt re-issuance.

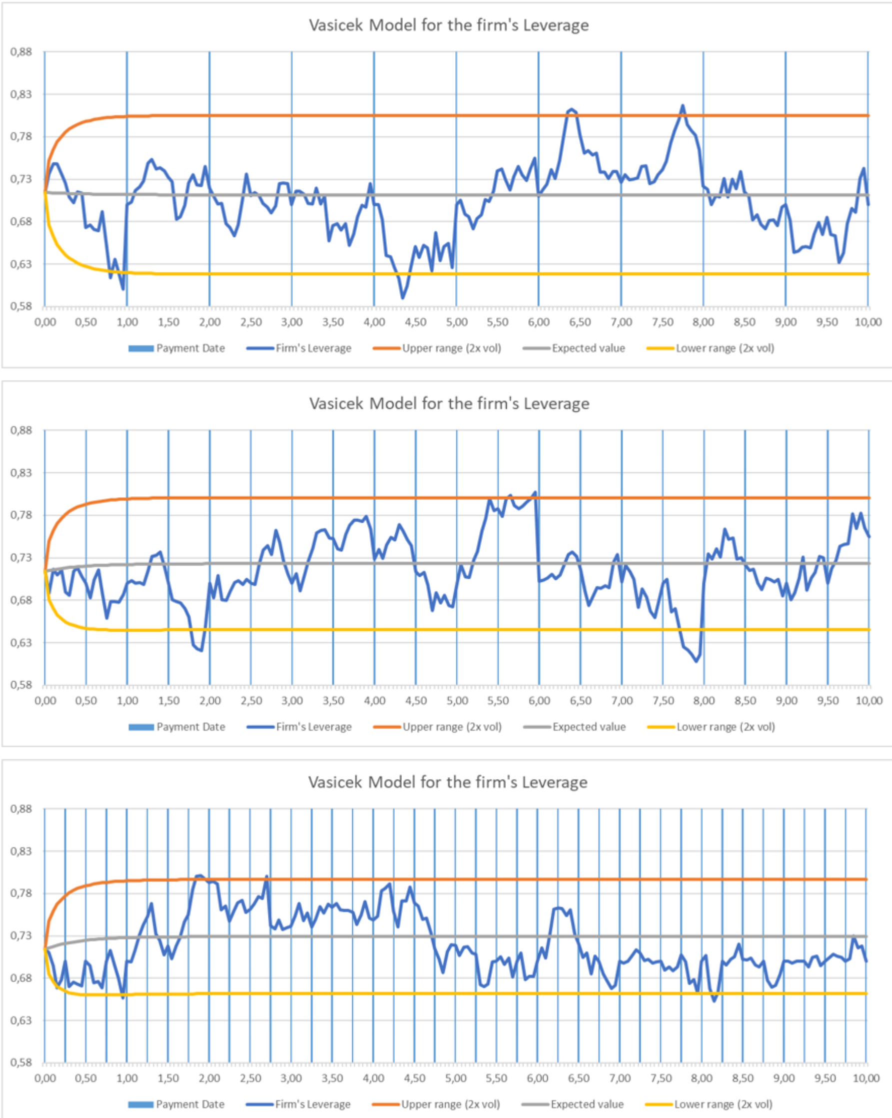


Figure 6.1: Selection of three random paths for the firm's leverage. All assume that the firm issued a DCL at $t = 0$, one with an annual frequency payment (upper plot), a second with a bi-annual payment (middle plot), and a third with a quarterly payment (lower plot).

As observed in Fig. 6.1, increasing the frequency of payments leads to less chaotic movements in addition to narrowing the 95% interval channel (defined as two standard deviations about the mean). The discretisation step is set at $\delta t = 0.05$ (i.e., 12.6 trading days).

To verify the assumption on the leverage channelling effect of the frequency payment, we introduce three simulations plus a benchmark. Each simulation relies on $n = 500$ distinct paths for the leverage and shows, based on the Vasicek model, the interest of DCL in controlling the issuer leverage based on the frequency payment and not on the two driving parameters L_{min} and L_c (as these influences are admitted).

- Benchmark, Simulation 0: the balance sheet **does not** incorporate the DCL and $T^* = 10$ years[2].
- Simulation 1: an **annual** frequency payment, $T^* = 10$ years, and the balance sheet incorporates the DCL.
- Simulation 2: a **bi-annual** frequency payment, $T^* = 10$ years, and the balance sheet incorporates the DCL.
- Simulation 3: a **quarterly** frequency payment, $T^* = 10$ years, and the balance sheet incorporates the DCL.

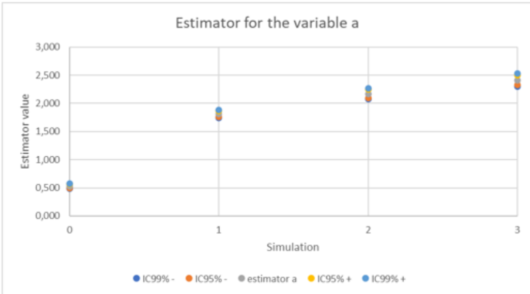
Figures 6.2, 6.3 and 6.5 provide estimators for the Vasicek model and the associated confidence intervals for every scenario, which provides the average calibrated parameters for every 500 paths simulated. The intervals are provided with $\alpha_1 = 5\%$ and $\alpha_2 = 1\%$ thresholds (i.e., 95% and 99% confidence intervals, respectively) and are computed by :

$$IC_{1-\alpha}(\hat{x}) = \hat{x} \pm Z_{\alpha/2}\frac{Std(x_i)}{n} = \hat{x} \pm N^{-1}\left(1 - \alpha + \frac{\alpha}{2}\right)\frac{Std(x_i)}{n} \qquad (6.8)$$

6.2.1 Speed of reversion estimator

First, we focus on the speed of reversion estimator $\hat{a}$, a parameter that acts as the retraction force for springs. The higher the estimator value, the faster the firm's leverage returns to its expected equilibrium value.

[2] This case is proxied by setting the frequency payment to once per θ days, with $\theta > T^*$.



Estim / Sim	0	1	2	3
$\widehat{a}^{-}_{99\%}$	0,481	1,740	2,072	2,299
$\widehat{a}^{-}_{95\%}$	0,492	1,757	2,095	2,326
$\hat{a}$	0,529	1,811	2,169	2,413
$\widehat{a}^{+}_{95\%}$	0,565	1,865	2,244	2,500
$\widehat{a}^{+}_{99\%}$	0,577	1,882	2,267	2,527

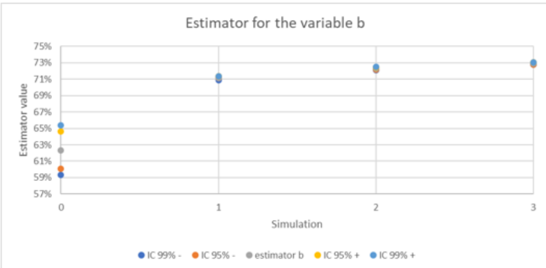
Figure 6.2: Vasicek calibration of the parameter a for the three simulations (1, 2, 3) and benchmark (0). The value is provided with its 95% and 99% confidence intervals. The left plot representation highlights the positive correlation between the estimator value and the number of payment dates per year. The data table is listed to the right of the figure.

In the benchmark experiment (Simulation 0), the firm's balance sheet does not include a DCL instrument nor a re-issuance process to guarantee a minimum leverage ratio over time. Instead, the debt is assumed constant, leaving the debt-to-asset ratio at the mercy of the random share value evolution as it became a one-argument function. Without any surprise in these circumstances, the mean-reversion phenomenon is not constrained by any instrument and results in a very low estimator $\hat{a}$, ranging (with a 1% error assumption) from 0.481 to 0.577. Also, the simulation suggests there is no control of capital because over the ten years, the average minimum leverage ratio hit varies between 47.9% and 51.5%. In contrast, the average maximum leverage ratio hit varies between 81.6% and 83.3% (also with a 1% error assumption).

The following three simulations demonstrate the non-negligible role of the DCL instrument in steering the leverage back to its expected value. Supposing a DCL proceeds to pay annually and increases the $\hat{a}$ estimator up to the range of 1.74 to 1.88. Switching to a bi-annual payment while maintaining the remaining factors unchanged increases the 99% confidence interval breadth to $[2.07; 2.27]$. This interval does not intersect with the one obtained through the benchmark simulation, demonstrating the positive correlation between the speed of the mean reversal value and the number of payment dates per year. The $\hat{a}$ interval obtained with a quarterly payment (Simulation 3) of $[2.30; 2.53]$ is higher and also does not intersect with the one obtained for a bi-annual payment (Simulation 2), suggesting that the relation holds pairwise.

6.2.2 Long-term mean estimator

We emphasise understanding the impact of frequency payments on the long-term mean leverage. This consideration is everything but marginal, as the overall aim of the DCL is to canalise the firm's leverage toward an optimum value that should be sufficiently high for the firm to remain competitive (with respect to its equity and asset volatility). In the meantime, the financial health of the company remains secure.



Estim / Sim	0	1	2	3
$\widehat{b}^{-}_{99\%}$	59,4%	70,9%	72,1%	72,7%
$\widehat{b}^{-}_{95\%}$	60,1%	71,1%	72,1%	72,8%
$\widehat{b}$	62,4%	71,1%	72,3%	72,9%
$\widehat{b}^{+}_{95\%}$	64,6%	71,3%	72,5%	73,0%
$\widehat{b}^{+}_{99\%}$	65,3%	71,4%	72,5%	73,1%

Figure 6.3: Vasicek calibration of the parameter b for the three simulations (1, 2, 3) and the benchmark (0). The value is provided with its 95% and 99% confidence interval. The left plot representation highlights the positive correlation between the estimator value and the number of payment dates per year. The data table is listed to the right of the figure.

Figs. 6.2 and 6.3 corroborate the hypothesis on the positive correlation between the long-term mean leverage and the number of payments per year. The 99% confidence intervals do not intersect, which once again may be justified by an economic reason. If the payment frequency (also referred to as the observation frequency) is increased, then the firm's leverage negligibly falls far below the maximum re-issuance level L_{min}. In such cases, at time t with $k-1 \leq t < k$, the leverage automatically readjusts to L_{min} through the issuance of Q_k. The faster this re-issuance occurs, the higher the average leverage through the same period considered.

The opposite procedure of an unlevered mechanism is observed in the case where the leverage breaches the conversion threshold L_c. Here, we can show the existence of a limit to which the parameter b tends when increasing the payment frequency.

The management board that intends to issue a DCL instrument must consider the shifting effect described previously, which involves the following statement: targeting a fixed long-term indebtedness ratio b requires decreasing the re-issuance leverage L_{min} for an increased number of payments (and reciprocally).

The absence of DCL in the firm's capital structure leaves the leverage free to evolve, resulting in a broader distribution of results. This outcome is reflected in the spread between the estimator $\hat{b}$ and its 95% and 99% lower and upper confidence bounds, respectively. Fig. 6.4 features a plot of the observed mean lowest and highest value hit by the leverage ratio through the ten years simulated for the n paths. Again, the instrument proves helpful in avoiding extreme leverage values that are detrimental to the firm's operation.

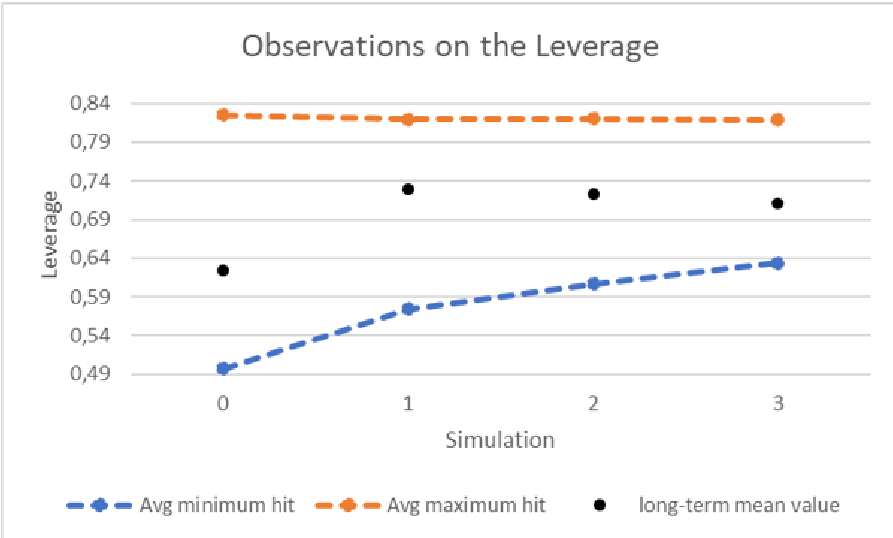
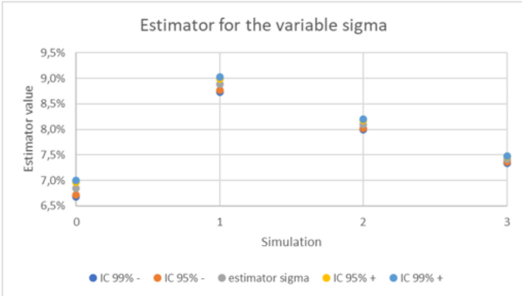


Figure 6.4: Long-term mean value b with respect to the average maximum and minimum values hit by the leverage. Whereas the average maximum hit is slightly decreasing with the number of payments due to the driving factor L_c (conversion), the average minimum hit is an increasing function of the payment frequency due to the second driving factor L_{min} (re-issuance).

6.2.3 Instantaneous volatility estimator

Finally, the instantaneous volatility is the single variable driving the amplitude of the market randomness, proxied by a Wiener process. This indicator describes how well the instrument can concentrate the leverage values around its expected mean, dwindling the incidence of extreme movements.



Estim / Sim	0	1	2	3
$\widehat{\sigma^-_{L_t,99\%}}$	6,68%	8,74%	7,99%	7,34%
$\widehat{\sigma^-_{L_t,95\%}}$	6,72%	8,77%	8,02%	7,36%
$\widehat{\sigma_{L_t}}$	6,84%	8,88%	8,10%	7,41%
$\widehat{\sigma^+_{t,95\%}}$	6,96%	8,99%	8,17%	7,47%
$\widehat{\sigma^+_{L_t,99\%}}$	7,00%	9,02%	8,20%	7,48%

Figure 6.5: Vasicek calibration of the parameter σ_L for the three simulations (1, 2, 3) and the benchmark (0). The value is provided with its 95% and 99% confidence interval. The left plot representation highlights the negative correlation between the estimator value and the number of payment dates per year. The data table is shown at the right of the figure. All values are annualised.

These findings are again verified by non-intersecting confidence intervals. The benchmark simulation shows the lowest instantaneous volatility $(6.84\%\pm0.16\%)_{\alpha=0.01}$, which is biased by the absence of a re-issuance or conversion event, restricting the amplitude of moves for the leverage. However, given the findings on the entire set of Vasicek parameters, favouring straight debt over the contingent convertible introduced here is not sufficient. Regarding the estimator behaviour in reaction to a DCL, an increase in the payment frequency is demonstrated to conduct fewer shocks and chaotic movements of leverage, displayed by a decrease in the instantaneous volatility. This effect is desired, as the DCL instrument should prevent excessive market randomness from entering the system.

6.3 Discussion of the Vasicek Model

This section introduces a discussion on the use of the Vasicek Model for leverage forecasting and looks closely at the conclusions drawn. The model demonstrates its efficiency on a problem that is not related to interest rate modelling but still assumes an underlying mean-reversion process. Therefore, why a more robust model, such as Cox-Ingersoll-Ross (CIR), is not used might be of concern. The Vasicek deficiencies in interest rate forecasting remain valid in this context, including the fact that the leverage values are constrained by a lower and upper bound of 0% and 100%, respectively. However, denoting P as the probability to invalidate the model, i.e., the probability that the process goes negative [102] or above 1 territory at the time $t + \delta t$, we write

$$P(L_{t+\delta t} < 0 \cup L_{t+\delta t} > 1) = P(L_{t+\delta t} < 0) + P(L_{t+\delta t} > 1)$$

because the two events are logically independent. Then, it followed that

$$\begin{cases} P(L_{t+\delta t} < 0) & = P\left(L_t + a(b - L_t)\delta t + \sigma_L\sqrt{\delta t}N(0;1) < 0\right) = 1 - \Phi\left(\frac{L_t + a(b-L_t)\delta t}{\sigma_L\sqrt{\delta t}}\right) \\ P(L_{t+\delta t} > 1) & = P(N(0;1) > 1 - L_t - a(b - L_t)\delta t - \sigma_L\sqrt{\delta t}) = \Phi\left(\frac{L_t - 1 + a(b-L_t)\delta t}{\sigma_L\sqrt{\delta t}}\right) \end{cases} \tag{6.9}$$

Using the same values as our example shows that the probability

$$P(L_{t+\delta t} < 0 \cup L_{t+\delta t} > 1)$$

tends in the least favourable case (annual payment) to $2 * 10^{-47}$.

With this as a justification for the use of Vasicek's model, we next spotlight the DCL instrument efficiency. Regarding the long-term standard deviation in a Vasicek process ($Std(L_t)$ with $t \to \infty$) derived in Section 6.1, the findings on the speed of the reversion parameter (Section 6.2.1), and the process volatility (Section 6.2.3), the DCL instruments are established to be efficient and successful in containing a firm's leverage around a targeted value. Also, managing the constraint applied to the firm's leverage by increasing the payment or observation frequency, among other solutions, is possible. This constraint is illustrated by the two-std interval being tightened when switching from annual to bi-annual and then to a quarterly payment (all else kept equal). We refer to Fig. 6.1 for our numerical example, where the lower and upper ranges use the following triple obtained in Section 6.2,

$$\begin{pmatrix} a \\ \hat{b} \\ \sigma_L \end{pmatrix}^f$$

with f as the payment frequency.

6.4 Conclusion

The concept of the DCL instrument helps an issuing entity target a long-term debt-to-asset level, which is achieved through the conversion of debt to new equity issues or debt re-issuance, depending on the firm's needs. Simulations on the instrument impact leave the footprint of an effective mean-reversal phenomenon. For this reason, this chapter proposed a new approach to quantifying efficiency based on the famous Vasicek Model, the adaption of which points to the existence of upper and lower bounds for the leverage that is not breached in 95% of the cases. The spread between these two limits appears to be modulated to constrain the debt-to-asset ratio into a relatively tight channel, leaving less room for uncertainty. This study focused on how the payment frequency played a critical role in this mechanism and exhibited, on the one hand, a positive correlation between the number of payments and the Vasicek parameters a and b. On the other hand, a negative correlation between the payment frequency and the process volatility σ_L was observed.

Considering the global picture, DCL instruments have been shown to allow for improved control of the leverage ratio compared to a firm not issuing any, which then constitutes an efficient tool to comply with regulatory requirements, such as those defined by Basel III. The re-issuance process could qualify this security as a perpetual instrument as well as Additional Tier 1 (AT1). This assumption must be discussed further but could lead to more incentives for the investors, as the callability risk component would disappear.

Chapter 7

Stability and Equilibrium Examination of Contingent Convertibles

"A banking system is an act of faith: it survives only for as long as people believe it will."

Michael Lewis [103].

7.1 Abstract

This chapter studies the stability and equilibrium in contingent convertible pricing. The effect is investigated for legacy CoCo instruments and for a class of contingent convertibles that allows for better capital control through dynamic payment or conversion to equity, including the Equity Recourse Note (ERN) or the recently developed Dynamic Control of Leverage (DCL) instrument with its leverage-based conversion.

Because of its structure and conditional payoff, the pricing of CoCo bonds is known to be a challenging problem. Still, few papers exclusively focus on the stability of CoCo pricing in the case of market triggers. However, the research on CoCos is not static, and some previous papers often cited by the industry include errors.

A framework is developed for CoCo stability when the conversion condition is monitored at discrete times instead of continuously. In this way, the threshold requirement can be violated between two observations without triggering the conversion of the debt into equity.

7.2 The equilibrium problem

Many recent papers focused on introducing new products that never increase in market interest because they are not suitable for direct issuance, making them valuable only from a theoretical perspective while revealing difficulties in their practical implementation. These include instruments that trigger a conversion as soon as a breach occurs in the examined variable and is limited to the required amount to restore the minimum non-triggering condition.

In 2002, Flannery [47] identified a lack of advance arrangements to restructure firms if significant losses occurred and proposed a "minimum viable product" (MVP) to prevent government intervention and facilitate the bank's recapitalisation. This RCD qualified by the term MVP comes with drawbacks. However, this product design offered a foundation of the traditional CoCo and paved the way for Glasserman & Nouri's CoCo, which featured a capital-ratio trigger [17] and offered a type of debt instrument that glosses over operability issues incurred by very small payments or conversion.

Dual-trigger products that depend on micro- and macro-parameters (i.e., at the firm level to the scale of the entire banking system) have been recommended by McDonald [61] and Calomiris and Herring [62]. In the best case, such features incur further delays in loss recognition, whereas in the worst case, a CoCo completely loses its core purpose to force the recapitalization of a bank experiencing financial trouble. If a bank underperforms compared to a benchmark, then dual triggers still might not prevent bankruptcy.

Few papers exclusively focus on the stability of CoCo pricing, while other research dealing with CoCo valuations featuring a market-based trigger observe a double equilibrium problem that occurs when varying the trigger [31, 35, 36]. Table 1.1 summarises the effects related to the conversion price. As the ongoing research on CoCo is not static, some researchers have identified the existence of a single solution even when the CoCo conversion penalizes the initial shareholder. Glasserman and Nouri showed that by having an instrument that is continuously traded and a conversion price C_p low enough, the existence and uniqueness of a share value can be found, despite the high dilution factor [37]. Pennacchi and Tchistyi (PT) [20] later extended the scope of this paper to illuminate an error that invalidates Theorem 1 in Sundaresan and Wang's (SW) research [35]. Compared to the Glasserman and Nouri article, PT derived closed-form equations for the price of the stock (in a unique equilibrium) and the contingent convertible security. They then showed that the requirement from Sundaresan and Wang was too strict and suggested the possibility of having a single and stable solution even if $C_p \neq S_\tau$. While this restriction is required only at conversion and not previously, as was stated by SW, they observed that multiple equilibrium behaviours can still manifest in deterministic models but not when asset modelling is continuous or stochastic.

On the multiple equilibrium problem, Glasserman and Nouri found its origin in the discrete-time design of CoCos, where prices that could not adapt progressively would shift value at conversion time only [37]. This challenge was parenthetically solved by ERNs (the same applies to our proposed DCLs) because the conversion occurs at scheduled times. Flannery pointed out that by converting the ERN coupon, only a small amount is really "at stake." Therefore, the initial shareholders lack commitment, which should not prevent them from operating at a high-risk level. Again, DCLs are slightly better in the sense that they are loans that pay equal payments over their lifetime. Furthermore, CoCos that rely on market-based triggers are flawed, but recurring forbearance and symmetrical errors occur either with a false conversion or in the absence of conversion when truly needed, which appears preferable [63]. We do not claim that DCL eliminates this symmetrical error. However, because conversion is automatic, relies on leverage, and is limited to the sole payment due, the absolute value of this error, if quantified, would be at least reduced significantly.

If the CoCo terms are particularly dilutive for the initial shareholders by setting the conversion price C_P low enough to issue a significant amount of new shares, there might be many share values that lead to an equilibrium [97]. However, if the conversion benefits the initial shareholder, then the researchers of PT and SW agree on the absence of equilibrium. This raises the question of the ideal conversion ratio C_r (alternatively, the conversion price C_p). The higher the conversion ratio (i.e., a lower conversion price), the more equity a CoCo bond will convert into. While this is a positive outcome from the debt/equity ratio perspective and reduces the probability of default on other obligations, it comes at the expense of diluting the ownership of existing equity holders and reducing their control rights. This may be the necessary cost for the bank's improved financial health and stability. In some way, a CoCo can be designed to punish the initial shareholders for the excessive risk they accepted before conversion. Upon triggering, the risk-shifting is reduced and prevents extreme debt overhang recourse [20].

Because of the unknowns regarding stability in market-based triggers, CoCos featuring such designs are not popular in the industry. Even so, they are intended to address concerns about opacity and latency in loss recognition implied by accounting triggers. Also, the multi-equilibrium problem does not manifest with a CoCo relying on a minimum asset value because the conversion does not affect this control variable [37].

7.3 Understanding the correction in Sundaresan and Wang

Many papers do not consider the pricing stability referred to by SW and draw hasty conclusions about the multi-equilibrium eventuality. The correction from PT derived closed-form equations for the CoCo value and pre-conversion stock price[1]. However, as stated in Proposition 2, if an equilibrium stock price exists, then the value is given by the equations in section F.2. The reciprocal does not hold, implying that depending on the inputs, the candidate results derived are not guaranteed to be valid or stable.

Thus, Theorem 1 by PT studies the condition for this equilibrium stock price to exist (and its uniqueness conditions). The cases are split into three scenarios. The first ensures the existence and a unique equilibrium, the second eliminates the existence of equilibrium, and the third might eliminate the existence if the maturity is sufficiently long. Mathematically, these are expressed as

- (i) Existence of a unique equilibrium if $\frac{mK}{n} \geq max\left(\overline{C}, \frac{c\overline{C}}{r}\right)$.
- (ii) No equilibrium if $\frac{mK}{n} < \overline{C}$.
- (iii) Potentially no equilibrium if the maturity is long enough and $\overline{C} \leq \frac{mK}{n} < max\left(\overline{C}, \frac{c\overline{C}}{r}\right)$.

Assuming the triggering price, CoCo nominal, coupon, interest rate, and initial number of shares are known, the existence of a unique equilibrium can be linked to the dilution factor. From the demonstration of Theorem 1, the fact that no equilibrium exists is not reflected in the equations from section F.2, as it will still output a value for S_t that violates some restrictions (taking the form of additional equations) and leads to an inconsistency with the equilibrium suggested. These constraints are interpreted as a pre-conversion candidate price of the share (S_t) that must remain above the trigger price for asset values (A_t) greater than the triggering asset threshold (A_{uc}).

For example, we use the numerical inputs suggested by the "Internet appendix" from the PT paper. Theorem 1 derives the condition on the variable m (the number of new shares in the case of conversion) to ensure the existence of a unique equilibrium. This value must be at least $m \geq 0.625$. To show the inconsistency, we first set the asset value to $A_t = 108.993$ to see that by setting m slightly lower than the minimum threshold, such as $m = 0.624$, the asset value that would trigger the conversion is $A_{uc} = 108.992$. We verify that for a given maturity, the candidate share price, also called pre-conversion share price (provided by Eq. 10), is below $K = 8$ and the conversion price ($S_t = 7.9967$). Then, even though the asset value is above the threshold that would imply a conversion ($A_t > A_{uc}$), the pre-conversion stock price is already lower than the conversion price. This result is a breach of

[1]See Appendix F

Proposition 1 that states if there is an equilibrium stock price, then conversion occurs when A_t falls to A_{uc} the first time. Here, we observe a candidate share price already below the trigger for $A_t > A_{uc}$. This happens when the conversion terms benefit shareholders by giving CoCo-holding investors less new equity than the CoCo principal (or its unconverted perpetuity value).

7.4 Effect on other CoCos

To the best of our knowledge, an interesting and unexplored area of research lies in the stability of contingent convertibles that convert and deliver the interest payment (either cash or through new shares) at prefixed times. This is a key difference with papers from Sundaresan and Wang, Pennacchi and Tchistyi, and Glasserman and Nouri that assume a conversion occurs the first time τ that the share value (alternatively, the asset value) falls below a given threshold, which is defined in Proposition 1.

When turning this design into a discrete "observation time" for the conversion, the underlying mechanism of ERN is similar to DCL, with only the current payment being subject to conversion at a prefixed time. Regarding the equilibrium problem for ERN, the authors Bulow and Klemper assert that their CoCo is not subject to it because the incentive that an early conversion might create vanishes with the payment limited to the current interest due (instead of the full nominal). Yet, they recognise the possibility of small, multiple equilibria, justified by the value of the stock being reduced by the conversion (i.e., a decrease of the debt-to-equity ratio implies an increase in the outstanding note value and then a decline in equity value). However, these authors rely on the inaccurate paper from SW, and considering the recent PT paper, such a multi-equilibrium feature turns out to be more of a problem of equilibria existence. Also, the Bulow and Klemper paper is not quantitative, but their multiple equilibria should come from how the equity must be conserved from the conversion time $\tau - dt$ to $\tau + dt$, similarly to the notion of energy conservation in a physical experiment.

To study the effect of a contingent convertible on the capital structure of a firm, we derive an equation for the share value based on a structural approach. A firms assets are comprised of a senior debt tranche (B), a contingent convertible buffer (CC), and NS shares valued at S_t each. If at observation time T the share value S_T falls below the threshold S_c, then the contingent convertible is converted into AS additional shares. In economic terms, the pre-conversion equity value is equal to

$$E_T^+ = \frac{A_t - B - CC}{NS}$$

representing the asset value less the debt instruments (senior debt B and contingent convertible debt CC) divided by the number of shares NS at time T, with

$T < \tau$. When simulating the asset value or the equity value as a geometric Brownian motion (gBm), there is continuity at the conversion time, allowing it to bridge with the post-conversion equity value E_T^+ as

$$E_T^+ = \frac{A_T - B}{NS + AS}$$

defined as the asset value less the senior debt divided by the total outstanding number of shares, which is determined by the initial number of shares NS plus the additional shares issued upon conversion. Therefore, we can now derive the value of each share at maturity (observation time) as

$$S_T = \frac{A_T - SD - CC \,.\, \mathbb{1}_{\{S_T > S_c\}}}{NS + AS \,.\, \mathbb{1}_{\{S_T < S_c\}}}$$
$$\iff S_T \left(NS + AS \,.\, \mathbb{1}_{\{S_T < S_c\}}\right) = A_T - SD - CC \,.\, \mathbb{1}_{\{S_T > S_c\}}$$
$$\iff E_t \left[S_T \left(NS + AS \,.\, \mathbb{1}_{\{S_T < S_c\}}\right)\right] = E_t \left[A_T - SD - CC \,.\, \mathbb{1}_{\{S_T > S_c\}}\right]$$
$$\iff E_t[S_T]NS + AS \,.\, E_t \left[S_T \mathbb{1}_{\{S_T < S_c\}}\right] = A_T - SD - CC \,.\, E_t \left[\mathbb{1}_{\{S_T > S_c\}}\right]$$

Additionally, we know that m

$$E_t \left[S_T \mathbb{1}_{\{S_T < S_c\}}\right] = e^{r(T-t)} S_t N\left(-d_1(S_t, S_c)\right)$$
$$E_t \left[\mathbb{1}_{\{S_T > S_c\}}\right] = N\left(d_2(S_t, S_c)\right)$$
$$E_t[S_T] = S_t e^{r(T-t)}$$

with

$$d_1(S_t, S_c) = \frac{log\left(\frac{S_t}{S_c}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2(S_t, S_c) = \frac{log\left(\frac{S_t}{S_c}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

The cumulative normal distribution function is defined as

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy$$

The number of additional shares AS can be written as a function of the conversion price CP, such as $AS = \frac{CC}{CP}$.

We next plot the asset value as a function of the underlying share price at inception ($t = 0$), which is defined by

$$A_t(S_t, S_c) = S_t e^{r(T-t)}(NS + AS \,.\, N(-d_1(S_t, S_c)) + B + CC \,.\, N(-d_2(S_t, S_c)) \quad (7.1)$$

This value for the assets is compared to the value of the assets at maturity, equivalent to studying the $\lim_{t\to T} A_t(S_t, S_c)$, such that

$$A_T = S_T(NS + AS \,.\, \mathbb{1}_{\{S_T < S_c\}}) + B + CC \,.\, \mathbb{1}_{\{S_T > S_c\}} \quad (7.2)$$

In Fig. 7.1, We draw different profiles of asset values, exhibiting various behaviours depending on the control variable that we change. Specifically, We study the effect of two: (a) the conversion price CP and (b) the time to maturity T. The baseline inputs used for our calculations are listed in Table 7.1.

Variable	**Value**
Senior Debt B	80$
Contingent Convertible BB	5$
Number of initial shares NS	1
Triggering share price S_c	10
Drift r	0%
Volatility of the equity σ	25%

Table 7.1: Baseline scenario variables.

These results show the "footprint" of the value transfer between the initial shareholders and CoCo-holders, which is also responsible for the discontinuity in the case of $C_p \neq S_p$. The evenness being lost, the asset value at maturity might have more value before (if $C_p < S_c$) or after the conversion (if $C_p > S_c$).

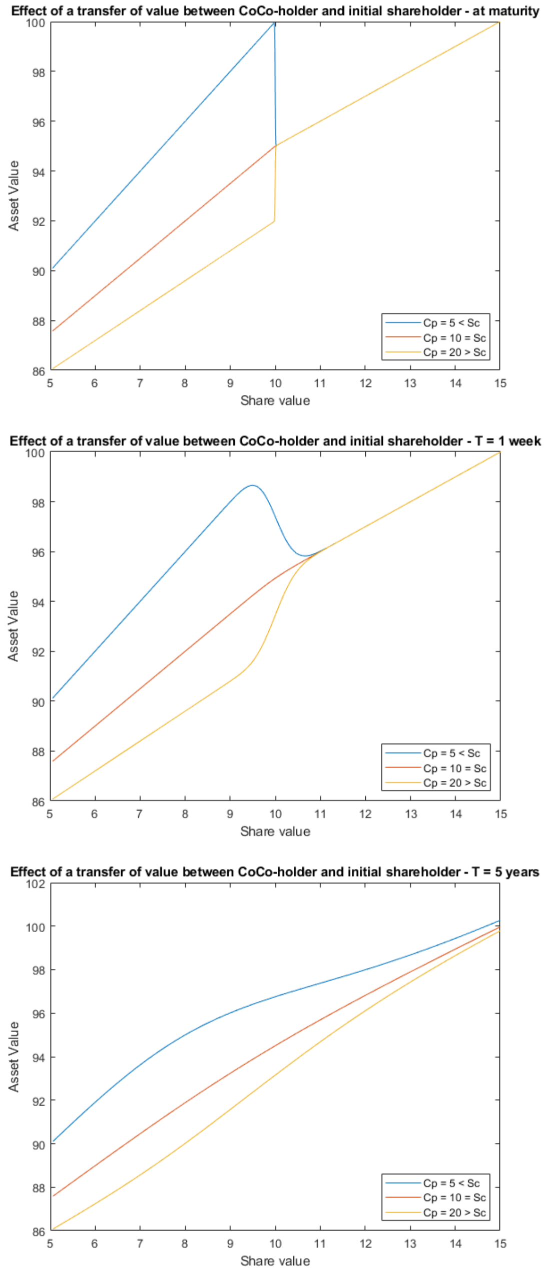


Figure 7.1: Plots of the asset values as a function of the share value for three conversion prices C_p, which create a transfer of value between the initial shareholders and CoCo-holder (blue and yellow) or no transfer of value (orange). The time to maturity is assumed to be 0 on the upper plot, one week on the middle plot, and five years on the lower plot.

The more the conversion price is widened from the par conversion price ($C_p = S_c$), the bigger the amplitude of the break between the pre- and post-conversion asset values (see the upper plot in Fig. 7.1). By relating with the world of options and with a reduced time to maturity, the magnitude of the assets 'delta', defined here as $\frac{\partial A}{\partial S}$, is amplified at the proximity of the triggering price. This phenomenon is exhibited in the middle plot of Fig. 7.1 when the maturity is set to one week. The concept of transferring value is often evoked without being plotted. Not surprisingly, when the share value at time T is above the triggering price, i.e., $S_T > S_c = 10$ in our example, the CoCo does not convert, so no transfer of value occurs between the initial shareholder and CoCo-holder. However, upon conversion, if the conversion price C_p is not set to create a conversion at par, then the following two phenomena might take place:

- (i) If $C_p < S_c$, then the number of additional shares being released to the CoCo-holder when multiplied by their current value S_T, exceed the nominal value that would have been paid as cash (the interest payment). By diluting the existing shareholders, it creates a positive transfer of value from the initial shareholders to the CoCo-holder. The investment from the CoCo-holder is equivalent to the shares being purchased at a discount, which is exhibited by the blue curve in Fig. 7.2.
- (ii) If $C_p > S_c$, then the number of additional shares being released to the CoCo-holder, when multiplied by their current value S_T, are lower than the nominal value that would have been paid as cash (the interest payment). By under-diluting the existing shareholders, it creates a positive transfer of value from the CoCo-holder to the initial shareholders. The investment from the CoCo-holder is absorbed by the capital structure in the form of shares purchased at a premium, which is exhibited by the orange curve in Fig. 7.2.

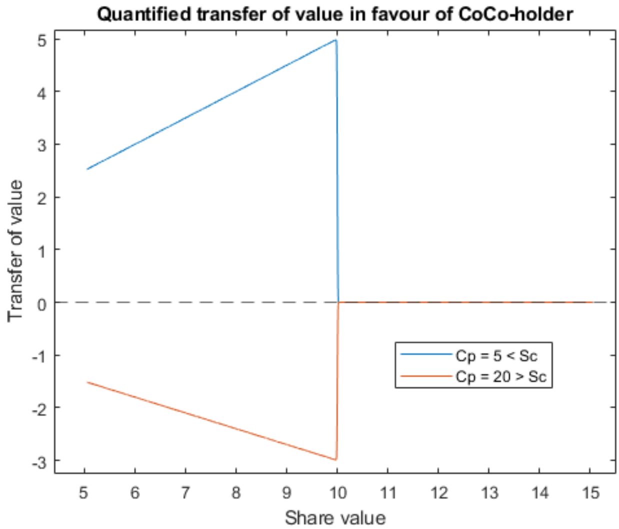


Figure 7.2: Quantification of the value transfer from the initial shareholder to the CoCo-holder at conversion time as a function of the share value at time T for two conversion prices C_p.

A jump in the asset value may only occur if there is a transfer of value between shareholders and CoCo-holders at maturity (i.e., at conversion time). However, turning Eq. 7.2 into the continuous Eq. 7.1 allows for the constant monitoring of the conversion risk and adjustment of the risk-neutral asset value. Assuming all inputs are constant, this results in a single solution for the asset value and is shown when overlaying the asset value as a function of the share price for different maturities. The price is observed to eventually converge at the extremities.

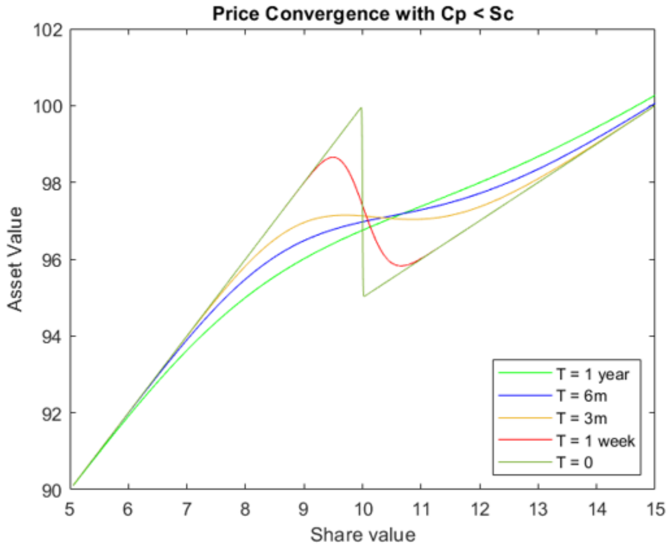


Figure 7.3: Plot of the asset value as a function of the share value for five times to maturity T. The conversion price is set at $C_p = 5$, benefiting the CoCo-holder in the case of conversion. Prices converge for "extreme" values of the share price, reflecting the increasing probability of conversion for low share prices and, reciprocally, the low probability of conversion for high share prices.

In Fig 7.3, when $S_t << S_c$, the asset values evolve as $\frac{\partial A(S_t,S_c)}{\partial S} \rightarrow 2$, which is the sum of $NS+AS$ when the share price is very low, the conversion occurs at maturity only, or the conversion is already "priced". Hence, if the share price increases by 1, then the asset values increase by $NS + AS$.

The asset values when $S_t >> S_c$ are still an increasing function of the share price, but only at the rate $\frac{\partial A(S_t,S_c)}{\partial S} \rightarrow NS = 1$, which is due to the lower probability of conversion and, as a result, new shares being issued. Finally, when the share price is close to the conversion price, the asset value behaviour is driven by the equity volatility and remaining time to maturity (i.e., the time before the observation date or automatic conversion decision). This aspect is addressed in Section 7.5 where the partial derivative of the asset values with respect to the share price is derived, which is the so-called delta function plotted in Fig. 7.4.

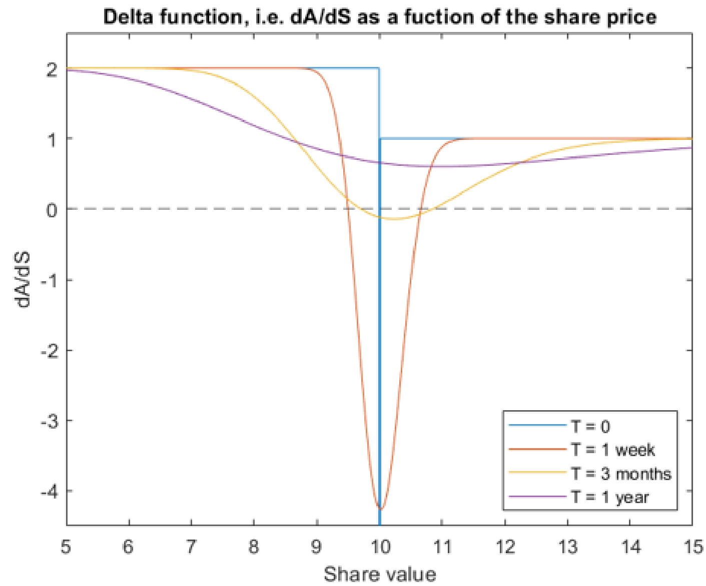


Figure 7.4: Delta function for the asset value of a firm with a DCL in its capital structure, i.e., $\frac{\partial A_t}{\partial S}$ function (Eq. 7.3 in Section 7.5), plotted for four times to maturity. We observe that the delta tends toward $NS + AS$ for low share values (high probability of conversion) and toward NS for high share values (low probability of conversion), which is due to the assets evolving with the same slope as the expected total equity value.

DCL instruments feature a superior design because leverage is the control variable, and by using it not as a proxy, we allow some jumps in this variable. Therefore, DCL instruments are not subject to multiple equilibria or equilibrium nonexistence. Either when paying down the loan cash or by proceeding to conversion of P_N at the conversion price C_P, the leverage suddenly reduces on the payment date. This allows the share price to be continuous without any sub-constraint being violated. The control variable (i.e., leverage) is a process with jumps at the payment date and defined as

$$L_t = \frac{D_t}{A_t} = \frac{B_t + CC_t}{B_t + CC_t + E_t} = \frac{B_t + CC_t}{B_t + CC_t + NS_t S_t}$$

The equity value S_t does not have to adjust to compensate for the decrease in CC_t outstanding nor increase in the number of shares NS. Continuous monitoring of the share price (and the leverage) is another advantage, as it provides the necessary information to know if the CC will convert the next interest payment at the prefixed date. Finally, Theorem 1 from PT is derived from the pre-conversion candidate stock price being able to fall below the conversion price. However, both in ERN and DCL, the share price is only monitored at the payment date k and can unambiguously reach values lower than the triggering threshold. This way, the equivocal equilibria problem can not manifest.

7.5 Capturing the non-monotonicity moment

An important aspect of this study captures the point of non-monotonicity in terms of the time to maturity as a function of the volatility. As seen in Fig. 7.3, more than one share value can potentially lead to the same asset value. A condition to avoid this stability issue is to have the asset value be a monotonic function of the share value. Ensuring this stability can be obtained by having the partial derivative of the asset value with respect to the share value be strictly above 0 on the entire interval of share prices, such that

$$\frac{\partial A_t(T,\sigma)}{\partial S_t} > 0$$

$$\Longleftrightarrow \frac{\partial}{\partial S_t}\left[S_t e^{r(T-t)}\left(NS + AS\,.\,N(-d_1(S_t,S_c)) + B + CC\,.\,N(d_2(S_t,S_c))\right)\right] > 0$$

$$\Longleftrightarrow \frac{\partial}{\partial S_t}\left[S_t e^{r(T-t)} NS\right] + \frac{\partial}{\partial S_t}\left[S_t e^{r(T-t)} AS\,.\,N(-d_1(S_t,S_c))\right] + \frac{\partial B}{\partial S_t} + \frac{\partial}{\partial S_t}\left[CC + N(d_2(S_t,S_c))\right] > 0$$

$$\Longleftrightarrow NSe^{r(T-t)} + AS\,.\,N(-d_1(S_t,Sc))e^{r(T-t)} + AS\,.\,e^{r(T-t)} * \frac{\partial(-d_1(S_t,S_c))}{\partial S_t}\frac{\partial N(-d_1(S_t,S_c))}{\partial(-d_1(S_t,S_c))} + CC\frac{\partial(d_2(S_t,S_c))}{\partial S_t}\frac{\partial N(d_2(S_t,S_c))}{\partial(d_2(S_t,S_c))} > 0$$

$$\Longleftrightarrow NSe^{r(T-t)} + AS\,.\,N(-d_1(S_t,S_c))e^{r(T-t)} - \frac{S_t AS\,.\,e^{r(T-t)}}{\sigma\sqrt{T-t}}N'(-d_1(S_t,S_c)) + \frac{CC}{S_t\sigma\sqrt{T-t}}N'(d_2(S_t,S_c)) > 0$$

with $N'(x)$ being the probability density function of the normal distribution, i.e.,

$$N'(x) = \frac{exp\left(-\frac{x^2}{2}\right)}{\sqrt{2\pi}}$$

that involves

$$\frac{\partial A_0(T,\sigma)}{\partial S_t} > 0 \iff$$

$$(NS + AS \,.\, N(-d_1))\, e^{rT} - \frac{S_t AS \,.\, e^{rT} \,.\, exp\left(-\frac{d_1^2}{2}\right)}{\sigma\sqrt{2\pi T}} + \frac{CC \,.\, exp\left(-\frac{d_2^2}{2}\right)}{S_0\sigma\sqrt{2\pi T}} > 0 \quad (7.3)$$

Because the non-monotonicity might only occur for $C_p < S_c$, we assume again $C_p = 5$ and we plot Equation 7.3 in the following two ways:

(a) A straight Delta function $\left(\Delta = \frac{\partial A_0(T,\sigma)}{\partial S_t}\right)$ depending on the share value, as seen in Fig. 7.4 where the existence of a unique solution is conditional on the function being above the $y = 0$ line.

(b) The minimum maturity T leading to monotonicity of the asset values, i.e., $\frac{\partial A_0(T,\sigma)}{\partial S_t} > 0$ is apparent. Fig. 7.5 displays this minimum maturity as a function of the equity volatility σ and for three initial share prices S of $S = 10.5$, $S = 11.0$, and $S = 11.06$ when S is maximized.

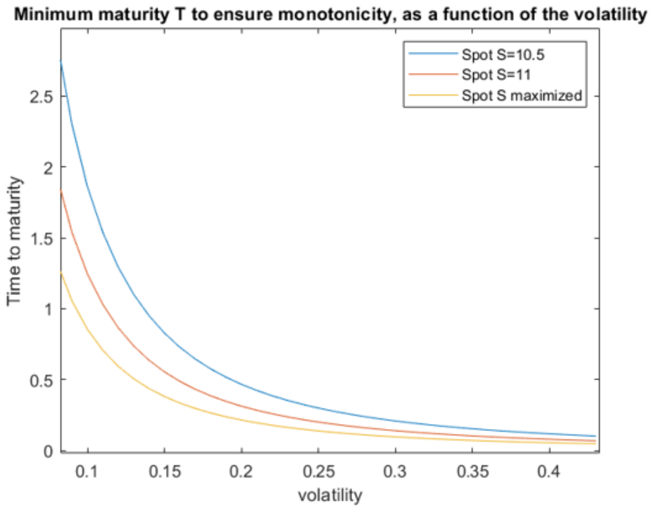


Figure 7.5: Plot of the minimum maturity T to ensure monotonicity of the asset values as a function of the equity volatility. We assume three possible spot values of $S = 10.5$ (blue), $S = 11.0$ (red), and S being maximized at ($S = 11.06$)

For any given share price, the minimum maturity is a decreasing function of the volatility. Higher volatility smooths Eq. 7.1, and lower volatility creates more abrupt variations close to the conversion price, as the triggering probability is increased. Therefore, in the case of $C_p < S_c$, the remaining time to maturity must be very little to skew the asset function enough to create a double equilibrium problem (respectively, a long time to maturity). This phenomenon is observable in Fig. 7.6 when comparing the asset value behaviour as a function of the share value for different times to maturities with volatilities of $\sigma = 10\%$ (upper panel) and $\sigma = 40\%$ (lower panel). Low volatility, even when coupled with a long maturity, can be seen to not always be enough to lead to monotonically increasing assets (as a function of share value). However, high volatility mixed with reduced maturity still smooths the asset function sufficiently to eliminate the local maximum and minimum (at the origin of the multiple equilibrium problem).

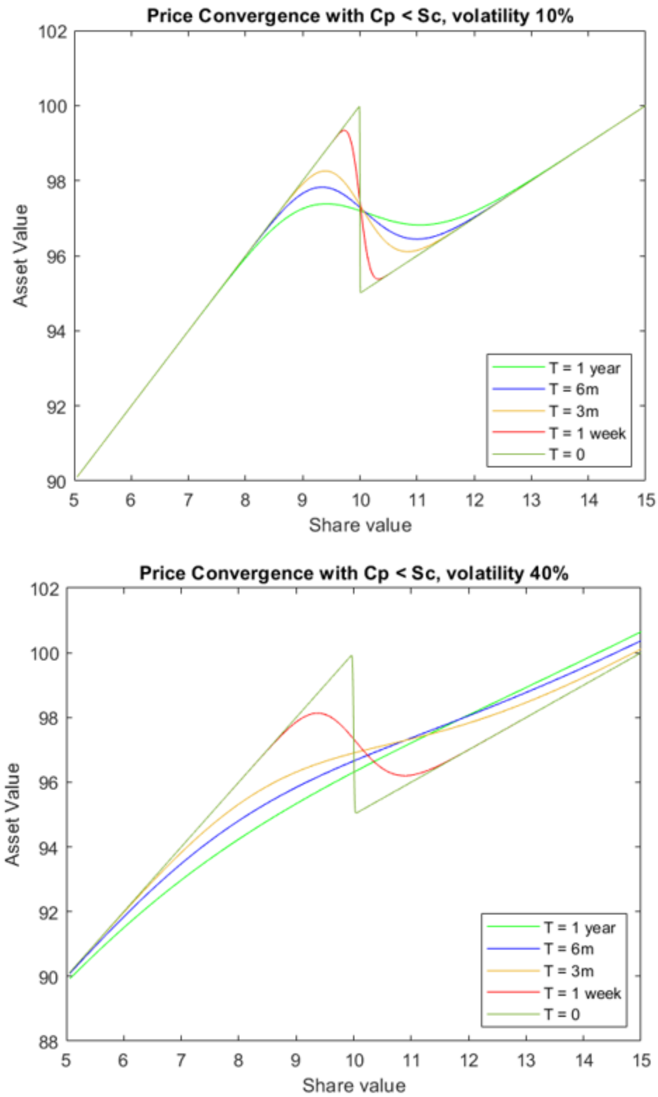


Figure 7.6: Comparison of the asset value behaviours as a function of the share value for different times to maturity with a volatility of $\sigma = 10\%$ (upper plot) and $\sigma = 40\%$ (lower plot). Lower volatility with a reduced time to maturity creates more uncertainty when the share value trades close to the triggering price.

7.6 Conclusion

In the PT paper, the mathematical framework relied on the value of the assets, defined as a geometric Brownian motion, which is characterised as continuous without jumps. Under some circumstances established in Theorem 1, the possibility exists that there is no equilibrium point for the candidate stock price. This chapter fills the research gap regarding CoCos that do not convert the first time the control variable falls below a given threshold. The literature counts some of these, such as ERN and the proposed DCL, which removes some potential constraints on the candidate share price, making the pricing framework more robust, even when the conversion price penalises the CoCo-holder. The asset value is a monotonically increasing function of the share value for any $S_t \geq S_c$ when one of the two following conditions is verified:

- $C_p \geq S_c$.
- $C_p < S_c$ with a maturity sufficiently high.

Now, calibrating an implied share price or implied asset value is possible based on the other input without facing any double equilibrium problem or absence of equilibrium. An issue raised by the figures presented in this chapter is multiple equilibria, such as that occurring in Fig. 7.3 in the case of $C_p < S_c$. Two distinct share values S_t could result in the same asset valuation (the function is not monotonic when $C_p > S_c$). This scenario results when the CoCo-holder benefits from the conversion, whereas Pennacchi and Tchistyi showed there is an absence of equilibrium only when there is a transfer of value from the CoCo-holder to the shareholder at conversion time.

A key difference remains between the model developed here and the basis of the CoCo framework used by Sundaresan and Wang and Pennacchi and Tchistyi. This refers to their Proposition 1 that assumes the contingent convertible turns to equity at the first instant τ the share price S_τ falls below the threshold S_c, which is not the case in the model we develop. Still, by turning the equation from a discrete to a continuous model, the market can continuously price the conversion risk and adjust to a change in the underlying share price.

Chapter 8

Overview of an Alternative Trigger for DCL

"Money is gold, and nothing else."

J.P. Morgan, [105].

This chapter is based on a paper co-published with Prof. Sverrir Ólafsson in Finance Research Letters [69].

8.1 Abstract

In this chapter, we suggest a novel trigger for Dynamic Control of Leverage (DCL). These instruments are analogous to conventional Contingent Convertible (CoCo) bonds, employed to constrain a firm's leverage within specified limits, thus minimizing the likelihood of default. Our proposed trigger enables continuous leverage monitoring, bringing in the desired flexibility and allowing the conversion to be triggered ahead of the cash payment. This alternative design, building upon the DCL mechanism, has been shown to further reduce the probability of default while preserving the desirable mean-reverting behaviour of the leverage dynamic. We provide evidence of the model's effectiveness and discuss the implications of its implementation.

8.2 Introduction

Dynamic Control of Leverage (DCL) instruments [56] represent a new type of CoCo bonds, designed for self-regulation or compliance with existing regulations.

These annuity bonds maintain optimal debt-to-assets levels by converting the interest payment to equity if the firm's leverage breaches a critical threshold L_c, or issuing additional debt when leverage falls beneath a certain level L_{min}. Notably, the original DCL instrument triggers conversions at discrete, pre-agreed times, contingent on the firm's leverage at the given observation date T_k.

The original DCL design offers significant flexibility, with multiple degrees of freedom granted to the issuer. Of these, the payment frequency f passively impacts the intrinsic risk. Increasing payment frequency enables enhanced capital protection by responding promptly to undesired leverage levels. This is contingent, however, on payment and leverage observation occurring simultaneously. The current "European-like" approach to leverage observation may delay loss recognition and the conversion trigger, an issue this chapter addresses.

Our contribution to the field lies in proposing an alternative triggering mechanism for DCL bonds, enhancing their effectiveness and thereby, minimizing further the default risk. Our proposed trigger enables continuous leverage monitoring (i.e. "American-like"), allowing conversion to occur ahead of cash payments, leading to more timely reactions to fluctuating leverage levels. Building upon our original research on the DCL model [56], this revised design offers significant potential for evolving CoCo bond designs.

The enhanced design for DCL we propose facilitates its integration within the existing regulatory framework. This is particularly relevant given some of the concerns the U.S. Securities and Exchange Commission (SEC) raised in a 2019 No-Action Letter [104]. As described previously, by making the leverage continuously monitored, our proposed trigger mechanism facilitates a more proactive and continuous conversion process for DCLs. This innovative approach enables early adjustments to leverage levels and mitigates the likelihood of sudden, drastic conversion events, fostering more consistent market behaviour and reducing the risk of a side-by-side public and private market for the underlying stock.

This chapter is structured in two parts. Section 8.3 introduces (a) the benefits of continuous observation for the leverage, (b) the adjusted investor position compared to the original DCL design, and (c) the probability of conversion. Section 8.4 deals with the new leverage dynamic and dilution for the initial shareholder. We conclude with comments on our results and findings.

8.3 DCL featuring American observations

The assumptions on the capital structure are kept unchanged from [56], where the firm is financed with a mixture of equity and debt in the form of a hybrid instrument of the type of a CoCo bond. In the original DCL proposal, the conversion condition was expressed as an inequality on the leverage at a time horizon T_k that is translated into a condition on the share price as

$$L_k = \frac{RQ_k}{RQ_k + NS_{k-1}S_k} > L_c \iff S_k < \left(\frac{1-L_c}{L_c}\right)\frac{RQ_k}{NS_{k-1}} = S_{c,k} \tag{8.1}$$

where L_k is the debt-to-asset, or leverage, ratio, RQ_k is the residual value of the bond, and S_k is the share price, all expressed at time T_k. Also, NS_{k-1} is the outstanding circulating number of shares, expressed at time T_{k-1}, and L_c is the critical leverage that triggers conversion.

From here, assuming the stock price follows a geometric Brownian motion (gBm) [106, 18], the probability of conversion or, alternatively, the probability of no-conversion, $P(S_k \geq S_{c,k})$ is straightforward to derive where

$$\mathbb{P}(S_k \geq S_{c,k}) = \Phi\left(\frac{\log\left(\frac{S_0 L_c NS_{k-1}}{RQ_k(1-L_c)}\right) + \left(\mu - \delta - \frac{\sigma^2}{2}\right)T_k}{\sigma\sqrt{T_k}}\right) \tag{8.2}$$

This probability must then be computed term to term, adjusting NS_{k-1} by the expected circulating number of shares and RQ_k by the total residual loan value. Here, $T_k - T_{k-1}$ is assumed constant and equal to the time step between two observation dates in the vector $\mathbf{T} = (T_0, T_1, ..., T_{N_n})$.

By increasing the payment frequency f, we prevent the share value (and, mechanically, the leverage) from running to values between payment dates that could be dangerous for the issuing entity's viability. Or, this at least excludes the case of operating at these levels for too long. As the payment frequency is related to the conversion and re-issuance feature, the observation process could hypothetically be performed every week, day, or even second. Still, this repetition would raise an operational issue, leading to a payment of very small amounts to the bondholders over a frequency unsuitable for a firm's operations.

Nevertheless, payment and observation dates might be separated through continuous observation of the firm's leverage, while payment (either as cash or in shares) is held at discrete times on a reasonable frequency. The conversion probability will increase as a result of this design, leading to a more prominent risk for the bondholder, who could expect a higher yield in return on the investment. By defining τ_k as

$$\tau_k = \left\{\inf t\ ;\ \min_{[T_{k-1},T_k]} S_t \leq S_c\right\} \tag{8.3}$$

the proposed mechanism of the American-like observation could be viewed as the following:

- If the maximum leverage $\max_{T_{k-1}<t\leq T_k} L_t = L_k^*$ over the period $[T_{k-1}, T_k]$ is above the L_c threshold, then the next payment is immediately converted to C_r new shares, delivered at time τ_k, with τ_k defined in Eq. 8.3.
- If the maximum leverage L_k^* over the period $[T_{k-1}, T_k]$ remains below the L_c condition, then the next payment is made in cash, at the pre-fixed date T_k.

The conversion condition on L_t is now expressed as $L_k^* > L_c$. As the process L_t remains unknown, the equivalent conversion condition might be written $S_k^* < S_{c,k}$ with $S_k^* = \min_{t\in[T_{k-1},T_k]} S_t$, i.e., in terms of the stock price S_t, modelled with a stochastic differential equation that is known to be [106]

$$dS_t = (\mu - \delta)S_t dt + \sigma S_t dW_t \tag{8.4}$$

where μ is the expected annual return on equity, δ is the annual dividend yield paid by the firm to equity holders, and σ is the annual volatility of returns. dW_t represents an infinitesimal change in the Wiener process W_t in the objective world.

At any pre-fixed payment time T_k, the cash amount received by the bondholders is $P_N(T_k)$, and defined in [56] (Eq. 4.1) as an equal quantity over the DCL's N-periods lifetime, such that

$$P_N(T_k) = \frac{rQ}{1-(1+r)^{-N}} \tag{8.5}$$

with Q being the DCL's nominal value and r the yield delivered by the instrument.

The value of the investment at time T_k is represented as

$$P_{T_k} = P_N(T_k)\mathbb{1}_{\left\{S_k^*>S_{c,k}\right\}} + C_r S_{\tau_k}\mathbb{1}_{\left\{S_k^*\leq S_{c,k}\right\}} \tag{8.6}$$

where S_{τ_k} is the price per share at the conversion time τ_k.

The conversion probability can still be estimated at the prior T_{k-1} by deriving the first hitting time equation (see Appendix G). This result follows the probability that the stock price S_t will not drop to the endogenous conversion level $S_{c,k}$ at any time between T_{k-1} and T_k (with $k \geq 1$). Assuming the conversion threshold is lower than the initial stock value, i.e., $(S_{k-1} > S_{c,k})$, we have

$$\mathbb{P}\left(\min_{k-1\leq t\leq k} S_t > S_{c,k}\right) = 1 - \mathbb{P}(S_{\tau_k} \leq S_{c,k} | T_{k-1} < \tau_k < T_k)$$
$$= \Phi\left(\frac{\log\left(\frac{S_{k-1}}{S_{c,k}}\right) + \left(\mu - \delta - \frac{\sigma^2}{2}\right)(T_k - T_{k-1})}{\sigma\sqrt{T_k - T_{k-1}}}\right) - \exp\left(2\log\left(\frac{S_{k-1}}{S_{c,k}}\right)\frac{\mu - \delta - 0.5\sigma^2}{\sigma^2}\right)$$
$$* \Phi\left(\frac{\log\left(\frac{S_{k-1}}{S_{c,k}}\right) - \left(\mu - \delta - \frac{\sigma^2}{2}\right)(T_k - T_{k-1})}{\sigma\sqrt{T_k - T_{k-1}}}\right) \tag{8.7}$$

As the probability is calculated at time T_{k-1} for the following term and due to the eventuality that a conversion is triggered before the payment date T_k, the conversion threshold $S_{c,k}$ requires in this American DCL version the use of RQ_{k-1} instead of RQ_k. The critical threshold $S_{c,k}$ is then defined as

$$S_{c,k} = \left(\frac{1 - L_c}{L_c}\right)\frac{RQ_{k-1}}{NS_{k-1}} \tag{8.8}$$

By setting the conversion rate $C_r = \frac{P_N(T_k)}{S_P}$, the price of this adapted DCL instrument, which can still switch between fixed coupon cash payments and conversion into shares while now being conditional on a breach of the minimum share price between two payment dates, is defined as the sum over the lifetime of the instrument of the discounted expected value of Eq. 8.6, such that

$$P(t, \mathbf{T}) = \sum_{k=1}^{N} \mathbb{E}_t\left[P_N(T_k)\mathbb{1}_{\{S_k^* > S_{c,k}\}}D(t, T_k) + \frac{P_N(T_k)S_{\tau_k}}{S_P}\mathbb{1}_{\{S_k^* \leq S_{c,k}\}}D(t, \tau_k)\right] \tag{8.9}$$

Evaluation of Eq. 8.9 requires the use of numerical methods and computational simulations.

	Q	y	N	S_0	μ	δ	σ	L_c	NS_0	dt	S_p
Inputs	5000	0,05	10	20	0,1	0,025	0,35	0,8	100	1	18
Time [k]	0	1	2	3	4	5	6	7	8	9	10
P_N	647,52										
RQ_k	5000,00	4602,48	4185,08	3746,81	3286,63	2803,44	2296,08	1763,37	1204,01	616,69	0,00
S_k	20,00	26,78	15,97	18,46	10,35	14,80	15,78	27,02	42,64	52,75	64,88
$S_{c,k}$		11,506	10,268	8,939	7,628	6,352	5,103	3,862	2,613	1,332	0,000
$\mathbb{P}(S_k > S_{c,k})$		**0,947**	**0,920**	**0,919**	**0,927**	**0,940**	**0,954**	**0,970**	**0,985**	**0,997**	**1,000**
NS_k	100	101,897	104,789	107,713	110,330	112,495	114,134	115,216	115,759	115,885	115,885
L_k	0,714	0,632	0,720	0,659	0,747	0,632	0,564	0,364	0,197	0,092	0,000
$S_{c,k}$		12,500	10,769	9,091	7,625	6,331	5,160	4,075	3,044	2,039	1,033
$\mathbb{P}(S_k > S_{c,k})$		**0,810**	**0,771**	**0,784**	**0,807**	**0,832**	**0,860**	**0,890**	**0,922**	**0,956**	**0,987**
NS_k	100	106,848	115,087	122,840	129,787	135,821	140,865	144,835	147,642	149,228	149,693
L_k	0,714	0,632	0,710	0,638	0,721	0,593	0,517	0,317	0,163	0,073	0,000

Table 8.1: For the case specified in the first numerical row, the **blue cells** overview the DCL behaviour under the base mechanism discussed in Chapter 4, such as the probability of conversion at time T_k, the expected number of circulating shares, and the resulting firm's leverage. The **yellow cells** offer the same overview but assume a continuous observation of the leverage. The survival probability appears to be lower from term to term as the conversion might occur at any time $t \in [T_{k-1}, T_k]$.

Visually assessing the impact of the American observation on the leverage over the original (European) version of DCL is available in Fig. 8.1, which presents (a) the survival probability from term to term for the two triggering mechanisms and (b) the outstanding circulating number of shares at different time horizons. The data is listed in Table 8.1.

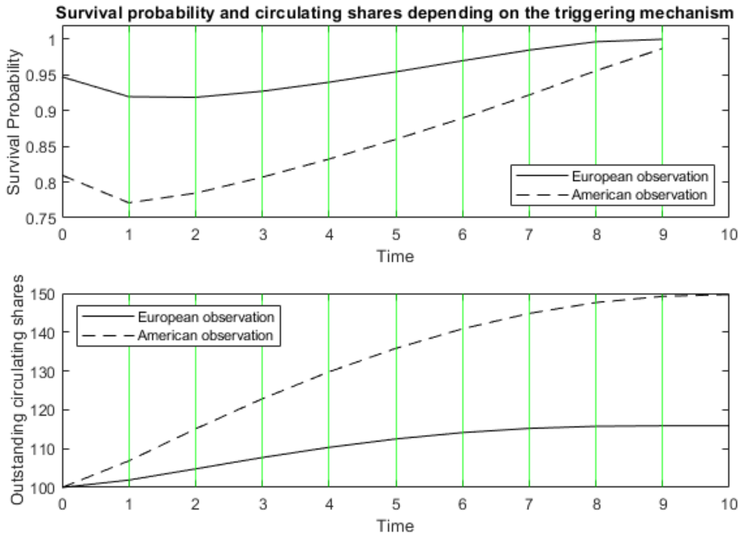


Figure 8.1: Overview of the survival probability (upper plot) and circulating number of shares (lower plot) depending on the triggering mechanism used in the DCL. The parameters and data are listed in Table 8.1.

The increased protective effect of the continuous monitoring of the leverage (with the American observation) is balanced by the increased dilutive impact, as the survival probability is lowered.

8.4 Leverage dynamic under continuous observation with the re-issuance feature

We reintroduce the re-issuance feature used in [56] to channel the leverage dynamic. This process prevents the firm's leverage from falling below a preset threshold L_{min} by issuing a new DCL in the capital structure of the same parameters but with an adjusted nominal Q_k, such as

$$Q_k = -RQ_k + \frac{L_{min} N S_{k-1} S_k}{(1 - L_{min})} \tag{8.10}$$

In other words, the outstanding debt is increased to take the leverage up to the minimum critical leverage level L_{min}.

As a company running on low leverage is less problematic than on high leverage (regarding the probability of default), taking adaptive measures is less urgent. So, the re-issuance process is maintained on interest payment dates only.

Fig. 8.2 shows a ten-year simulation of a firm's leverage behaviour, assuming its capital structure comprises equity and debt as a DCL instrument featuring an American observation of the leverage. The resulting leverage process with a continuous observation is path-dependent, contrary to the discrete observation model.

On the one hand, in the simulation Fig. 8.2, the number of shares is observed to increase at any time t and is not restricted to payment dates k. The conversion of coupon to equity occurs as soon as the leverage reaches the critical boundary $L_c = 80\%$ but remains limited to one conversion per period $[T_{k-1}, T_k]$. On the other hand, the re-issuance mechanism remains only conditional on the leverage being lower than the threshold $L_{min} = 50\%$ at the pre-fixed times k. An example of re-issuance is observable at time $k = 9$.

Fig. 8.3 displays the leverage distribution of a DCL with continuous observation as well as a comparison of the triple Gaussian fits (Eq. 8.11) for the leverage dynamics for a firm with no DCL, original (European) DCL, and (American) DCL. The leverage distribution is fitted with the probability density function of

$$f_T(t, \mathbf{a}, \mathbf{b}, \mathbf{c}) = \sum_{i=1}^{3} \frac{a_i}{\sqrt{2\pi c_i^2}} \exp\left(-\frac{1}{2}\left(\frac{t - b_i}{c_i}\right)^2\right) \tag{8.11}$$

where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are three component real-valued vectors (triple Gaussian). The calibrated parameters are listed in Table 8.2.

Fit Parameters	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3
without DCL	0.3078	0.6359	0.0563	0.5134	0.6652	0.7409	0.0417	0.0989	0.0487
with European DCL	0.1102	0.4816	0.4082	0.5032	0.551	0.713	0.0246	0.0653	0.0591
With American DCL	0.4795	0.3866	0.1339	0.5156	0.6591	0.7430	0.0478	0.0609	0.0303

Table 8.2: Calibrated parameters for the triple Gaussian for a firm with and without DCL (European and American-like, respectively) in its capital structure.

The effect of implementing this trigger to dictate the conversion is non-negligible, as it prevents, most of the time, the firm from operating at leverage above L_c. The only remaining case where L_t could exceed L_c is by having leverage increase again above L_c after a conversion over the short period of time $[\tau_k, T_k]$. In this scenario, the right tail of the debt-to-asset distribution is truncated.

From the upper simulation seen in Fig. 8.3, we can draw the empirical and theoretical probabilities that the leverage values are below L_{min}, above L_c, or between the values L_{min} and L_c for the three capital structures considered, respectively. The DCL instrument acts as a protective barrier and fulfils its role by providing the firm with a source of ready cash to draw upon in times of financial need, ensuring that the firm can remain liquid and solvent in the long term. This phenomenon is enhanced by an American observation of leverage.

	Empirical			Theoretical		
Probabilities	**Am DCL**	**EU DCL**	**No DCL**	**Am DCL**	**EU DCL**	**No DCL**
$\mathbb{P}(L_t < L_{min})$	17.9%	15.30%	14.34%	18%	15.42%	14.5%
$\mathbb{P}(L_t > L_c)$	**0.6%**	**2.85%**	**6.31%**	**0.8%**	**2.88%**	**6.13%**
$\mathbb{P}(L_{min} \leq L_t \leq L_c)$	81.5%	81.85%	79.35%	81.2%	81.7%	79.37%

Table 8.3: Empirical and theoretical probabilities for the leverage values satisfying different inequalities. Three scenarios are considered: a firm with European (EU) DCL, American (Am) DCL, and without DCL in its capital structure.

A counter-intuitive situation has been observed to arise in the case of continuous observation that is not represented in discrete-time modelling, wherein an increase of f increases the probability of being above L_c (Table 8.4). The explanation for $\frac{\partial P(L_t > L_c)}{\partial f}|_{L_c = cst} > 0$ for the continuous conversion model and $\frac{\partial P(L_t > L_c)}{\partial f}|_{L_c = cst} < 0$ for the discrete conversion model is that with the higher the frequency, a lower payment amount $P_N(T_k)$ results at each period k, leading to lower cash (or equity) payments, which does not reduce the leverage as much as when a smaller f is used (Eq. 4.1). The threshold is then infringed many times, but the duration of each breach is relatively short. However, for a given frequency, the conversion probability has been seen to remain lower in the case of the American observation due to faster loss recognition and improved responsiveness to the conversion decision.

	Empirical Probabilities			
	Continuous observations		**Discrete** observations	
Payment Frequency	$P(L_t < L_{min})$	$P(L_t > L_c)$	$P(L_t < L_{min})$	$P(L_t > L_c)$
$f = 0.2$ **(Once per 5 years)**	39.8%	0.02%	18.5%	11.64%
$f = 1$ **(annually)**	21%	0.2%	17.4%	3.7%
$f = 12$ **(monthly)**	9.81%	1.8%	8.25%	2.7%

Table 8.4: Comparison of the empirical results for the continuous and discrete observations.

The dilutive effect of the DCL might be measured by looking at the expected number of shares at the time horizon $T = 10$ years. For each of the three frequencies f studied above, we look at the probability that at least one conversion occurs (formally, the probability of having $NS_{10} > 100.0$) and the probability that the number of shares exceeds 200, i.e., a dilution of a factor greater or equal to 2.

	Empirical Probabilities ($NS_0 = 100$)			
	Continuous observations		**Discrete observations**	
Payment Frequency	$P(NS_{10} > 100)$	$P(NS_{10} > 200)$	$P(NS_{10} > 100)$	$P(NS_{10} > 200)$
$f = 0.2$ **(Once per 5 years)**	56.04%	56.04%	7.81%	7.81%
$f = 1$ **(annually)**	40.12%	7.42%	22.04%	3%
$f = 12$ **(monthly)**	34.76%	2.1%	32.37%	1.81%

Table 8.5: Comparison of the DCL instrument's dilutive effects depending on the type of observation and payment frequency. The probabilities are obtained through a Monte Carlo simulation and refer to the likelihood of (a) having at least one conversion occurring over the DCL lifetime (10 years) and (b) having a dilution factor of at least 2.

As previously highlighted, a lower payment frequency f is mechanically increasing the payment amount P_N, and, by extension, for a given conversion price S_P, the number of new shares issued in case of conversion also increases. The initial shareholder interests are then protected by combining a high f and high S_P. Such a DCL design would raise the cost of debt for the firm as an increase in S_P (alternatively, a decrease in the conversion ratio C_r) results in increasing the market expectations in terms of yield.

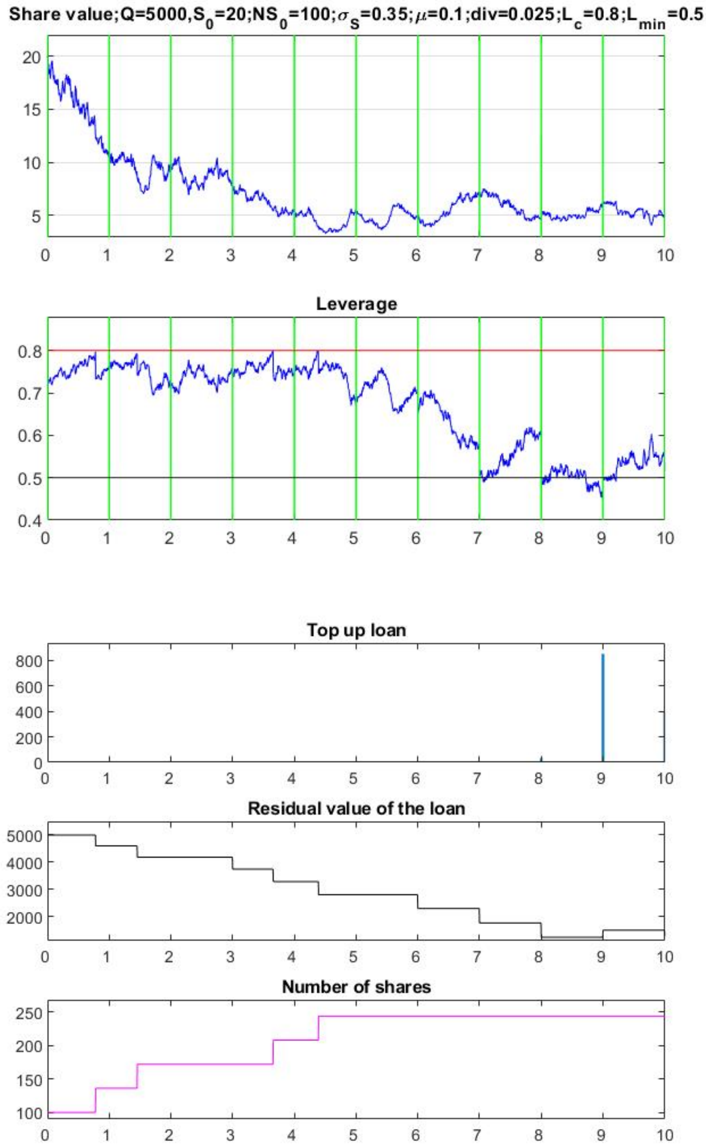


Figure 8.2: The upper stack of two plots presents the share price and the leverage levels, where the discrete times for the re-issuance and payments are shown in green. The lower stack of three plots presents, from top to bottom, the top-up loans, residual value of all outstanding loans, and number of shares.

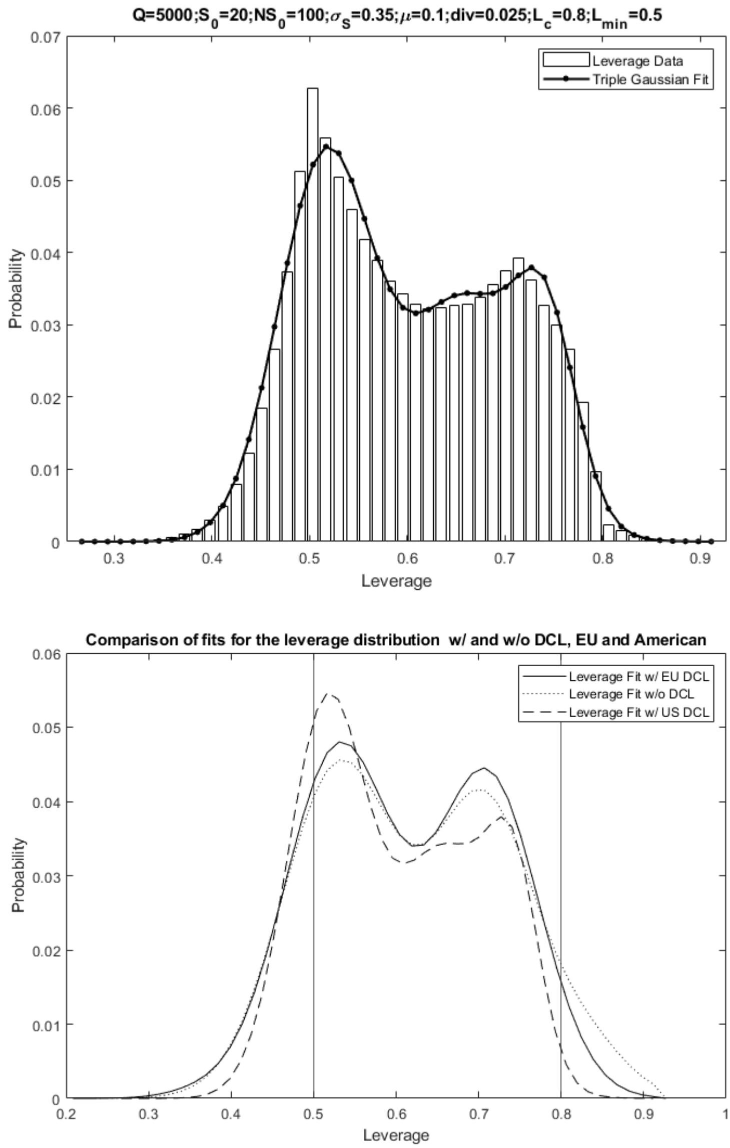


Figure 8.3: Upper plot: Distribution of the leverage for fixed values of $L_{min} = 50\%$ and $L_c = 80\%$ in the case of **continuous** observation and its probability distribution function fit through triple Gaussians. Lower plot: Three triple Gaussian fits for a firm with no DCL in its structure, an EU DCL, and American DCL, respectively. The input variables are listed in Table 8.1. $N_{experiments} = 5000$.

8.5 Conclusion

In *Design of a self-adaptive model for leverage* [56], a new type of contingent convertible was introduced, answering a call for additional flexibility, transparency, and automaticity in the CoCo design and triggering process. DCL works as a load-bearing instrument, such that in the eventuality of conversion, the debt is progressively turned to equity, reducing the leverage ratio of the issuing firm. The probability of default *PD* is an increasing function of the leverage *L*. By mechanically reducing this leverage, the probability of default also reduces. Because rating agencies rely on the probability of default to grade issuers, a lower *PD* level implies a better rating, and the issuing entity can then access capital at a lower cost (i.e., better funding).

One of the original model's drawbacks relates to the responsiveness to a breach in the conversion condition $L_k > L_c$, i.e., the time required to account for the loss recognition and to trigger the conversion of the next coupon into equity. This downside is due to the flexibility offered by this new instrument in terms of design and the payment frequency that acts at the same time as the leverage-observation frequency.

The objective of this chapter presented an alternative trigger for DCL that retains transparency, automaticity, and direct observability for all market participants. We proposed a continuous observation of leverage (an American-like mechanism) as an approach that allows, on the one hand, for the conversion to be activated before the cash payment date if necessary. On the other hand, it facilitates integration within the existing regulatory framework. This is significant in light of the initial concerns raised by the SEC in their 2019 No-Action Letter.

We simulated the leverage dynamics for the capital structures of three firms, having their debt in the form of (i) the original DCL, (ii) the American DCL (established in this scope), and (iii) regular debt. Beyond providing a liquidity buffer, DCL featuring an American observation of leverage, have been shown to reshape the firm's leverage dynamics by redistributing the right tail of its distribution onto healthier levels. This chapter extends the existing literature by providing a new option available to the firm's management when designing an appropriate DCL that can respond to the concerns of the initial shareholders, bond investors, and regulators, all of whom, despite their potentially conflicting interests, share the objective of averting bankruptcy until the capital buffer is fully depleted.

Chapter 9

Semi-analytical Framework for the Pricing of DCL Instruments

"All models have faults - that doesn't mean you can't use them as tools for making decisions."

Myron Scholes [107].

9.1 Abstract

This chapter provides a framework for the pricing of Dynamic Control of Leverage (DCL) instruments. DCL is a new type of hybrid debt working under a similar mechanism to traditional CoCo bonds that aims to better maintain a firm's leverage within pre-defined boundaries, ensuring a low probability of default. In its original form, the interest payment could switch from cash to equity upon a breach in the issuing firm's leverage ratio at the pre-fixed payment date. In Chapter 8, the continuous monitoring of the leverage was shown to improve the model's reactiveness and the use of the capital buffer. We rely on known exotic derivatives to replicate the cash flow delivered by this novel instrument and value two different design alternatives.

9.2 Introduction

DCL functions similarly to an annuity bond, where the interest payment can be paid in cash or converted into shares based on the issuer's debt-to-asset ratio. Our previous discussions demonstrated that this leverage requirement can be interpreted as a minimum share price requirement (Chapter 4). When the leverage falls below a certain threshold, sub-loans are issued bearing the same characteristics as those of the initial DCL. The resulting dynamics makes pricing these instruments feasible only in a semi-analytical form. Simulations thus become essential for evaluating the critical share price that triggers the conversion and the re-issuance threshold from term to term. This chapter builds on the two established DCL variants and lays the foundation for a pricing framework for these structures. In one variant, the interest payment can be converted if the firm's leverage breaches a certain limit, observable on the payment date. This mechanism resembles the European-like model [56]. In the other American-like model, conversion might occur at any time between two payment dates [69] (Chapter 8).

In this chapter, we provide a semi-analytical pricing framework for DCL instruments by relying on replication portfolios and (existing) exotic derivative options. We consider both design possibilities of European-like models (Section 9.3) and American-like models (Section 9.4). By replicating the cash flow delivered, Sections 9.3.1, 9.3.2, and 9.3.3 value the components in the European-like DCL. Section 9.3.4 combines the previous findings to form a "long-only" portfolio, replicating the EU DCL. Then, in Sections 9.4.1 and 9.4.2, the pricing of the American DCL is considered. Two scenarios are assumed where the converted shares are delivered at the next payment time (9.4.2.1) or at conversion time (9.4.2.2). The replication portfolio is then constructed in 9.4.3. Finally, we conclude with our findings in Section 9.5.

9.3 European mechanism

For the case of DCL bonds featuring a European observation mechanism, the conversion is triggered whenever the leverage exceeds the threshold L_c on a payment date. The price of this DCL can be replicated by a portfolio of financial instruments that reproduce the same payoff on a payment date. This portfolio could be simplified and, under specific conditions, paralleled with Merton's findings that a risky debt is made of the sum of a riskless bond and a short put option [24].

A DCL behaves similarly to hybrid debt but automatically converts the interest payment into equity upon a breach of the trigger requirement. Its value might be derived by discounting the different cash flow delivered. Also, the leverage-based triggering condition is converted into a condition on the share price that must remain above an equivalent threshold at payment times to ensure a cash payment. Assuming the original DCL has a maturity of T years, with f payments per year, amounting each to P_N, either paid in cash or converted into shares at a conversion price S_P, the cash flow from term to term (i.e., from issuance $k = 0$ to the first payment time $k = 1, \ldots$, from $k = Tf - 1$ to $k = Tf$), is the sum of the following three components [56]:

- (a) a Zero Coupon Bond (ZCB) with the face value equal to the interest payment value P_N and maturing on the next term T_k.
- (b) P_N times a Short Binary Put Option, which will cancel the ZCB payoff (P_N) in the eventuality of conversion. Indeed, upon conversion, the DCL-holder is entitled to receive an interest payment in shares and not in cash. The strike price is then set at the critical share price level ($S_{c,k}$).
- (c) $\frac{P_N}{S_P}$ Asset-or-Nothing Put Options, aiming to replicate the conversion to $\frac{P_N}{S_P}$ new shares, when the share price is below the triggering level $S_{c,k}$ at time T_k.

In Section 9.3.4, we demonstrate that in the specific case of $S_P = S_{c,k}$, i.e., the conversion price S_P being equal to the triggering share price, the portfolio could be simplified to only two sets of binary options, a long cash-or-nothing call and a long asset-or-nothing put. With this approach, replication can be achieved through **long-only** financial instruments.

We summarize the focus of Section 9.3 in Fig. 9.1.

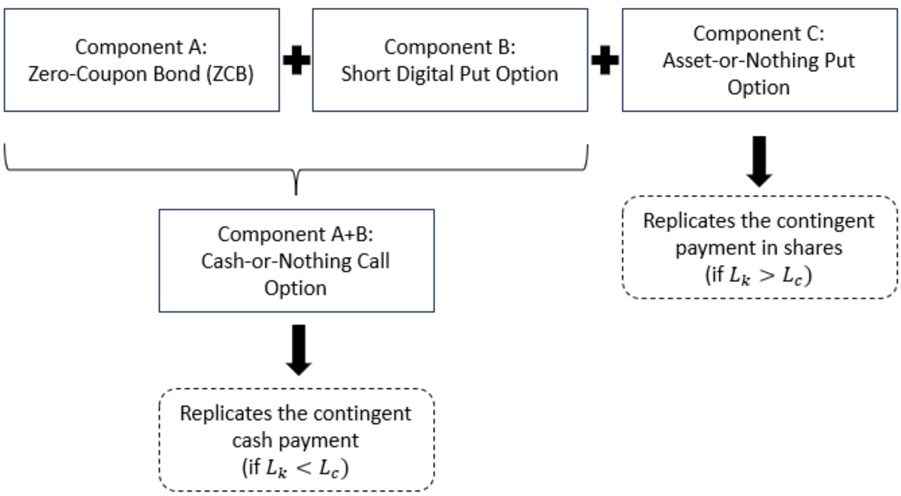


Figure 9.1: Construction of the portfolio replicating the European-like DCL mechanism.

9.3.1 Component A: Zero-Coupon Bond (ZCB)

The ZCB replicates the cash payment that would occur on a payment date of k. The bond nominal equals the interest payment P_N. The price today (T_{k-1}) of component A is obtained by discounting its payoff at the risk-free rate r by

$$P_A(T_{k-1}) = P_N \exp\left(-r(T_k - T_{k-1})\right) \tag{9.1}$$

9.3.2 Component B: Short Binary Put Option

The binary put option's role is to offset for the eventuality of a conversion occurring at T_k. In case of conversion, the DCL does not pay a cash coupon P_N, so component A must be cancelled. The coupon cancellation is proxied by the sale of P_N binary put options, paying each one unit of currency if the value of the share drops below the conversion threshold $S_{c,k}$ at time T_k (the option strike), zero otherwise. Hence, the payoff is represented as

$$P_{B,k}(T_k) = -P_N \mathbb{1}_{\{S_{T_k} < S_{c,k}\}} \tag{9.2}$$

By taking the expected value, we derive the price of component B at time T_{k-1} as

$$\begin{aligned} P_{B,k}(T_{k-1}) &= \mathbb{E}_t[P_{B,k}(T_k)] \exp\left(-r(T_k - T_{k-1})\right) \\ &= -P_N \exp\left(-r(T_k - T_{k-1})\right)\Phi(-d_2) \end{aligned} \tag{9.3}$$

where Φ stands for the standard normal cumulative distribution and d_2 is defined as

$$d_2 = \frac{\log\left(\frac{S_{T_{k-1}}}{S_{c,k}}\right) + \left(r - \delta - \frac{\sigma^2}{2}\right)(T_k - T_{k-1})}{\sigma\sqrt{T_k - T_{k-1}}} \tag{9.4}$$

with r representing the risk-free rate, δ the annual dividend yield, and σ the annual volatility of returns. Also, the inequality in leverage $L_k < L_c$ is translated into the following inequality on the share price

$$S_{T_k} \geq \left(\frac{1 - L_c}{L_c}\right) \frac{RV_k}{N^s_{k-1}} = S_{c,k}$$

requiring knowledge of the residual loan value RV_k, number of outstanding shares N^s_{k-1}, and critical leverage triggering the conversion L_c.

9.3.3 Component C: Asset-or-Nothing Put Option

The third instrument in the replication portfolio is an Asset-or-Nothing put option with the role to pay out one unit of equity if, at maturity (T_k), the share value (S_{T_k})

is valued below the option strike $S_{c,k}$. The investor is entitled to receive $\frac{P_N}{S_P}$ shares upon conversion (i.e. the conversion ratio C_r). The payoff is then defined as

$$P_{C,k}(T_k) = C_r S_{T_k} \mathbb{1}_{\{S_{T_k} < S_{c,k}\}} \tag{9.5}$$

The price of the Asset-or-Nothing put option is then obtained by discounting the expected value of $P_{C,k}(T_k)$ at the present time T_{k-1} as

$$\begin{aligned} P_{C,k}(T_{k-1}) &= \mathbb{E}_t\left[P_{C,k}(T_k)\right]\exp\left(-r(T_k - T_{k-1})\right) \\ &= C_r\mathbb{E}_t\left[S_{T_k}\mathbb{1}_{\{S_{T_k} < S_{c,k}\}}\right]\exp\left(-r(T_k - T_{k-1})\right) \\ &= C_r\left[\exp\left((r-\delta)(T_k - T_{k-1})\right)S_{T_{k-1}}\Phi(-d_1)\right]\exp\left(-r(T_k - T_{k-1})\right) \\ &= C_r S_{T_{k-1}}\exp\left(-\delta(T_k - T_{k-1})\right)\Phi(-d_1) \end{aligned} \tag{9.6}$$

with $d_1 = d_2 + \sigma\sqrt{T_k - T_{k-1}}$.

9.3.4 Simplification of the portfolio

The replication portfolio for a European-like DCL is separated into three known financial instruments. One possible price for the European DCL is given in a semi-analytical form, where knowledge of $S_{c,k}$ is required at the prior time T_{k-1}. By summing the sub-components A, B, and C from term to term, we obtain the value of the CoCo bond after one complete simulation (denoted by the the superindex label ^{ex}):

$$\begin{aligned} DCL_{EU}^{ex}(t,\mathbf{T}) &= \sum_{k=1}^{Tf} P_A(T_k) + P_{B,k}(T_k) + P_{C,k}(T_k) \\ &= \sum_{k=1}^{Tf} P_N\exp\left(-r(T_k - T_{k-1})\right) - P_N\exp\left(-r(T_k - T_{k-1})\right)\Phi(-d_2) \\ &\quad + C_r\left[\exp\left((r-\delta)(T_k - T_{k-1})\right)S_{T_{k-1}}\Phi(-d_1)\right] \end{aligned} \tag{9.7}$$

If, in the bond covenant, the conversion price S_P is not pre-fixed but set equal to the triggering share price $S_{c,k}$, then the replication portfolio can be simplified. The pre-condition for that to hold is

$$S_P = S_{c,k}\ ,\forall k \tag{9.8}$$

By using $P_N = C_r S_P$, the DCL instrument turns to the sum across all payment dates of a riskless bond maturing at T_k and C_r short put options on the stock, with strike S_P. Denoting $h = T_k - T_{k-1}$, this can be demonstrated below:

$$\begin{aligned} DCL^{ex}_{EU}\Big|_{S_P=S_{c,k}}(t,\mathbf{T}) &= \sum_{k=1}^{Tf} P_N \exp(-rh) \\ &- \left[P_N \exp(-rh)\Phi(-d_2) - C_r S_{T_{k-1}} \exp(-\delta h)\Phi(-d_1)\right] \\ &= \sum_{k=1}^{Tf} P_N \exp(-rh) \\ &- C_r \left[S_P \exp(-rh)\Phi(-d_2) - S_{T_{k-1}} \exp(-\delta h)\Phi(-d_1)\right] \\ &= \sum_{k=1}^{Tf} P_N D(t,T_k) - C_r \mathrm{Put}(S_{T_{k-1}}, S_P, h) \end{aligned} \tag{9.9}$$

As d_1 and d_2 are functions of $S_{c,k}$, we emphasise the $S_{c,k} = S_P$ requirement for this simplification to hold. A direct consequence of this simplification is the possible parallel with Merton's findings [24] on the valuation of a risky debt, equal to a risk-free bond and a short put option on the firm's assets.

Alternatively, a "long-only" approach exists for the replication portfolio expressed in Eq. 9.7 by noticing that

$$P_A(T_{k-1}) + P_B(T_{k-1}) = P_N \exp(-rh)\Phi(d_2) \tag{9.10}$$

which is the Black and Scholes valuation of a cash-or-nothing call. This remains consistent with the payoff of a European DCL as the cash-or-nothing call models the eventual coupon payment at the payment date T_k (instead of accounting for the possible coupon cancellation with the sum of a ZCB and short binary put). This simplification, leading to the following valuation, allows for using long instruments only, which could be a convenient choice for some investors who may be limited in their ability to engage in short selling. In such a case, the DCL price is expressed as the sum of Eq. 9.6 and Eq. 9.10.

$$DCL^{ex}_{EU}(t,\mathbf{T}) = \sum_{k=1}^{Tf} P_N \exp(-rh)\Phi(d_2) + C_r S_{T_{k-1}} \exp(-\delta h)\Phi(-d_1) \tag{9.11}$$

Then, the expected price of this European-like DCL is approximated by

$$DCL_{EU}(t,\mathbf{T}) = \frac{1}{N_{ex}} \sum_{i=1}^{N_{ex}} DCL^i(t,\mathbf{T}) \tag{9.12}$$

where N_{ex} is the number of simulations.

The sum on the right-hand side of Eq. 9.12 is to be understood as the average over a large number of simulations of the $DCL_{EU}(t,\mathbf{T})$ value.

9.4 American mechanism

In the DCL variant featuring a continuous observation of the leverage (American-like), the pricing of DCL becomes similar to that of traditional CoCo bonds, with some adjustments. Upon conversion, the DCL limits the conversion to equity only to the next interest payment (in opposition to the full nominal value of existing CoCo bonds). This section introduces the required replication portfolio components and their payoff. Then, we investigate the differences with the Equity Derivatives Model (EDM) used for pricing regular CoCos [31].

As exhibited in Chap. 8, the payoff Π_k at a payment time T_k for a DCL featuring a continuous observation (American-like) is represented as

$$\Pi_k = P_N(T_k)\mathbb{1}_{\left\{\min_{[T_{k-1},T_k]} S_t > S_{c,k}\right\}} + C_r S_{T_k}\mathbb{1}_{\left\{\min_{[T_{k-1},T_k]} S_t \leq S_{c,k}\right\}} \tag{9.13}$$

Here, the conversion ratio is set to $C_r = \frac{P_N(T_k)}{S_P}$ shares.

To replicate the cash flow from an American-like DCL, where the interest payment method is path-dependent, we rely on two elements:

(a) a strip of P_N long down-and-out digital call options, each with a delayed conditional knock-out period (protective barrier) running from the issuance to the payment date T_{k-1}, and maturing at the following payment date T_k. The down-and-out digital option pays one unit of currency if the barrier is not hit during its monitoring period and replicates the conditional coupon payment.

(b) a strip of C_r down-and-in digital shares, with the same delayed barrier monitoring period. The down-and-in digital shares pay out one unit of the underlying equity if the share price hits the barrier during its monitoring period. It replicates the contingent delivery of shares in place of the cash interest payment.

The instruments respectively represent the cash flow expected on every payment date. Compared to the instruments used for pricing DCLs featuring a European mechanism (Section 9.3), we now rely on exotic derivatives that are path-dependent due to the delayed knock-out/-in period. The need for such instruments (a) and (b) is explained below and illustrated in Fig. 9.2.

As the replication portfolio is purchased at inception time ($t = 0$), the knocking window prevents the entire strip of down-and-in digital shares (Component b) from converting into new shares upon a breach of the triggering condition. Similarly, it prevents coupon cancellation on the entire strip of down-and-out digital options. In other words, the knocking window on the option maturing at T_k allows for barrier monitoring only during the period $[T_{k-1}; T_k]$, which is a sub-period of the option lifetime $[0, T_k]$.

Now, the triggering condition is defined by

$$\min_{[T_{k-1};T_k]} S_t \leq S_{k,c} \tag{9.14}$$

i.e., the nature of the payment k at time T_k depends on the path followed by the share price S_t on the time interval $[T_{k-1};T_k]$. Furthermore, Eq. 9.13 involves a deferred delivery of converted shares to the next payment date. So, if Eq. 9.14 holds, then the conversion is acknowledged, but the shares are received at the payment date T_k.

Alternatively, the converted shares can be delivered immediately following a breach of the triggering price (at time $\tau_k \in [T_{k-1};T_k]$). Each share would then be valued S_{τ_k}, with $S_{\tau_k} \leq S_{c,k}$.

This American-like DCL instrument can still switch between fixed coupon cash payments and conversion into shares but is now conditional on the lowest level reached by the share price between two payment dates. Its value is the sum over the instrument lifetime of the following discounted expected payoffs [69] calculated as

$$\begin{aligned} DCL_{Am}(t,\mathbf{T}) = &\sum_{k=1}^{Tf} P_N(T_k)D(t,T_k)\mathbb{P}\left(\min_{[T_{k-1},T_k]} S_t > S_{c,k}\right) \\ &+ \sum_{k=1}^{Tf} C_r E_t\left[S_{T_k}\mathbb{1}\left(\min_{[T_{k-1},T_k]} S_t \leq S_{c,k}\right)\right] \end{aligned} \tag{9.15}$$

The non-deferred case, where new shares are received at time τ_k instead of T_k, is treated independently in Section 9.4.2.1.

The pricing of existing CoCo bonds using EDM slightly differs from the pricing of the American DCL. The EDM is the sum of (i) a corporate bond, (ii) a strip of down-and-in binary put options (to replicate the coupon cancellation), and (iii) a down-and-in forward contract (to replicate the issuance of new shares) [31]. In our case, we have the following two comparisons:

- Replication portfolio component (a) differs from (i) and (ii) because of the nature of DCL, paying down the debt into N_n equals payments, conversely to regular CoCos paying a coupon C and refunding the nominal value N at the maturity/call-date (if no conversion event). This involves the binary down-and-in put option being used for replicating the missed-coupon eventuality in the regular CoCo pricing that can no longer be used. Otherwise, the entire strip of binary puts would be knocked-in upon a single breach of the trigger requirement, cancelling all the DCL future cash interest payments. Instead, we use a long strip of down-and-out digital options featuring a protective barrier (where the knock-out period monitoring only runs from the payment $k-1$ to k).

- Replication portfolio component (b) differs from (iii) as the DCL limits the conversion to the current interest payment instead of the full nominal value, involving the issuance of C_r shares at a time. A similar protective period is required on the barrier monitoring, avoiding conversion to equity of all future payments.

This specific protection in the barrier monitoring, starting at a time t and ending at the option maturity T, is referred to in the literature as a *protected barrier option* [108], *Forward-Start (barrier) option* [109, 18], *American partial barrier option* [110], *rear-end time-dependent barrier option* [111], *type B*, or *Partial-Time-End Barrier* [112].

Fig. 9.2 illustrates the barrier monitoring timeline for the conversion.

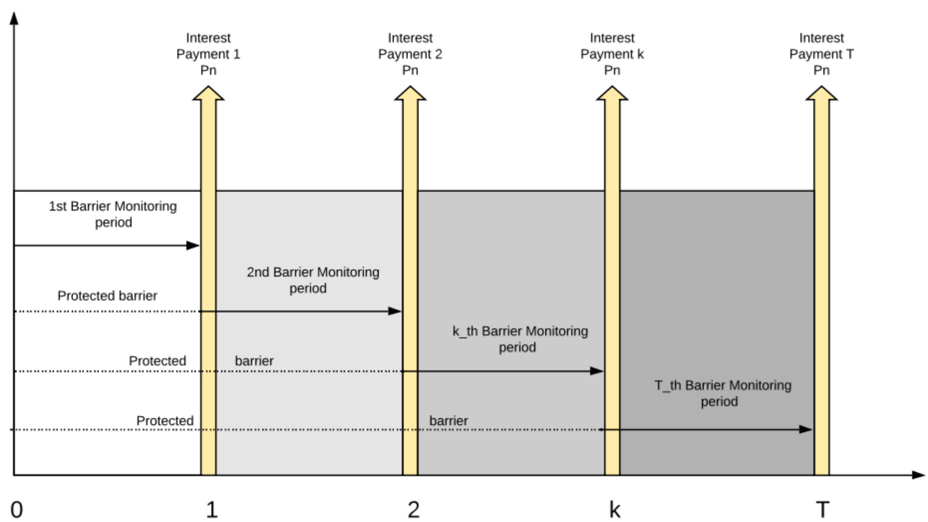


Figure 9.2: Representation of the working mechanism for a window barrier monitoring in the context of American DCL pricing. The replication portfolio is purchased at time $t=0$. The set of digital options and digital shares all have a barrier monitoring period stretching from a payment date T_{k-1} to their respective maturity T_k.

In the replication portfolio, each combination of options, i.e. down-and-**out** digital option and down-and-**in** digital share is maturing at T_k, with $k \in [1, T_N]$.

Then, each combination represents respectively the contingent cash payment and the contingent conversion to shares; both payment methods are not possible on the same payment date T_k. As observed on Fig. 9.2, upon a breach of the conversion condition, the protective barrier prevents the whole replication portfolio to be suddenly knocked-in or knocked-out.

We summarize the focus of Section 9.4 in Fig. 9.3.

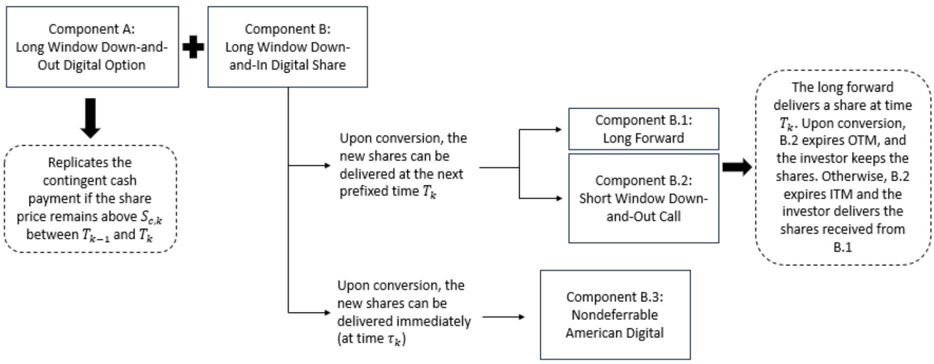


Figure 9.3: Construction of the portfolio replicating the American-like DCL mechanism, in case the shares are delivered at the next payment date T_k or immediately at time τ_k.

The next sections value the replication portfolio components (a) and (b).

9.4.1 Digital Window Barrier Option (DWBO)

As observed in Fig. 9.2, for each DWBO, the barrier is monitored only during a time window $h = 1/f$ running from $[T_{k-1}; T_k]$ over the DCL lifetime $[0; T_N]$. More specifically, the DWBO features an opening protection period (from T_0 to T_{k-1}). During this "protection" time, the DWBO cannot be knocked-in/-out.

By writing Pr_{DO} as the price of the digital window down and **out** call option, T_{k-1} the initial barrier monitoring time, τ_k the first hitting time given that $\tau_k \geq T_{k-1}$, i.e., $\tau_k = \inf(u|S_u \leq S_{c,k}, u \geq T_{k-1})$, S_t the share price at t, and $S_{c,k}$ the conversion threshold (barrier) that is valid between T_{k-1} and the time horizon T_k, we express the price at time $t = 0$ for the DWBO that delivers the cash payment P_N at time T_k upon the condition $\tau_k > T_k$ being verified as

$$Pr_{DO}(0, T_k) = P_N e^{-rT_k} \mathbb{1}_{\{\min_{t\in[T_{k-1},T_k]} S_t > S_{c,k}\}} = P_N e^{-rT_k} \mathbb{1}_{\{\tau_k > T_k\}} \tag{9.16}$$

We split the valuation into two components, starting by evaluating the likelihood of the event $\mathscr{E}_k = \{S_{T_k} > S_{c,k} \cap S_{T_{k-1}} > S_{c,k}\}$. A solution to $\mathbb{P}(\mathscr{E}_k)$ is provided[1] in [113] as the following

$$\mathbb{P}(\mathscr{E}_k) = N_2\left(\frac{\ln\left(\frac{S_{T_k}}{S_{c,k}}\right) + (r-\delta-0.5\sigma^2)T_k}{\sigma\sqrt{T_k}}; \frac{\ln\left(\frac{S_{T_k}}{S_{c,k}}\right) + (r-\delta-0.5\sigma^2)T_{k-1}}{\sigma\sqrt{T_{k-1}}}; \sqrt{\frac{T_{k-1}}{T_k}}\right) \tag{9.17}$$

where $N_2(y_1; y_2; \rho)$ is the bi-variate standard normal distribution function characterized by

$$N_2(y_1; y_2; \rho) = \int_{-\infty}^{y_1}\int_{-\infty}^{y_2} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{\frac{1}{2(1-\rho^2)}\left[z_1^2 - 2\rho z_1 z_2 + z_2^2\right]\right\} dz_1 dz_2 \tag{9.18}$$

Eq. 9.17 calculates the probability that the conversion threshold is not hit on the first observation date (T_{k-1}) or the last (T_k). In the context of an American-like DCL structure, the probability of receiving a cash payment at time T_k (i.e., the absence of conversion) is the probability of the event $\mathscr{E}_k$ less the probability to hit the $S_{c,k}$ threshold at any time between T_{k-1} and T_k. By identification from a rear-end down-and-out option equation[2] in [112] , it appears that

$$\begin{aligned} Pr_{DO}(0, T_k) = \; & P_N e^{-rT_k}\left[N_2\left(\frac{\ln\left(\frac{S_{T_k}}{S_{c,k}}\right) + gT_k}{\sigma\sqrt{T_k}}; \frac{\ln\left(\frac{S_{T_k}}{S_{c,k}}\right) + gT_{k-1}}{\sigma\sqrt{T_{k-1}}}; \sqrt{\frac{T_{k-1}}{T_k}}\right)\right. \\ & \left. - \left(\frac{S_{c,k}}{S_{T_k}}\right)^{2\mu} N_2\left(\frac{\ln\left(\frac{S_{c,k}}{S_{T_k}}\right) + gT_k}{\sigma\sqrt{T_k}}; -\frac{\ln\left(\frac{S_{c,k}}{S_{T_k}}\right) + gT_{k-1}}{\sigma\sqrt{T_{k-1}}}; -\sqrt{\frac{T_{k-1}}{T_k}}\right)\right] \end{aligned} \tag{9.19}$$

where $g = r - \delta - \frac{1}{2}\sigma^2$ and $\mu = \frac{g}{\sigma^2}$.

$Pr_{DO}(0, T_k)$ is the price at time $t = 0$ of a future cash payment (P_N) at time T_k, conditional on the share price S_t remaining higher than $S_{c,k}$ during the barrier monitoring period $[T_{k-1}, T_k]$. Otherwise, the payment is zero.

---

[1] Eq. 41

[2] Eq. 4.62

9.4.2 Window down-and-in Digital Share

The continuous monitoring of the barrier turns the potential conversion into a path-dependent process. For regular CoCo pricing, the contingent shares are represented with a down-and-in synthetic forward. This contract is known to be the combination of a down-and-in short position in a put and a long position in a down-and-in call option, both maturing at T with a strike K and the barrier B [19]. As mentioned in Section 9.4.1, in the case of the American-like DCL, the option barrier should not be monitored over the entire option lifetime. To this effect, we consider digital shares to be more adequate.

We envision two conversion mechanisms described as the following options:
Option 1: The bondholder is entitled to receive C_r shares at the next pre-fixed payment time T_k if the share price S_t is lower than the critical value $S_{c,k}$ between times T_{k-1} and T_k, i.e., if $\min_{[T_{k-1}:T_k]} S_t \leq S_{c,k}$.
Option 2: The bondholder is entitled to receive C_r shares at time τ_k when the breach is observed.
It could be argued that by delaying the payment of shares to the next interest payment date, Option 1 limits panic sell movements while preventing further increases in the firm's leverage, by leaving time to the market to absorb the inflow of sell orders. Option 2 might appear closer to existing market practices in the field of contingent convertibles.

In the next two subsections, we consider the pricing of the window down-and-in digital share option in the context of these two conversion options.

9.4.2.1 Window Digital Share delivered at pre-fixed time

This approach enhances the issuer's restructuring capacity by postponing the issuance of new shares and preemptively cancelling the coupon payment. It also lends flexibility to the pricing model by incorporating dividends, thereby challenging the $\delta = 0$ assumption, as explored in [49]. This assumption was initially rationalized by the consideration that an issuer facing conversion of its hybrid instrument would probably face constraints to dividend distribution.

If the shares are set to be delivered at the next pre-fixed payment time T_k for a conversion that occurred at a time $\tau_k \in [T_{k-1}, T_k]$, then the price of a Window Digital Share (WDS) is obtained by constructing a sub-portfolio made of the following components (see Fig. 9.3):

- A set of C_r **Long** forward contracts on the underlying shares, purchased at time $t = 0$ and maturing at time T_k with $k \in [1; Tf]$.

- A set of C_r **Short** Partial-Time-End Barrier, down-and-out calls (referred to as *type B2* in [112]), where the strike X is set as $X \to 0^+$ and the barrier B, such as $B_k = S_{c,k}$, for the k^{th} option.

If the conversion condition is not verified at the time horizon T_k, then the portfolio holder loses the right on the newly acquired shares because the short down-and-out call will expire in the money (ITM). Alternatively, if the conversion condition is met, then the payoff from the down-and-out call is 0, and the forward contract delivers the shares. The holder of this sub-portfolio is not entitled to any dividend until T_k (as it is built on forward contracts at time $t = 0$).

On the one hand, the value of the forward, discounted at the $t = 0$, is straightforward and written as

$$\begin{aligned} \text{FWD}(0, T_k) &= F(0, T_k) D(0, T_k) = S_0 \exp\left((r - \delta) T_k\right) \exp(-r T_k) \\ &= S_0 \exp(-\delta T_k) \end{aligned} \tag{9.20}$$

On the other hand, the valuation of the exotic option c_{doB2} delivering C_r shares at time T_k, contingent on the stock price not falling below $S_{c,k}$ between time T_{k-1} and T_k is derived[3] in [112], by setting the call strike $X \to 0^+$ as

$$\begin{aligned} & c_{doB2}(0, T_k) = \\ & C_r S_0 e^{(b-r)T_k} \left[N_2\left(\frac{\ln(S_0/S_{c,k}) + (b + \sigma^2/2) T_k}{\sigma\sqrt{T_k}}; \frac{\ln(S_0/S_{c,k}) + (b + \sigma^2/2) T_{k-1}}{\sigma\sqrt{T_{k-1}}}; \rho \right) \right. \\ & \left. - \left(\frac{S_{c,k}}{S_0} \right)^{2(\mu+1)} N_2\left(\frac{\ln(S_{c,k}/S_0) + (b + \sigma^2/2) T_k}{\sigma\sqrt{T_k}}; -\frac{\ln(S_{c,k}/S_0) + (b + \sigma^2/2) T_{k-1}}{\sigma\sqrt{T_{k-1}}}; -\rho \right) \right] \end{aligned} \tag{9.21}$$

where $\mu = \frac{b - \sigma^2/2}{\sigma^2}$, $b = r - \delta$, and $\rho = \sqrt{\frac{T_{k-1}}{T_k}}$.

By combining FWD and c_{doB2}, we conclude on the price of the WDS option (written $Pr_{DS}(0, k)$), delivering each C_r share at time T_k if the underlying equity

[3] Eq. 4.62

price is below $S_{c,k}$ on the time interval $[T_{k-1};T_k]$ to be

$$Pr_{DS}(0,T_k) = C_r S_0 \Bigg\{ \exp(-\delta T_k) \\ - \exp\left((b-r)T_k\right) \left[N_2\left(\frac{\ln\left(\frac{S_0}{S_{c,k}}\right) + (b+\sigma^2/2)T_k}{\sigma\sqrt{T_k}} ; \frac{\ln\left(\frac{S_0}{S_{c,k}}\right) + (b+\sigma^2/2)T_{k-1}}{\sigma\sqrt{T_{k-1}}} ; \rho \right) \right. \\ \left. - \left(\frac{S_{c,k}}{S_0}\right)^{2(\mu+1)} N_2\left(\frac{\ln\left(\frac{S_{c,k}}{S_0}\right) + (b+\sigma^2/2)T_k}{\sigma\sqrt{T_k}} ; - \frac{\ln\left(\frac{S_{c,k}}{S_0}\right) + (b+\sigma^2/2)T_{k-1}}{\sigma\sqrt{T_{k-1}}} ; -\rho \right) \right] \Bigg\} \tag{9.22}$$

$Pr_{DS}(0,T_k)$ is the price at time $t=0$ for an option delivering C_r shares at time T_k, conditional on the share price S_t trading below the conversion threshold $S_{c,k}$ at any time during the barrier monitoring window $[T_{k-1};T_k]$. Otherwise, the payoff is zero.

9.4.2.2 Window Digital Share delivered at hitting time

Considering now the option with immediate delivery of shares following conversion, it appears analytically impossible to construct a portfolio similarly to Section 9.4.2.1 by adjusting the forward contract $F(0,T_k)$ to $F(0,\tau_k)$, where $\tau_k \in [T_{k-1};T_k]$. The evaluation of the expected value

$$\mathbb{E}_t[S_0 \exp(-\delta\tau_k) \mid T_{k-1} \leq \tau_k \leq T_k]$$

is required, which appears only possible with numerical methods. Indeed, where the **first**-hitting time density function is known, it is not the case for $f(\tau_k)$, where τ_k is a random stopping time. Instead, we use an analytical approximation of the value delivered to the investor at hitting time by relying on Nondeferrable American Digitals (NAD).

Zhang introduced in [109] the price of a NAD as the present value of the rebate of a window out barrier option. The below instruments pays one unit of currency at the time τ an asset S hits a barrier H monitored between the time τ_1 and τ_e, with

$0 < \tau_1 < \tau_e \leq T$, such that

$$NAD(0,T_k) = \left(\frac{H}{S}\right)^{q_1} e^{-(r+\nu q_1 - \sigma^2 q_1^2/2)\tau_1} \left[N_2\left(D_1; -DD_1; \rho\right) + N_2(-D_1; DD_1; \rho)\right]$$
$$+ \left(\frac{H}{S}\right)^{q_{-1}} e^{-(r+\nu q_{-1} - \sigma^2 q_{-1}^2/2)\tau_1} \left[N_2\left(D_{-1}; -DD_{-1}; \rho\right) + N_2(-D_{-1}; DD_{-1}; \rho)\right] \tag{9.23}$$

where

$$\begin{aligned}
\upsilon &= r - q - \sigma^2/2 \\
D_\nu &= \frac{ln(S/H) + \upsilon\tau_1}{\sigma\sqrt{\tau_1}} - \sigma q_\nu \sqrt{\tau_1} \\
DD_\nu &= \frac{ln(S/H) + \upsilon(\tau_1 + \tau_e)}{\sigma\sqrt{\tau_1 + \tau_e}} - \sigma q_\nu \sqrt{\tau_1 + \tau_e} \\
\rho &= -\sqrt{\frac{\tau_1}{\tau_1 + \tau_e}} \\
\psi(r) &= \sqrt{\upsilon^2 + 2r\sigma^2} \\
q_\upsilon(r) &= \frac{\upsilon + \nu\psi(r)}{\sigma^2} \\
\nu &= 1 \text{ or } -1
\end{aligned}$$

In the context of an American-like DCL structure, we proceed with the following change in parameters:

- The barrier H is set equal to $S_{c,k}$.
- The initial monitoring time τ_1 is set equal to T_{k-1}.
- The final monitoring date τ_e is set equal to T_k.
- The number N_{NAD} of nondeferrable American digitals to be purchased is set equal to the expected payment in the case of conversion, i.e., the market value of C_r shares at the conversion time. Because the conversion occurs when the share price hits the threshold $S_{c,k}$, we have

$$N_{NAD}(k) = C_r S_{c,k} = \frac{P_N}{S_P} S_{c,k} \tag{9.24}$$

This approach for evaluating a WDS delivered at hitting time has a noticeable limitation for low payment frequencies f. If the payments are spaced in time, then the DCL is less effective (Chap. 8) and the probability of having the share price below its triggering level ($S_t < S_{c,k}$) at the beginning of a new monitoring period (e.g. T_{k-1}) increases.

In this scenario of an immediate conversion (on the first date T_{k-1} of a new monitoring period), the bondholders are entitled to C_r shares, each valued at $S_{T_{k-1}}$ with $S_{T_{k-1}} < S_{c,k}$. Yet, the above approximation that relies on NAD would lead to the payoff $C_r S_{c,k}$, which is higher than the market value of the C_r shares received by the investor.

9.4.3 Portfolio construction

The American-like DCL can now be valued through a replication portfolio, combining the components priced in the previous sections. As the value for $S_{c,k}$ is unknown until the time T_{k-1}, the semi-analytical equation approximates the hybrid instrument price.

Denoting $DCL^{ex}_{Am,1}(0, \mathbf{T})$ the first variant of the instrument, where new shares are delivered at the following pre-fixed payment date, one possible price is given by

$$DCL^{ex}_{Am,1}(0, \mathbf{T}) = \sum_{k=1}^{Tf} Pr_{DO}(0, T_k) + Pr_{DS}(0, T_k) \tag{9.25}$$

If the delivery of the shares is not deferred in time (Option 2), then one possible price for the American-like DCL, denoted $DCL^{ex}_{Am,2}$, is obtained according to section 9.4.2.2 instead of 9.4.2.1 and expressed as

$$DCL^{ex}_{Am,2}(0, \mathbf{T}) = \sum_{k=1}^{Tf} Pr_{DO}(0, T_k) + N_{NAD} \,.\, NAD(0, T_k) \tag{9.26}$$

For both designs of the American-like DCL, the pricing of the instrument is obtained by discounting its future cash flow over the instrument lifetime, i.e. the interest payments, either in cash or in shares, conditional on the share price S_t between two interest payment dates (T_{k-1} and T_k).

The expected price of the DCL can then be approximated by

$$\begin{aligned} DCL_{Am,1}(0, \mathbf{T}) &= \frac{1}{N_{ex}} \sum_{i=1}^{N_{ex}} DCL^{i}_{Am,1}(0, \mathbf{T}) \\ DCL_{Am,2}(0, \mathbf{T}) &= \frac{1}{N_{ex}} \sum_{i=1}^{N_{ex}} DCL^{i}_{Am,2}(0, \mathbf{T}) \end{aligned} \tag{9.27}$$

respectively for the share deliveries by Options 1 and 2, respectively, given a number of experiments N_{ex}.

9.5 Conclusion

When designing new financial instruments, pricing is an essential step for the issuers and potential investors. It delivers crucial information about the fair price, depending on the expected outcome. The possible cash flow are weighted by their occurrence likelihood and discounted to the present value.

In the case of the DCL instrument, the cash flow are replicated with known exotic derivative instruments. DCL was first introduced in [56] with a discretionary "European-like" mechanism. This payment behaviour is the sum of three strips of long ZCB, short digital put option, and long asset-or-nothing put option. A long-only alternative exists made of cash-or-nothing call options and asset-or-nothing put options.

Chapter 8 suggested the use of an alternative trigger to improve the DCL overall efficiency in keeping the firm at healthier levels of leverage. If the firm's debt-to-asset ratio is continuously observed, and the conversion can be triggered between two interest payment dates T_{k-1} and T_k, then the DCL is said to feature an American-like mechanism. The instrument valuation becomes more challenging and involves window barrier options, with a so-called *protection period* disabling the barrier. The pricing relies on the joint normal distribution of the logarithms of stock prices on dates T_k and T_{k-1}.

This chapter covered two design possibilities where, upon triggering the conversion, the new shares can be delivered at the next payment date or immediately. On the one hand, the American-like DCL can be replicated with strips of long window down-and-out digital call options, long forwards, and short window down-and-out call options. On the other hand, when the conversion is immediately effective, the instrument is priced with a strip of long window down-and-out digital call options (representing the cash payments), while the cash flow of the window down-and-in digital shares is approximated by a strip of nondeferrable American digitals.

Bibliography

[1] Darrow, C. D. (2000) Monopoly: the property trading board game. Ed. Millennium Edition. Hasbro / Parker.

[2] United States. Congress. (1984) *Inquiry Into Continental Illinois Corp. and Continental Illinois National Bank : Hearings Before the Subcommittee on Financial Institutions Supervision, Regulation, and Insurance of the Committee on Banking, Finance, and Urban Affairs, House of Representatives*, Ninety-Eighth Congress, Second Session, September 18, 19 and October 4, 1984, Washington: U.S. Government Printing Office.

[3] Statistical Series. (2008) Gross Domestic Product 2007. Vol 93, Issue 17. ISSN: 1670-4665

[4] Glitnir. (2008) Condensed Consolidated Interim Financial Statements. 30 June 2008.

[5] Landsbanki. (2008) Condensed Consolidated Interim Financial Statements. 1 January - 30 June 2008.

[6] Kaupthing. (2008) Condensed Consolidated Interim Financial Statements. 1 January - 30 June 2008.

[7] Fiordelisi, F., G. Pennacchi & O. Ricci (2019) Are Contingent Convertibles Going-Concern Capital? Journal of Financial Intermediation.

[8] Minsky, Hyman P. (1992) : The Financial Instability Hypothesis, Working Paper, No. 74, Levy Economics Institute of Bard College, Annandale-on-Hudson, NY

[9] Basel III: Finalising post-crisis reforms, Basel Committee of Banking Supervision, Dec. 2017

[10] Pennacchi, G., T. Vermaelen, and C. Wolff, Contingent Capital: The Case of COERCs, Journal of Financial and Quantitative Analysis, 49(3), pp541-574, 2014

[11] Raviv, A. Bank Stability and Market Discipline: Debt for Equity Swap versus Subordinated Notes, The Hebrew University Business School, Working Paper 2004

[12] P. Veronesi, Fixed Income Securities, John Wiley & Sons, 2010

[13] Saa Popovi and Ana Mugoa, Pricing Contingent Convertible Bonds An Empirical Approach, MIC 2017, Monastier di Treviso, Italy

[14] Glasserman, P. and E. C. Perotti. The Unconvertible CoCo Bonds. In D. D. Evanoff, A. Leonello and S. Manganelli (Eds.), Achieving Financial Stability : Challenges to Prudential Regulation (pp. 317-329). World Scientific (2018).

[15] Avdjiev, S., A. Kartasheva & B. Bogdanova. Coco's: A Primer. BIS Quarterly Review, 43-56. 2013

[16] Segal, M., and Ólafsson, S. Make CoCo Great Again: Designing Capital-Ratio Triggers for Contingent Convertibles. Working paper, 2019.

[17] Glasserman, P. and B. Nouri., Contingent Capital with a Capital-Ratio Trigger, Management Science, Vol. 58, No. 10, Published on line 27 Apr 2012

[18] Chin, E., D. Nel and S. Ólafsson. (2017), Problems and Solutions in Mathematical Finance, Volume II: Equity Derivatives, Wiley.

[19] Hull, J. C., Options, Futures, and Other Derivatives. Global Edition, 9th Ed. Pearson, 2018

[20] Pennacchi, G. & A., Tchistyi. (2019). On Equilibrium When Contingent Capital Has a Market Trigger: A Correction to Sundaresan and Wang Journal of Finance (2015). The Journal of Finance. 74. 10.1111/jofi.12762.

[21] Wildau, G and Y. Jia. "China banks face huge capital hole as stimulus spurs lending". March 19, 2019. The Financial Times.

[22] Kolchin, K. & J. Podziemska. 2019. Capital Markets Fact Book. SIFMA Research, Securities Industry and Financial Markets Association.

[23] Brennan, M. J. & E. S. Schwartz (1978) Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure. *The Journal of Business,* 51**,** 103-114.

[24] Merton, R. C. (1974) On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance,* 29**,** 449-470.

[25] Bundgaard, J. 2016. *Hybrid Financial Instruments in International Tax Law.* Kluwer Law International.

[26] Chan, S. (2017). Wake Me Up Before You CoCo: Implications of contingent convertible capital for financial regulation.

[27] Modigliani, F. & M. H. Miller (1958) The Cost of Capital, Corporation Finance and the Theory of Investment. *The American Economic Review,* 48**,** 261-297.

[28] Liberadzki, Marcin and Kamil Liberadzki. }}Contingent Convertible Bonds, Corporate Hybrid Securities and Preferred Shares - Instruments, Regulation, Management.ε *Palgrave Macmillan*, Cham : 2019. DOI: 10.1007/978-3-319-92501-1. ISBN: 978-3-319-92500-4.

[29] Spiegeleer, J. D., W Schoutens and C. Van Hulle, The Handbook of Hybrid Securities, Concertible Bonds, CoCo Bonds and Bail-In, Wiley 2014

[30] Kiewiet, Gera and Iman van Lelyveld, Sweder van Wijnbergen. "Contingent convertibles: Can the market handle them?." *DNB Working Papers 572, Netherlands Central Bank, Research Department*. September 2017. doi: 10.2139/ssrn.3047123

[31] Erismann, M. 2015. Pricing Contingent Convertible Capital - A Theoretical and Empirical Analysis of Selected Pricing Models. In School of Management, Economics, Law, Social Sciences and International Affairs. University of St. Gallen.

[32] Pennacchi, G. (2010) A Structural Model of Contingent Bank Capital. *Federal Reserve Bank of Cleveland, Working Paper.*

[33] Brigo, D., J. Garcia & N. Pede (2013) CoCo Bonds Valuation with Equity- and Credit-Calibrated First Passage Structural Models. *Papers 1302.6629*, arXiv.org.

[34] Black, F. and J. C. Cox, Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, The Journal of Finance, Vol. XXXI, No. 2, May 1976

[35] Sundaresan, S. and Wang, Z. 2015. On the Design of Contingent Capital with a Market Trigger. The Journal of Finance, 70: 881-920. 2015. Access to the article

[36] Prescott, E. (2011) Contingent Capital: The Trigger Problem. *Federal Reserve Bank of Richmond Economic Quarterly,* 98.

[37] Glasserman, P. & N., Behzad. (2016). Market-Triggered Changes in Capital Structure: Equilibrium Price Dynamics. Econometrica. 84. 2113-2153. 10.3982/ECTA11206.

[38] Bulow, J. and Klemperer, P., Equity Recourse Notes: Creating Countercyclical Bank Capital. Econ J, 125: F131-F157. 2015. Access to the article

[39] Deutsche Bundesbank. 2018. Contingent convertible bonds: design, regulation, usefulness. In *Monthly Report*, 53-64.

[40] Haldane, A. G., Speech given at the American Economic Association, Denver, Colorado, 9 January 2011, Access to the speech transcript

[41] Nouri, B. 2012. Contingent Capital: Valuation and Risk Implications Under Alternative Conversion Mechanisms. In *Industrial Engineering and Operations Research*. Columbia University.

[42] Leland, H. E., Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, Journal of Finance, Volume 49, Issue 4 (Sep., 1994), 1213-1252.

[43] Leland, H. E. & K. B. Toft (1996) Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. *The Journal of Finance,* 51**,** 987-1019.

[44] Barucci, E. & L. Del Viva (2010). Dynamic Capital Structure and the Contingent Capital Option. *Annals of Finance***,** 9.

[45] Eisenbeis, Robert, Bailouts, Capital, or CoCos: Can Contingent Convertible Bonds Help Banks Cope with Financial Stress? (July 30, 2019). Cato Institute Policy Analysis, No. 877, 2019, Available at SSRN: https://ssrn.com/abstract=3501157

[46] Lee, C.-F., A. Lee & J. Lee. 2010. *Handbook of Quantitative Finance and Risk Management*.

[47] Flannery, Mark J. No pain, no gain? Effecting market discipline via reverse convertible debentures. Capital adequacy beyond Basel: Banking, securities, and insurance, pp. 171-196, 2005.

[48] Tanous, P.J. and Tobias, A. and New York Institute of Finance and Breeden, R.C. (1997). "Investment Gurus: A Road Map to Wealth from the World's Best Money Managers". Robert Miller. New York Institute of Finance. ISBN:9780132607209.

[49] Spiegeleer, Jan & Schoutens, Wim. (2012). Pricing Contingent Convertibles: A Derivatives Approach. The Journal of Derivatives, 20(2), 2736

[50] Albul, Boris, Jaffee, Dwight M. and Tchistyi, Alexei, Contingent Convertible Bonds and Capital Structure Decisions December 14, 2015. http://dx.doi.org/10.2139/ssrn.2772612

[51] Chin, E., D. Nel & S. Ólafsson. (2014). "Problems and Solutions in Mathematical Finance: Stochastic calculus." Volume 1, John Wiley & Sons.

[52] Arion Bank hf., 2020. Information Memorandum. Issue of Fixed Rate Reset Perpetual AT1 Convertible Notes. Link to document

[53] Arion Bank hf., 2021. Consolidated Financial Statements 2020. Link to document

[54] McDonald, Robert L. (2012). Derivatives Markets. 3rd ed. Upper Saddle River, NJ: Pearson.

[55] United States. Congress. (1947) *Direct Purchases of Government Securities By Federal Reserve Banks*, Eightieth Congress, First Session, March 3, 4 and 5, 1947, Washington: U.S. Government Printing Office.

[56] M. Segal and S. Ólafsson, Design of a self-adaptive model for leverage. Finance Research Letters, Volume 54, June 2023. doi: https://-doi.org/10.1016/j.frl.2023.103721.

[57] Ingersoll, J. E. A contingent-Claims Valuation of Convertible Securities, Journal of Financial Economics 4, pp289-322, 1977

[58] Flannery, Mark J. Stabilizing Large Financial Institutions with Contingent Capital Certificates, Quarterly Journal of FinanceVol. 06, No. 02, 1650006 (2016)

[59] Duffie, D., Contractual methods for out-of-court restructuring of systemically important financial institutions, Submission requested by the US Treasury Working Group on Bank Capital, 9 November 2009.

[60] Flannery, M., Market-Valued Triggers Will Work for Contingent Capital Instruments, Solicited Submission to U.S. Treasury Working Group on Bank Capital, November 2009.

[61] Mcdonald, R., Contingent Capital with a Dual Price Trigger. Journal of Financial Stability. 9. 2010.

[62] Calomiris, C.W. and Herring, R.J., How to Design a Contingent Convertible Debt Requirement That Helps Solve Our Too-Big-to-Fail Problem*. Journal of Applied Corporate Finance, 25: 39-62. 2013.

[63] Flannery, M., Contingent Capital Instruments for Large Financial Institutions: A Review of the Literature. Annual Review of Financial Economics 2014 6:1, 225-240. 2014.

[64] Gleason, K., Bright, S., Martinez, F., & Taylor, C. (2017). Europe's CoCos provide a lesson on uncertainty. OFR WP, 17-02.

[65] R. Goncharenko, "Fighting Fire with Gasoline: CoCos in Lieu of Equity". Journal of Money, Credit and Banking, Volume 54, Issue 2-3, March-April 2022.

[66] The Economist. "Credit Suisse's takeover causes turmoil in a $275bn bond market". March 20, 2023. The Economist Group Limited. London.

[67] Fabozzi, F. J., Bond Markets, Analysis and Strategies 10th Ed., Pearson 2021

[68] R. Goncharenko, S. Ongena and A. Rauf, "The agency of CoCos: Why contingent convertible bonds are not for everyone". Journal of Financial Intermediation, Volume 48, October 2021.

[69] M. Segal and S. Ólafsson, "Overview of an alternative trigger for DCL". Finance Research Letters, Volume 58, Part A, December 2023. doi: https://-doi.org/10.1016/j.frl.2023.104281

[70] Powell, J., *Treasury Markets and the TMPG*, Remarks by Jerome H. Powell Member at TMPG Best Practices @ 10: A Look Back and a Look Ahead Federal Reserve Bank of New York. October 5, 2017. Access to the speech transcript

[71] European Central Bank, Banking Supervision. "Methodology Booklet 2016 edition." *SSM SREP*, 2016.

[72] Sweden. "2017 Budget Bill Reform and Financing Table. Impact on general government net lending." *Government Offices of Sweden*. September 2017. Last modified September 20$^{\text{th}}$, 2016. Accessed January 2020. Access to the table

[73] The Netherlands. "CoCos not tax deductible anymore as from 1 January 2019." *Government of the Netherlands website.* Last modified June 29th, 2018. Accessed January 2020. Access to the article

[74] Avdjiev, S., B. Bogdanova, P. Bolton, W. Jiang and A. Kartasheva. "CoCo issuance and bank fragility." Bank for International Settlements (BIS), No 678, November 2017.

[75] Niedrig, T., H. Gründl. (2015) : The effects of Contingent Convertible (CoCo) bonds on insurers' capital requirements under solvency II, SAFE Working Paper, No. 98, Goethe University Frankfurt, SAFE - Sustainable Architecture for Finance in Europe, Frankfurt a. M.

[76] European Union, European Parliament. "Solvency II, Title I, Chapter VI, Section 4 Solvency capital requirement, Article 101-104." *Official Journal of the European Union Directive 2009/138/EC*. Last Modified December 17, 2009. Accessed January 2020. Access to the regulation

[77] European Banking Authority (EBA). "EBA standardised templates for Additional Tier 1 (AT1) instruments - Final." *EBA reports*, October 10, 2016.

[78] European Banking Authority (EBA). "On the Monitoring of Additional Tier 1 (AT1) Instruments of European Union (EU) Institutions - Update". *EBA reports*. EBA/REP/2021/19. June 24, 2021. Access to the report

[79] European Central Bank, Banking Supervision. "Methodology Booklet 2018 edition." *SSM SREP*, 2018.

[80] Vossos, T. and J. Glover. "Santander Gives Bond Investors a Nasty Surprise." *Bloomberg*. Last modified February 12, 2019. Accessed February 2020. Access to the article

[81] European Union, European Parliament. "Regulation (EU) 2019/876 of the European Parliament and of the Council of 20 May 2019" *Official Journal of the European Union Directive 2019/876*. Last Modified June 27, 2020. Accessed June 2021. Access to the regulation

[82] Moody's Analytics. "EU Publishes Capital Requirements Regulation II in Official Journal". June 07, 2019. Access June 2021. Access to the electronic source

[83] Scope Ratings. "AT1 Quarterly: legacy instruments, infection risk and usability of capital buffers". *AT1 Quarterly*. November 24, 2020.

[84] Scope Ratings. "AT1 Quarterly: regulatory backdrop evolves on Covid-19 lessons, final Basel 3 push". *AT1 Quarterly*. March 31, 2021.

[85] Scope Ratings. "AT1 Quarterly: normalisation process begins; EBA stands firm on ESG capital instruments". *AT1 Quarterly*. July 2, 2021.

[86] Ben Slimane, M., Dany D. F. and Vivek M., "Facts and Fantasies about the Green Bond Premium". *Amundi Asset Management*. Working Paper 102-2020. December 2020.

[87] Squam Lake Group (2010). The Squam Lake Report: Fixing the Financial System. Princeton: Princeton University Press.

[88] Zeppa, S. L. and A. S. Kreider. "SAFEs and KISSes Poised to Be the Next Generation of Startup Financing" *The National Law Review*. Last modified May 6, 2015. Accessed July 2021. Access to the article

[89] Standard & Poors. "Hybrid Capital: Methodology And Assumptions." *S&P Global Ratings, RatingsDirect*, July 1, 2019.

[90] Moody's. Rating Symbols and Definitions. Moody's Investors Service. January 2011.

[91] Liberadzki, Kamil, and Marcin Liberadzki. "Hybrid Securities. Structuring, Pricing and Risk Assessment." *Palgrave Macmillan*, New York and Basingstoke, 2016. ISBN: 9781137589705.

[92] Standard & Poors. "Default, Transition, and Recovery: 2020 Annual Global Corporate Default And Rating Transition Study." *S&P Global Ratings, RatingsDirect*, April 7, 2021.

[93] Lloyds Banking Group (2009). Invitation by LBG Capital No.1 plc to Offer to Exchange the Existing Securities for Enhanced Capital Notes issued by LBG Capital No.1 plc or LBG Capital No.2 plc unconditionally and irrevocably guaranteed by Lloyds Banking Group plc or Lloyds TSB Bank plc or the relevant Exchange Consideration Amount payable in New Shares and/or cash and/or Additional Enhanced Capital Notes. Practical Law, Thomson Reuters. Accessed on March 14th, 2022. Link to Prospectus

[94] Scott, Hal S., Capital Study Report: Use of Market Discipline. Committee on Capital Markets Regulation, Macroeconomics: Monetary & Fiscal Policies eJournal. (January 10, 2014).

[95] Standard & Poors. "Default, Transition, and Recovery: 2011 Annual Global Corporate Default And Rating Transition Study." *S&P Global Ratings, RatingsDirect*, March 21, 2012.

[96] Standard & Poors. "Bank hybrid capital methodology and assumptions." *Global Credit Portal, RatingsDirect*, November 1, 2011.

[97] Pennacchi, G. & A., Tchistyi. (2019). Contingent Convertibles with Stock Price Triggers: The Case of Perpetuities. The Review of Financial Studies, Vol. 32, Issue 6, 23022340, June 2019. 10.1093/rfs/hhy092

[98] Pressler, J. (2015). "Michael Burry, Real-Life Market Genius From The Big Short, Thinks Another Financial Crisis Is Looming" New York Magazine. Last modified December 28, 2015. Accessed April 8, 2021. Access to the article

[99] Vasicek, O. (1977). "An equilibrium characterisation of the term structure." Journal of financial economics, 5(2), 177-188

[100] Uhlenbeck, G. E., & Ornstein, L. S. (1930). "On the theory of the Brownian motion. Physical review." 36(5), 823.

[101] Fergusson, K., & Platen, E. (2015). "Application of Maximum Likelihood Estimation to Stochastic Short Rate Models." Annals of Financial Economics, Vol.10, No. 02.

[102] Bernal, V. (2016). Calibration of the Vasicek Model: An Step by Step Guide. Access to the article

[103] Lewis Michael. 2011. Boomerang : Travels in the New Third World. 1st ed. New York: W.W. Norton.

[104] U.S. Securities and Exchange Commission. "Re: Eligibility of certain contingent convertible capital securities for an offering under Securities Act Rule 144A". (2019)

[105] Morgan, J.P. (1912) "J.P. Morgan's Testimony. The justification of Wall Street". Bank and Currency Committee of the House of Representatives at Washington, D. C. Appointed for the Purpose of Investigating an Alleged Monet Trust in "Wall Street.". December 18 and 19, 1912.

[106] Jung, HyeYoon, "Pricing of Contingent Convertibles" (2012). Wharton Research Scholars. 90.

[107] Fisher, L. M. (1986) "Options Proposals Criticized." The New York Times. Edition December 27, 1986, Section 1, Page 29. Last Accessed April 9, 2021. Access to the online article

[108] P. Carr (1995) Two extensions to barrier option valuation, Applied Mathematical Finance, 2:3, 173-209, DOI: 10.1080/13504869500000010

[109] Peter G Zhang, (1998), Exotic Options: A Guide to Second Generation Options, World Scientific Publishing Co. Pte. Ltd.

[110] Jun, D., Ku, H. Valuation of American partial barrier options. Rev Deriv Res 16, 167191 (2013). https://doi.org/10.1007/s11147-012-9081-1

[111] Hui, Cho-Hoi. (1997). Time-Dependent Barrier Option Values. Journal of Futures Markets. 17. 667688. 10.1002/(SICI)1096-9934(199709)17:63.0.CO;2-C.

[112] Haug, E.G. (2007) The Complete Guide To Option Pricing Formulas. 2nd Edition, McGraw-Hill, New York.

[113] Ingersoll, Jr., J. (2000). Digital Contracts: Simple Tools for Pricing Complex Derivatives. The Journal of Business, 73(1), 67-88.

Appendix A

Demonstration of Quanto Adjustement in Cross-currency Valuation

This appendix demonstrates Eqs. 3.10, 3.11, and 3.12, and is based on the work of [54] (Section 23.5) and [18] (Section 6.2.1).

A.1 Black-Scholes equation in the case of a cross-currency option

We start from the probability space $(\Omega, F, \mathbb{P})$ and the two $\mathbb{P}$-standard Wiener processes $\{W_t^{stock} : t \geq 0\}$ and $\{W_t^{FX} : t \geq 0\}$, such as $dW_t^{stock} . W_t^{FX} = \rho_{stock/FX} dt$ with

$$-1 \leq \rho_{stock/FX} \leq +1$$

.

Assuming S_t^d is the underlying share price denominated in its market currency (domestic), this process follows as

$$\frac{dS_t^d}{S_t^d} = (\mu^d - \delta^d)dt + \sigma_{stock} dW_t^{stock}$$

Similarly, the exchange rate $FX^{f/d}$ between the equity currency and an investment currency B following the process as

$$\frac{dFX_t^{f/d}}{FX_t^{f/d}} = \mu^{FX} dt + \sigma_{FX} dW_t^{FX}$$

In these two equations, we introduce the new variables μ^{stock} and μ^{FX} as the drifts followed by the processes S_t^d (the underlying equity price) and $FX_t^{f/d}$ (the foreign exchange conversion rate), respectively.

To demonstrate the stochastic differential equation applicable to cross-currency options $V(S_t^d, FX_t, t)$, we first eliminate the risks associated with the underlying and FX rates.

We create an immunization portfolio Π_t that includes a long position in the quanto option V, a long position in Δ_1 shares converted in the foreign investment currency, and a short position in Δ_2 units of $FX_{f/d}$. In other words, we have

$$\Pi_t = V(S_t^d, FX_t, t) + \Delta_1(S_t^d FX_t) - \Delta_2 FX_t \tag{A.1}$$

Accounting for both the dividends and the domestic risk-free rate, we evaluate the change in the portfolio Π_t between two times dt as

$$d\Pi_t = dV + \Delta_1(d(S_t^d FX_t) + \delta^d S_t^d FX_t dt) - \Delta_2(dFX_t + r^d FX_t dt)$$

Given $d(S_t^d FX_t) = S_t^d dFX_t + FX_t dS_t^d + dS_t^d dFX_t$ and using Ito's lemma to derive $dS_t^d dFX_t = \rho_{stock/FX}\sigma_{stock}\sigma_{FX} S_t^d FX_t$, we simplify $d\Pi_t$ to

$$\begin{aligned} d\Pi_t = dV + \Delta_1(S_t^d dFX_t + FX_t dS_t^d + \rho_{stock/FX}\sigma_{stock}\sigma_{FX} S_t^d FX_t dt \\ + \delta^d S_t^d FX_t dt) - \Delta_2(dFX_t + r^d FX_t dt) \end{aligned} \tag{A.2}$$

We now use Taylor's theorem to expand $V(S_t^d, FX_t, t)$ to

$$\begin{aligned} dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S_t^d}dS_t^d + \frac{\partial V}{\partial FX_t}dFX_t \\ + \frac{1}{2}\left[\frac{\partial^2 V}{\partial {S_t^d}^2}(dS_t^d)^2 + \frac{\partial^2 V}{\partial FX_t^2}(dFX_t)^2 + 2\frac{\partial^2 V}{\partial S_t^d \partial FX_t}(dS_t^d dFX_t)\right] + \ ... \end{aligned} \tag{A.3}$$

In Eq. A.3, we replace dS_t^d and dFX_t by their original definitions, i.e.. $dS_t^d = (\mu^d - \delta^d)S_t^d dt + \sigma_{stock} S_t^d dW_t^{stock}$ and $dFX_t = \mu^{FX} FX_t dt + \sigma_{FX} FX_t dW_t^{FX}$, respectively.

Then, it follows that

$$dV = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_{stock}^2 S_t^{d\,2} \frac{\partial^2 V}{\partial S_t^{d\,2}} + \frac{1}{2}\sigma_{FX}^2 FX_t^2 \frac{\partial^2 V}{\partial FX_t^2} + \rho_{stock/FX}\sigma_{stock}\sigma_{FX} S_t^d FX_t \frac{\partial^2 V}{\partial FX_t \partial S_t^d} \right.$$
$$\left. +(\mu_{stock} - \delta^d) S_t^d \frac{\partial V}{\partial S_t^d} + \mu_{FX} FX_t \frac{\partial V}{\partial FX_t} \right] dt + \sigma_{stock} S_t^d \frac{\partial V}{\partial S_t^d} dW_t^{stock} + \sigma_{FX} FX_t \frac{\partial V}{\partial FX_t} dW_t^{FX} \tag{A.4}$$

Inserting Eq. A.4 and rearranging in Eq. A.2 leads to

$$d\Pi_t = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_{stock}^2 S_t^{d\,2} \frac{\partial^2 V}{\partial S_t^{d\,2}} + \frac{1}{2}\sigma_{FX}^2 FX_t^2 \frac{\partial^2 V}{\partial FX_t^2} + \rho_{stock/FX}\sigma_{stock}\sigma_{FX} S_t^d FX_t \frac{\partial^2 V}{\partial FX_t \partial S_t^d} \right.$$
$$+(\mu_{stock} - \delta^d) S_t^d \frac{\partial V}{\partial S_t^d} + \mu_{FX} FX_t \frac{\partial V}{\partial FX_t} + \Delta_1(\mu_{FX} + \mu_{stock} + \rho_{stock/FX}\sigma_{stock}\sigma_{FX}) S_t^d FX_t$$
$$\left. -\Delta_2(\mu_{FX} + r^d) FX_t \right] dt + \sigma_{stock} \left(\frac{\partial V}{\partial S_t^d} + \Delta_1 FX_t \right) S_t^d dW_t^{stock}$$
$$+\sigma_{FX} \left(\frac{\partial V}{\partial FX_t} + \Delta_1 S_t^d - \Delta_2 \right) FX dW_t^{FX} \tag{A.5}$$

To eliminate the two Wiener process terms dW_t^{stock} and dW_t^{FX}, we set

$$\frac{\partial V}{\partial S_t^d} + \Delta_1 FX_t = 0$$
$$\Longleftrightarrow \Delta_1 = -\frac{1}{FX_t}\frac{\partial V}{\partial S_t^d} \tag{A.6}$$

We also set

$$\frac{\partial V}{\partial FX_t} + \Delta_1 S_t^d - \Delta_2 = 0$$
$$\Longleftrightarrow \Delta_2 = \frac{1}{FX_t}\left(FX_t \frac{\partial V}{\partial FX_t} - S_t^d \frac{\partial V}{\partial S_t^d} \right) \tag{A.7}$$

enabling the result of

$$d\Pi_t = \Bigg[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_{stock}^2 S_t^{d^2} \frac{\partial^2 V}{\partial S_t^{d^2}} + \frac{1}{2}\sigma_{FX}^2 FX_t^2 \frac{\partial^2 V}{\partial FX_t^2} + \rho_{stock/FX}\sigma_{stock}\sigma_{FX} S_t^d FX_t \frac{\partial^2 V}{\partial FX_t \partial S_t^d}$$
$$+ (r^d - \delta^d - \rho_{stock/FX}\sigma_{stock}\sigma_{FX}) S_t^d \frac{\partial V}{\partial S_t^d} + \mu_{FX} FX_t \frac{\partial V}{\partial FX_t} - r^d FX_t \frac{\partial V}{\partial FX_t} \Bigg] dt \tag{A.8}$$

Assuming Π_t is invested in the foreign numeraire on a risk-free account to prevent any arbitrage opportunities, the growth is expected to be

$$d\Pi_t = r^f \Pi_t dt = r^f \left[B_t^d, FX^{f/d}, t) + \Delta_1 (S_t^d FX_t) - \Delta_2 FX_t \right] dt$$
$$d\Pi_t = r^f \left[V(S_t^d, FX^{f/d}, t) - FX_t \frac{\partial V}{\partial FX_t} \right] dt \tag{A.9}$$

As justified above, the expressions of $d\Pi_t$ derived in Eqs. A.8 and A.9 should be set equal. After simplification, we obtain

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_{stock}^2 S_t^{d^2} \frac{\partial^2 V}{\partial S_t^{d^2}} + \frac{1}{2}\sigma_{FX}^2 FX^{f/d^2} \frac{\partial^2 V}{\partial FX^{f/d^2}} +$$
$$\rho_{stock/FX}\sigma_{stock}\sigma_{FX} S_t^d FX^{f/d} \frac{\partial^2 V}{\partial S_t^d \partial FX^{f/d}} +$$
$$(r^d - \delta^d - \rho_{stock/FX}\sigma_{stock}\sigma_{FX}) S_t^d \frac{\partial V}{\partial S_t^d} +$$
$$(r^f - r^d) FX^{f/d} \frac{\partial V}{\partial FX^{f/d}} - r^f V(S_t^d, FX^{f/d}, t) = 0$$

This is Eq. 3.10 as stated in Chapter 3.

A.2 Quanto-adjusted dividend

We now assume the result derived in the previous section and pursue an approach based on the partial differential equation to value the price of a European Quanto Put option in the risk-neutral world.

We first recall the payoff equation for a European Quanto Put as

$$\Psi(S^d(T), FX^{f/d}(T)) = \overline{FX^{f/d}} \, . \, \max(K^d - S^d(T), 0)$$

Writing $P(S_T^d, T) = \max(K^d - S^d(T), 0)$ allows to express the various terms in the following as

$$\begin{cases} \frac{\partial V}{\partial t} = \overline{FX^{f/d}} \frac{\partial P}{\partial t} \\ \frac{\partial V}{\partial S_t^d} = \overline{FX^{f/d}} \frac{\partial P}{\partial S_t^d} \\ \frac{\partial V}{\partial FX_t} = 0 \\ \frac{\partial^2 V}{\partial {S_t^d}^2} = \overline{FX^{f/d}} \frac{\partial^2 P}{\partial {S_t^d}^2} \\ \frac{\partial^2 V}{\partial FX_t^2} = 0 \\ \frac{\partial^2 V}{\partial {S_t^d}^2 \partial FX_t^2} = 0 \end{cases} \tag{A.10}$$

Inserting the set of Eq. A.10 into Eq. 3.10 simplifies to

$$\frac{\partial P}{\partial t} + \frac{1}{2}\sigma_{stock}^2 {S_t^d}^2 \frac{\partial^2 P}{\partial {S_t^d}^2}(S_t^d, t) + (r^d - \delta^d - \rho_{stock/FX}\sigma_{stock}\sigma_{FX}){S_t^d}^2 \frac{\partial P}{\partial {S_t^d}^2}(S_t^d, t) - r^f P(S_t^d, t) = 0 \tag{A.11}$$

We identify above a Black-Scholes equation with the parameters of

- Volatility σ_{stock}.
- Risk-free interest rate r^f.

The parameter related to the continuous dividend yield is obtained by setting $(r^d - \delta^d - \rho_{stock/FX}\sigma_{stock}\sigma_{FX})$ equal to $r^f - \delta^f$, which leads to

$$\delta^f = r^f - r^d + \delta^d + \rho_{stock/FX}\sigma_{stock}\sigma_{FX}$$

This expression is Eq. 3.12 as stated in Chapter 3.

A.3 Valuation of a European quanto put option

From the payoff expression $P(S_T^d, T)$, and by using Eq. A.11, the European put price at inception t follows

$$P(S_t^d, t) = K^d \exp(-r^f \,.\, (T-t))N(-d_2) - S_t^d \exp(-\delta^f \,.\, (T-t))N(-d_1) \tag{A.12}$$

that leads directly to the expression of a European quanto put option as

$$Put^f(K^d, t, T) = \overline{FX^{f/d}}\left[K^d \exp\left(-r^f \, . \, (T-t)\right)N\left(-d_2\right) - S_t^d \exp\left(-\delta^f \, . \, (T-t)\right)N\left(-d_1\right)\right]$$

with

$$d_1 = \frac{\log\left(\frac{S_0^d}{K^d}\right) + (r^f - \delta^f + 0.5\sigma^2_{stock})(T-t)}{\sigma_{stock}\sqrt{T-t}}$$

and

$$d_2 = d_1 - \sigma_{stock}\sqrt{T-t}$$

This is Eq. 3.11 as stated in Chapter 3.

Appendix B

Loan Payment and Residual Value

The scenario assumes a company that takes a loan with a nominal Q, which it pays down with N equal payments P_N. The interest rate the firm needs to pay on this debt is defined by r.

The loan balance is defined by RQ_t at time t. Then, at $t = 0$, we have $RQ_0 = Q$. At the first payment *observation*, the balance is accrued by the the interest r and reduced by the payment value P_N, such that

$$RQ_1 = Q + rQ - P_N = Q(1 + r) - P_N$$

Following the same process, at the second and third payment dates, the residual values of the loan, respectively, are

$$RQ_2 = RQ_1(1 + r) - P_N = Q(1 + r)^2 - P_N(1 + r) - P_N$$

and

$$RQ_3 = RQ2(1 + r) - P_N = Q(1 + r)^3 - P_N(1 + r)^2 - P_N(1 + r) - P_N$$

Deriving the general form at any discrete time $k \leq N$ is now possible. To do so, for $k = N$, we first observe that

$$RQ_k = Q(1 + r)^k - P_N \sum_{i=0}^{N-1} (1 + r)^i$$

Recognizing the general summation formula,[1] we simplify the equation as

[1] $\forall r \neq, \sum_{i=0}^{N} ar^k = a\frac{r^{N+1}-1}{r-1}$.

$$RQ_k = Q(1+r)^k - \frac{P_N}{r}\left[(1+r)^N - 1\right]$$

To derive the amount of each payment P_N, we consider the last payment event. As for $t = N$, the residual value of the loan falls to zero, so it follows that

$$Q(1+r)^N - \frac{P_N}{r}\left[(1+r)^N - 1\right] = 0$$

$$\Longleftrightarrow \quad P_N = \frac{rQ(1+r)^N}{(1+r)^N - 1} = \frac{rQ}{1 - \left(\frac{1}{1+r}\right)^N} \tag{B.1}$$

Finally, we simplify the residual value of the loan at any time k by replacing P_N. Therefore, we have

$$RQ_k = Q\left[(1+r)^k - \frac{(1+r)^k - 1}{1 - \left(\frac{1}{1+r}\right)^N}\right] \tag{B.2}$$

Appendix C

On the Design of DCL instruments: A hedging-based Experiment

From our initial observations regarding the current CoCo market, we understand that designing the DCL parameters appropriately prior to issuance is a key contributor to the success of this new financial product and will extensively contribute to growing the interest of investors. In this section, we provide guidelines on the conversion price S_p setup within the scope of a hedging experiment. The results help issuers design the instrument appropriately to balance the transfer of wealth at conversion time[1] as well as limit perverse incentives potentially existing for short-sellers if S_p is set too low.

We bound our study to a single DCL issuance, with a price that evolves independently from any sub-issuance it might have created, such as if the leverage L_k falls below L_{min} on a payment date k.

In Chapter 4, we saw that the price of such an instrument should be expressed as

$$P_{CoCo}(t,T) = \sum_{k=1}^{N_n} P_{Nn}(T_k) D(t,T_k) \mathbf{E}_t\left[\mathbb{1}_{\{S_k > S_{c,k}\}}\right] + \sum_{k=1}^{N_n} \left(\frac{P_{Nn}(T_k)}{S_p}\right) \mathbf{E}_t\left[S_k \mathbb{1}_{\{S_k \leq S_{c,k}\}}\right] D(t,T_k) \tag{C.1}$$

A simplified hedging experiment

Before considering a generalized hedging case applicable to DCL instruments, we scrutinize a scenario of a single 0-coupon DCL, maturing at the next period $k = T$.

[1] On every payment date k.

In this specific scheme, setting the DCL payment frequency to $f = T$ is equivalent to turning the down payment P_{N_n} into the entire face value Q.

The instrument valuation is simplified as

$$P^*_{CoCo}(t,T) = Q \,.\, D(t,T)\mathbf{E}_t\left[\mathbb{1}_{\{S_T>S_c\}}\right] + \left(\frac{Q}{S_p}\right)\mathbf{E}_t\left[S_T\mathbb{1}_{\{S_T\leq S_c\}}\right]D(t,T) \tag{C.2}$$

We next build a simple hedging portfolio Π^* by adding a Δ number of put options to the DCL. Specifically, the number of put options to purchase is set to

$$\Delta = \frac{Q}{S_p}$$

to perfectly hedge the downside (occurring in the case of conversion). Also, the put strike is set equal to the underlying share level that forces the conversion of the DCL into new equity, i.e., S_c.

From the well-known put payoff, the portfolio can be written as

$$\begin{aligned}\Pi^*(t,T) = Q \,.\, D(t,T)\mathbf{E}_t\left[\mathbb{1}_{\{S_T>S_c\}}\right] + \left(\frac{Q}{S_p}\right)\mathbf{E}_t\left[S_T\mathbb{1}_{\{S_T\leq S_c\}}\right]D(t,T) + \\ \Delta\mathbf{E}_t\left[(S_c - S_T)\mathbb{1}_{\{S_T\leq S_c\}}\right]D(t,T)\end{aligned} \tag{C.3}$$

We can simplify this expression to reveal the standard form of a corporate bond with

$$\Pi^*(t,T) = Q \,.\, D(t,T)\mathbf{E}_t\left[\mathbb{1}_{\{S_T>S_c\}}\right] + \left(\frac{Q}{S_p}\right) .\, S_c \,.\, \mathbf{E}_t\left[\mathbb{1}_{\{S_T\leq S_c\}}\right]D(t,T) \tag{C.4}$$

In case the share price falls below S_c at maturity, the value recovered by the investor is

$$RV = \frac{Q \,.\, S_c}{S_p}$$

, which leads to a recovery ratio of $R = RV/Q$.

The importance of setting appropriately the triggering price S_c, the conversion price S_p, and the put strike $K = S_c$ in our example is reflected in the following scenarios:

- If $S_c = S_p$, then the hedging portfolio becomes $\Pi^*(t,T) = Q \,.\, D(t,T)$, i.e., a risk-free governmental bond.

- If $S_c < S_p$, then the recovery ratio R is below 1 in the case of conversion ($S_T < S_c$), but still deterministic, which makes the position not variable on the final share price S_T.
- If $S_c > S_p$, then the recovery ratio R is above 1 in the case of conversion ($S_T < S_c$), but still deterministic, which makes the position not variable on the final share price S_T.

An alternative consists in moving the strike level for the hedging put options. Such a case is considered in Fig. C.2 in the general DCL hedging experiment, with the structure of either

- a bull spread appears if K is set lower than S_c, or
- a bear spread appears if K is set higher than S_c.

A standard use-case of such a demonstration applies on the markets, where the trading price of the structures listed above might differ from their theoretical $DCL + put$ value. Upon verifying this condition, a risk-free profit can be locked in under the general arbitrage hypotheses.

A generalised hedging experiment

Based on the price of a DCL in the general case (Eq. C.1), we now set a portfolio Π comprised of the above DCL instrument as well as a strip of Δ vanilla European put options[2]. The maturity is fixed to the next payment date k. The strike is adjusted to $S_{c,k}$, forcing the hedger to buy the strip of Δ puts at every payment date for the next one, depending on the variation of $S_{c,k}$ (following straight down debt payment, re-issuance, or conversion). Given the S_c-strike option payoff at maturity k being

$$Put(K = S_{c,k}, t = k, T = k) = (S_{c,k} - S_k)\mathbb{1}_{\{S_k \leq S_{c,k}\}}$$

, it then appears that

$$\Pi(t,T) = \sum_{k=1}^{N_n} P_{Nn}(T_k) D(t,T_k) \mathbf{E}_t \left[\mathbb{1}_{\{S_k > S_{c,k}\}} \right] + \sum_{k=1}^{N_n} \left(\frac{P_{Nn}(T_k)}{S_p} \right) \mathbf{E}_t \left[S_k \mathbb{1}_{\{S_k \leq S_{c,k}\}} \right] D(t,T_k) + \sum_{k=1}^{N_n} \Delta \mathbf{E}_t \left[(S_{c,k} - S_k) \mathbb{1}_{\{S_k \leq S_{c,k}\}} \right] D(t,T_k) \tag{C.5}$$

[2] In the case of a different currency denomination between the DCL and the underlying shares, the use of a strip of quanto puts will be required. This valuation and *modus operandi* is explained in Chapter 3.

For the number of put options to purchase, we use

$$\Delta = \frac{P_{Nn}(T_k)}{S_p}$$

that is actually a constant, from the definition of $P_{Nn}(T_k)$, $\forall k$. This number of options allows for a perfect hedge of the downside risk (conversion) with a 1:1 ratio.

When developing the above equation, it appears that

$$\iff \Pi(t,T) = \sum_{k=1}^{N_n} P_{Nn}(T_k) D(t,T_k) \mathbf{E}_t\left[\mathbb{1}_{\{S_k > S_{c,k}\}}\right] + \sum_{k=1}^{N_n} \Delta S_{c,k} \mathbf{E}_t\left[\mathbb{1}_{\{S_k \leq S_{c,k}\}}\right] D(t,T_k) \tag{C.6}$$

In Fig. C.1, we use any payment date $k-1$ as a reference for the spot time and the maturity set to the next payment date (k). This allows a focus on the flux qualitatively rather than the quantities really transferred.

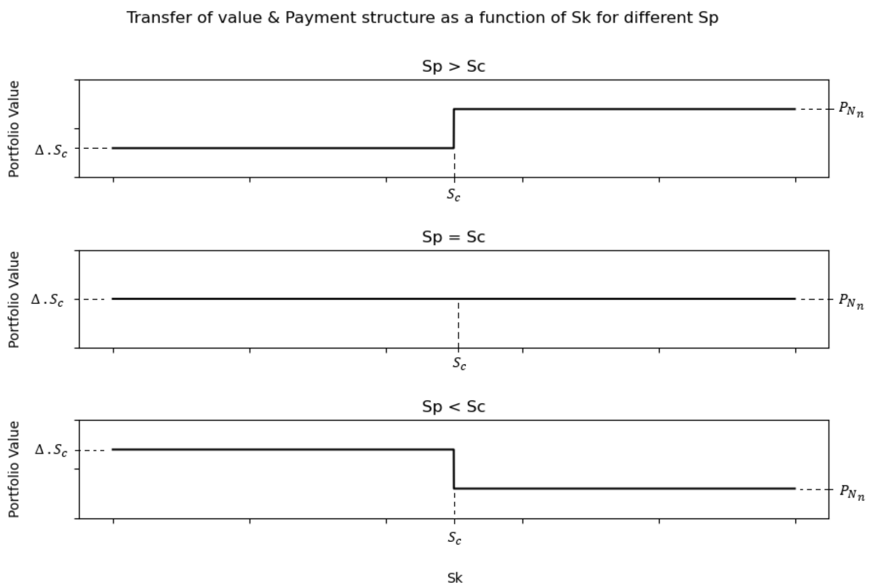


Figure C.1: Market value paid to the holder of a DCL + put option contract on any payment date k, either in cash (if $S_k > S_{c,k}$) or in new shares (if $S_k \leq S_{c,k}$). The plots are split to represent the importance of the conversion price S_p in the payment structure.

A few observations are drawn from Fig. C.1:

- When $S_p < S_c$, it appears that $\Delta \,.\, S_c = \frac{P_{Nn}}{S_p} \,.\, S_c > P_{Nn}$, creating a perverse incentive from the DCL + put holder to drive the underlying share price down to benefit from the conversion. As it is a zero-sum game, the cost is taken on by the initial shareholders through heavy dilution of their company ownership and control rights, which is equivalent to a recovery of $R > 1$ for the DCL investor.
- When S_p is not deterministic and set equal to S_c, the portfolio Π becomes fully deterministic and equivalent to holding a straight corporate bond.

This hedging experiment can also be applied to study the fair pricing of the DCL. Assuming $S_p \neq S_c$ and by shifting down or up the strike of the put, the portfolio structure should be priced at the same value of a bear or bull spread, respectively. We demonstrate this by using a strike for the put $K \neq S_c$, which adjusts the Π-equation to

$$\Pi(t,T) = \sum_{k=1}^{N_n} P_{Nn}(T_k) D(t,T_k) \mathbf{E}_t \left[\mathbb{1}_{\{S_k > S_{c,k}\}} \right] + \sum_{k=1}^{N_n} \left(\frac{P_{Nn}(T_k)}{S_p} \right) \mathbf{E}_t \left[S_k \mathbb{1}_{\{S_k \leq S_{c,k}\}} \right] D(t,T_k) + \sum_{k=1}^{N_n} \Delta (K - S_k) \mathbf{E}_t \left[\mathbb{1}_{\{S_k \leq K\}} \right] D(t,T_k) \tag{C.7}$$

This equation can be simplified into the two following cases:

- If $K < S_c$, then we have

$$\Pi(t,T) = \sum_{k=1}^{N_n} P_{Nn}(T_k) D(t,T_k) \mathbf{E}_t \left[\mathbb{1}_{\{S_k > S_{c,k}\}} \right] + \sum_{k=1}^{N_n} \Delta \,.\, K \,.\, \mathbf{E}_t \left[\mathbb{1}_{\{S_k \leq K\}} \right] D(t,T_k) + \sum_{k=1}^{N_n} \left(\frac{P_{Nn}(T_k)}{S_p} \right) \mathbf{E}_t \left[S_k \mathbb{1}_{\{K < S_k \leq S_{c,k}\}} \right] D(t,T_k) \tag{C.8}$$

  i.e., the structure of a bull spread.

- If $K > S_c$, then we have

$$\Pi(t,T) = \sum_{k=1}^{N_n} P_{Nn}(T_k) D(t,T_k) \mathbf{E}_t \left[\mathbb{1}_{\{S_k > S_{c,k}\}} \right] + \sum_{k=1}^{N_n} \Delta \,.\, K \,.\, \mathbf{E}_t \left[\mathbb{1}_{\{S_k \leq K\}} \right] D(t,T_k) + \sum_{k=1}^{N_n} \left(\frac{P_{Nn}(T_k)}{S_p} \right) \mathbf{E}_t \left[S_k \mathbb{1}_{\{S_k < K \leq S_{c,k}\}} \right] D(t,T_k) \tag{C.9}$$

  i.e., the structure of a bear spread.

Assuming $\Delta = \frac{P_{Nn}}{S_p}$, we plot Eq. C.8 in the upper panel and Eq. C.9 in the lower panel of Fig. C.2.

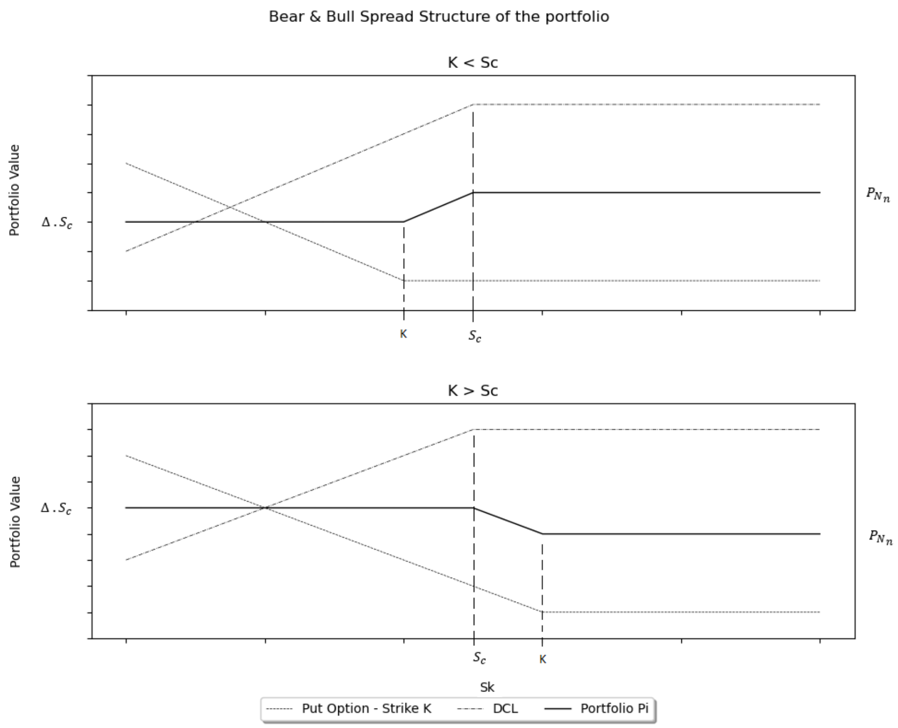


Figure C.2: Market value paid to the holder of a DCL + put option contract on any payment date k, either in cash (if $S_k > S_{c,k}$) or in new shares (if $S_k \leq S_{c,k}$). The plots are split to represent a put option with a strike k that is lower and higher than the conversion threshold S_c. We set $\Delta = \frac{P_{Nn}}{S_p}$.

If the DCL is not priced at the bear or bull spread strip value minus the strip of put options, then arbitrage could be exercised to levy on the price discrepancy. This will result in a risk-free profit for the investor (long or short) under the usual arbitrage hypotheses. Finally, including a static floor price FP to the conversion price is conceivable so as to protect the initial shareholders against heavy dilution.

Conclusion

Avoiding the existence of arbitrage by directly altering the stability of a company is primordial when originating a CoCo bond (with the purpose is to strengthen the capital structure of the firm). Aligned with this target, this appendix provided further recommendations regarding an appropriate design for DCL instruments.

By using a hedging-based approach, we also demonstrated the simplicity and transparency of the DCL mechanism, which can be replicated with well-known derivatives, such as bond + put or bear and bull spreads. The robustness of such a technique allows for the creation of a framework for the systematic identification of price discrepancies. Benefiting from these discrepancies can be achieved in a less opaque way than with regular CoCo bonds, contributing in the long run to achieving efficiency in a slowly maturing market.

Appendix D

Rating Caps as a function of a firm's rating and expected spread between the capital-ratio and trigger

The table presented here provides an equivalence between the credit profile of a firm issuing a hybrid instrument with a going-concern capital trigger and the associated emission depending on the spread between the trigger and the capital ratio projected 18-24 months forward.

	Governing Credit Profile									
Projected level of capital ratio on top of trigger	AA- or higher	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-
> 400 bps	BBB+	BBB+	BBB	BBB-	BB+	BBB	BB-	BB+	B-	CCC+
301 - 400 bps	BBB	BBB	BBB-	BB+	BB	BB-	B+	B-	CCC+	CCC+
201 - 300 bps	BBB-	BB+	BB+	BB	BB-	B+	B	CCC+	CCC+	CCC+
101 - 200 bps	BB	B	B	B	B-	B-	B-	CCC+	CCC+	CCC+
0 - 100 bps	CCC	CCC	CCC	CCC	CCC	CCC	CCC	CCC	CCC	CCC

Figure D.1: Rating caps for hybrid capital instruments with a going-concern capital trigger. These lead to a mandatory write-down or conversion to equity. Data reproduced from Table 3a in [95].

Appendix E

Detailed Calculation for the Estimators of the Vasicek Model

We first recall the log-likelihood function (Eq. 6.6) used in Ch. 6 as

$$
\begin{aligned}
\ell(L_i, a, b, \sigma_L) &= \log \prod_{i=0}^{N-1} f(L_{i+1} \mid L_i \,;\, a \,;\, b \,;\, \sigma_L) \\
&= \sum_{i=0}^{N-1} \log \left(f\left(L_{i+1} \mid L_i \,;\, a \,;\, b \,;\, \sigma_L\right)\right) \\
&= \sum_{i=0}^{N-1} \log \left(\frac{1}{\sigma_L \sqrt{2\pi \delta t}} \exp\left(- \frac{1}{2\sigma_L^2 \delta t} \left(\Delta L_i\right)^2 \right) \right) \\
&= \sum_{i=0}^{N-1} \log \left(\frac{1}{\sigma_L \sqrt{2\pi \delta t}} \right) - \frac{\Delta L_i^2}{2\sigma_L^2 \delta t} \\
&= \left[\frac{1}{\log(\sigma_L \sqrt{2\pi \delta t})} \right]^N - \sum_{i=0}^{N-1} \frac{\Delta L_i^2}{2\sigma_L^2 \delta t} \\
&= -\frac{N}{2} \log\left(\sigma_L^2 2\pi \delta t\right) - \sum_{i=0}^{N-1} \frac{\Delta L_i^2}{2\sigma_L^2 \delta t}
\end{aligned}
\tag{E.1}
$$

where $\Delta L_i = L_{i+1} - L_i - a(b - L_i)\delta t$ denotes the adjusted change in leverage at time i, which accounts for the mean-reversion process described by the Vasicek model.

The log-likelihood function $\ell(L_i, a, b, \sigma_L)$ represents the likelihood of observing a specific leverage path $L_t = \{L_t / t \in [0; T^*]\}$ given the parameters a, b, and

σ_L. By denoting b as the average leverage observed from the dataset $L_i = \{L_t/t \in [0; T^*]\}$, we demonstrate in this appendix the Eq. 6.7, i.e.,

$$\begin{cases} a &= \frac{\sum_{i=0}^{N-1}(L_{i+1}-L_i)(b-L_i)}{\delta t \sum_{i=0}^{N-1}(b-L_i)^2} \\ \hat{b} &= \frac{1}{N}\sum_{i=0}^{N-1}\frac{L_{i+1}-L_i(1-a\delta t)}{a\delta t} \\ \sigma_L &= \sqrt{\frac{1}{N}\sum_{i=0}^{N-1}\frac{(L_{i+1}-L_i-a(b-L_i)\delta t)^2}{\delta t}} \end{cases} \tag{E.2}$$

by setting the partial derivatives of the log-likelihood function with respect to the parameters equal to 0.

Regarding the estimator a

$$\begin{aligned} \frac{\partial \ell(L_i, a, b, \sigma_L)}{\partial a} &= -\sum_{i=0}^{N-1}\frac{[L_{i+1}-L_i-a(b-L_i)\delta t](b-L_i)\delta t}{\sigma_L^2\delta t} \\ &= -\sum_{i=0}^{N-1}\frac{[L_{i+1}-L_i-a(b-L_i)\delta t](b-L_i)}{\sigma_L^2} \\ &= -\sum_{i=0}^{N-1}\frac{[(L_{i+1}-L_i)(b-L_i)-a(b-L_i)^2\delta t)]}{\sigma_L^2} = 0 \end{aligned} \tag{E.3}$$

Hence,

$$\begin{aligned} &\sum_{i=0}^{N-1}(L_{i+1}-L_i)(b-L_i) = \sum_{i=0}^{N-1}a(b-L_i)^2\delta t \\ \iff & a = \frac{\sum_{i=0}^{N-1}(L_{i+1}-L_i)(b-L_i)}{\delta t\sum_{i=0}^{N-1}(b-L_i)^2} \end{aligned} \tag{E.4}$$

where a describes how quickly the leverage reverts to the long-term mean. In this equation, the numerator is the sum of the products of the change in leverage and the deviation from its long-term mean. The denominator is a scaling factor representing the sum of the squared deviations multiplied by the time difference. Thus, this estimator captures the reversion well by accounting for the relationship between leverage changes and their deviations from the long-term mean.

Regarding the estimator $\hat{b}$

$$\begin{aligned}\frac{\partial \ell(L_i, a, b, \sigma_L)}{\partial b} &= -\sum_{i=0}^{N-1} \frac{2[L_{i+1} - L_i - a(b - L_i)\delta t].a.b.\delta t}{2\sigma_L^2 \delta t} \\ &= -\sum_{i=0}^{N-1} \frac{[L_{i+1} - L_i(1 - a.\delta t) - a.b.\delta t].a.b}{\sigma_L^2} = 0\end{aligned} \tag{E.5}$$

Hence,

$$\begin{aligned}&\sum_{i=0}^{N-1} [L_{i+1} - L_i(1 - a.\delta t) - a.b.\delta t] = 0 \\ \iff &\sum_{i=0}^{N-1} a.b.\delta t = \sum_{i=0}^{N-1} [L_{i+1} - L_i(1 - a.\delta t)] \\ \iff &\hat{b} = \frac{1}{N} \sum_{i=0}^{N-1} \frac{L_{i+1} - L_i(1 - a\delta t)}{a\delta t}\end{aligned} \tag{E.6}$$

where $\hat{b}$ describes an adjusted average of the changes in leverage divided by the product of the mean reversion speed and the time difference. The adjustment in the numerator also includes the term $(1 - a\delta t)$, which encompasses the effects of the mean reversion speed on the change in leverage L_i. This is consistent with the concept that the long-term mean leverage b is an average value towards which the leverage tends to revert.

Regarding the estimator σ_L

$$\begin{aligned}\frac{\partial \ell(L_i, a, b, \sigma_L)}{\partial \sigma_L} &= -\frac{4N\pi\sigma_L \delta t}{4\pi\sigma_L^2 \delta t} - \sum_{i=0}^{N-1} \frac{-2[L_{i+1} - L_i - a(b - L_i)\delta t]^2}{2\sigma_L^3 \delta t} \\ &= -\frac{N}{\sigma_L} + \sum_{i=0}^{N-1} \frac{[L_{i+1} - L_i - a(b - L_i)\delta t]^2}{\sigma_L^3 \delta t} = 0\end{aligned} \tag{E.7}$$

Hence,

$$\begin{aligned}&\frac{N}{\sigma_L} = \sum_{i=0}^{N-1} \frac{[L_{i+1} - L_i - a(b - L_i)\delta t]^2}{\sigma_L^3 \delta t} \\ \iff &\sigma_L = \sqrt{\frac{1}{N} \sum_{i=0}^{N-1} \frac{(L_{i+1} - L_i - a(b - L_i)\delta t)^2}{\delta t}}\end{aligned} \tag{E.8}$$

where σ_L describes the variability in the leverage changes around its mean-reverting values. This also includes the effects of the mean reversion speed a and the long-term mean leverage b.

Appendix F

Mathematical References in the Correction to Sundaresan and Wang (Pennacchi & Tchistyi)

We provide here a few propositions referred to in the PT paper [20]. The pricing (Proposition 2) relies on a structural approach for CoCos, assuming conversion occurs the first time the threshold is violated (Proposition 1).

F.1 Proposition 1

If there is an equilibrium stock price, then conversion happens when A_t falls to A_{uc} for the first time. In other words,

$$\tau = inf\{t \in [0; T] : A_t \leq A_{uc}\}$$

F.2 Proposition 2

If there is an equilibrium stock price, then the pre-conversion date t values of the stock and CC equals

$$S(A_t, A_{uc}, q) = \frac{1}{n}\left(A_t - \overline{B} - C(A_t, A_{uc}, q)\right)$$

$$C(A_t, A_{uc}, q) = \frac{c\overline{C}}{r} + e^{-rq}\left(\overline{C} - \frac{c\overline{C}}{r}\right)(1 - F(q, A_t, A_{uc})) + \left(\frac{mK}{n} - \frac{c\overline{C}}{r}\right)G(q, A_t, A_{uc})$$

where $q \equiv T - t$ is the CC's time until maturity, such that

$$F(q, A_t, A_{uc} = \Phi(x_{1t}(q)) + \left(\frac{A_t}{A_{uc}}\right)^{-2\alpha} \Phi(x_{2t}(q))$$

$$G(q, A_t, A_{uc} = \left(\frac{A_t}{A_{uc}}\right)^{-\alpha+z} \Phi(y_{1t}(q)) + \left(\frac{A_t}{A_{uc}}\right)^{-\alpha-z} \Phi(y_{2t}(q))$$

and

$$y_{1t}(q) = \frac{-h_t - z\sigma^2 q}{\sigma\sqrt{q}}\ , y_{2t}(q) = \frac{-h_t + z\sigma^2 q}{\sigma\sqrt{q}}$$

$$x_{1t}(q) = \frac{-h_t - \alpha\sigma^2 q}{\sigma\sqrt{q}}\ , x_{2t}(q) = \frac{-h_t + \alpha\sigma^2 q}{\sigma\sqrt{q}}$$

Finally,

$$h_t = ln\left(\frac{A_t}{A_{uc}}\right)\ , \alpha = \frac{\mu - \frac{1}{2}\sigma^2}{\sigma^2}\ , z = \frac{\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2}}{\sigma^2}$$

where $\Phi(.)$ is the cumulative standard normal distribution.

Appendix G

First-passage Equation for a gBm

Let $m_t = \min_{[0;t]} S_u$ be the minimum value achieved by the process S_t before maturity T. Our stock price is defined by the process:

$$S_t = s + \alpha t + \sigma W_t$$

We assume that the CoCo conversion is triggered when the stock value drops below an implied level S_c. Thus, there is equality between the relations

$$P(m_t \leq S_c) = P(\min_{[0;t]} S_u \leq S_c) = P(t \geq \tau)$$

As described in Section 4.2.2, paragraph 15 of [51], we have

$$P(m_t \leq S_c) = \phi\left(\frac{S_c - s - \alpha t}{\sigma\sqrt{T}}\right) + \exp\left(\frac{2\alpha\,(S_c - s)}{\sigma^2}\right)\phi\left(\frac{S_c - s + \alpha t}{\sigma\sqrt{T}}\right)$$

The function $\phi(.)$ should be interpreted as the cumulative distribution function.

Applying Ito's lemma to the log of the process $dS_t = \mu S_t dt + \sigma S_t dW_t$ reveals an adjusted drift, such as in

$$d\log(S_t) = \left(\mu + \frac{\sigma^2}{2}\right)dt + \sigma dW_t$$

By integrating, we obtain

$$\log(S_t) = \log(S_0) + \left(\mu + \frac{\sigma^2}{2}\right)t + \sigma W_t$$

Allowing to identify $s = \log(S_0)$ and $\alpha = \left(\mu + \frac{\sigma^2}{2}\right)$ and noticing that

$$\left\{\min_{[0;T]} S_u \leq S_c\right\} = \left\{\min_{[0;T]} \log(S_u) \leq \log(S_c)\right\}$$

we conclude with the cumulative distribution function (CDF) for the first-hitting time in the case of a gBm, where

$$P(m_t \leq S_c) = \phi\left(\frac{\log\left(\frac{S_c}{S_0}\right) - \alpha T}{\sigma\sqrt{T}}\right) + \left(\frac{S_c}{S_0}\right)^{\frac{2\alpha}{\sigma^2}} \phi\left(\frac{\log\left(\frac{S_c}{S_0}\right) + \alpha T}{\sigma\sqrt{T}}\right) \tag{G.1}$$

This equation provides the probability that the stock price S_t can hit the threshold S_c at any moment during the period $[0;T]$ based on the information vector $\boldsymbol{I} = (S_0, S_c, \mu, \sigma, T)$. In our case, the conversion probability is calculated from term to term, so we adjust Eq. G.1 by setting $t = T_{k-1}$ and $T = T_k$ to obtain

$$P\left(\min_{k-1 \leq t \leq k} S_t \leq S_{c,k}\right) = P(S_\tau < S_{c,k} | T_{k-1} < \tau < T_k)$$
$$= \Phi\left(\frac{\log\left(\frac{S_{c,k}}{S_{k-1}}\right) - \left(\mu - \delta - \frac{\sigma^2}{2}\right)(T_k - T_{k-1})}{\sigma\sqrt{T_k - T_{k-1}}}\right) + \exp\left(2\log\left(\frac{S_{k-1}}{S_{c,k}}\right)\frac{\mu - \delta - 0.5\sigma^2}{\sigma^2}\right)$$
$$* \, \Phi\left(\frac{\log\left(\frac{S_{c,k}}{S_{k-1}}\right) + \left(\mu - \delta - \frac{\sigma^2}{2}\right)(T_k - T_{k-1})}{\sigma\sqrt{T_k - T_{k-1}}}\right) \tag{G.2}$$